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Statistical Simulation of Boxing Day Tsunami of The Indian Ocean and a Predictive Equation for Beach Run up Heights Based on Work-Energy Theorem

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The tsunami similar to the one that has occurred in December 26, 2004 (Boxing Day Tsunami) in the Indian Ocean is simulated using the expression derived from Modified Weibull Distribution (for maximum wave height simulation) for extreme wave height predictions. The tuning coefficient plays a significant role in estimating the tsunami heights at various stages. It follows well defined mathematical laws at different stages. It is time dependent in the first three stages and depth dependent in the last two stages. The beach run-up heights estimated by the expression derived from the work-energy relation are comparable with observed values with reasonable accuracy.

Keywords Tsunami, modified weibull, extreme wave height, turning coefficient, beach run-up, height prediction

The great earthquake on December 26, 2004, launched tsunami waves across the Indian Ocean. Tsunami waves are routinely recorded by tide gauges, and, at least in principle, by DART or a similar type measuring instruments wherever they are located. Of the total number of tsunamis generated so far in historical time, only a few of them were destructive (25%). Unnecessary tsunami alert warnings (false alarms) and temporary evacuation of coastal inhabitants will create great panic both in the government and with the public. Hence, an accurate estimation of the tsunami behavior in coastal waters is highly recommended.

The Indian Ocean is seismically active, but with less frequency than the Pacific Ocean. Accurate prediction of tsunamis in coastal waters is highly essential for India as well as the rim nations of the Indian Ocean. The simulation of the Indian Ocean Tsunami similar to the one that occurred on December 26, 2004, is considered in this work (for details on this tsunami see US Geological Survey’s website, http://www.seed.slb.com/en/scictr/watch/living_planet/tsunami.htm).

The distribution of the tsunami wave heights had 5 stages (Figure 1). In the initial stage (time of generation, Figure 2), the wave heights decreased very rapidly (39.03 m to 6.03 m) as time elapsed and depth and wave length remained constant at 4000 m (average depth) and 500 km, respectively. In the second stage, the wave heights again diminished...
quickly (2.97 m to 1.65 m) with little change in the initial depth and wave length (Figure 3).

Tsunami heights started declining further but slowly (1.57 m to 0.81 m) in the third stage shown in Figure 4. Figure 5 depicts increase of heights gradually from 0.82 m to 2.81 m along with reduction in depth and wave length in the fourth stage.

In the final stage, tsunami heights started increasing drastically (3.23 m to 19.79 m) as it propagates toward the shore with great decrease in depth and wave length as shown in Figure 6. The run-up heights for different beach angles by numerical experiments and for a real shore profile in Japan (Marchuk et al. 2001) are utilized to check the reliability of the expression derived for tsunami beach run-up height calculations.

Materials and Methods

We are able to simulate the Indian Ocean Tsunami heights using the expression

\[
h_L = a \cdot \left[ \alpha^b - \ln \left\{ 1 - \left( 1 - \frac{1}{t} \right)^{\alpha} \right\} \right]^{\frac{1}{b}} - \alpha
\]  

(1)

\[\alpha = 0.6051e^{0.1873t}\]  

\[R^2 = 0.9444\]  

Exponential

Figure 2. Distribution and functional relationship of tuning coefficient (\(\alpha\)) with respect to time elapsed at each stage.
Figure 3. Distribution and functional relationship of tuning coefficient ($\alpha$) with respect to time elapsed at each stage.

derived from Modified Weibull for maximum wave height distribution for predicting extreme wave heights (Mulagaleti et al. 2004). “a” and “b” are the scale and shape parameters of the model and are calculated by the method of maximum likelihood (MLE). N-total number of observations; t-time step in minutes and $\alpha$-real positive number are the

Figure 4. Distribution and functional relationship of tuning coefficient ($\alpha$) with respect to time elapsed at each stage.
Figure 5. Distribution and functional relationship of tuning coefficient ($\alpha$) with respect to depth at each stage.

Figure 6. Distribution and functional relationship of tuning coefficient ($\alpha$) with respect to depth at each stage.
three calibration coefficients and out of these $\alpha$ (the tuning coefficient) plays a significant role in simulating the Indian Ocean tsunami heights.

The work-energy relation is applied to develop the fundamental mechanism of the beach run-up heights by the tsunami. A brief retrospect in this regard is discussed here.

From physics, the work-energy theorem is stated by the following equation

$$TME_i + W_{ext} = TME_f$$  \hspace{1cm} (2)

The equation states that the initial amount of total mechanical energy ($TME_i$) plus the work done by the external forces ($W_{ext}$) is equal to the final amount of total mechanical energy ($TME_f$). The mechanical energy can be either potential energy (due to tsunami height in this case) or kinetic energy.

The above equation is often said to be an expression of the work energy theorem. This has formed the basis of the conceptual aspect of our study.

A tsunami wave approaching the coast contains both kinetic and potential energy. The external work done (by wind) is considered to be negligible. As it advances shallower waters its speed decreases ($\text{velocity} = \sqrt{\text{g} \cdot \text{d}}$, $\text{g}$-acceleration due to gravity, $\text{d}$-water depth) and height increases due to shoaling. Tsunami waves usually won’t break at zero meter depth. The work is being done by the wave as it climbs the beach (due to terminal speed) and the energy of the wave is gradually dissipated. Ultimately, the volume of water comes to a rest position. The work done serves to reduce the flow depth and thereby remove the energy from the wave. This relation is given in physical terms as:

$$W_{ext} + PE_i = 0$$  \hspace{1cm} (3)

Since the final mechanical energy is equal to zero, RHS of eq. (3) is zero.

Eq. \hspace{0.1cm} (4) \Rightarrow F \cdot d \cdot \cos \theta = -M \cdot g \cdot H_s$$  \hspace{1cm} (4)

where, $d$-beach slope run-up from 0m depth, ie. point O to point S, $\theta$-Beach angle, $H_s$-tsunami height at 0m depth, $M$-mass of the volume of water at 0m depth (Figure 7).

Let $t$ be the time taken by the shallow water tsunami wave to travel from a small depth 1m to 0m depth. Then

$$F = -M \cdot a = -M \cdot \sqrt{\frac{g}{d_1}} / t$$

$$H_r = \sqrt{\frac{g}{d_1}} \cdot H_s \cdot t \cdot \tan \theta$$

$\theta < 90^\circ$  \hspace{1cm} (5)

where $a$ is the deceleration.

![Figure 7. Schematic representation of the beach angle, beach run-up, direction of force, and run-up height.](image-url)
The time tuning coefficient \( t \) depends upon the offshore slope \( \theta_1 \) (<90) (Figures 8 and 9). This expression for tsunami run-up heights developed from work-energy theorem is dimensionally valid.

**Results and Discussion**

From the distribution of the calibration coefficient at various stages given in Figures 2–6, it is interesting to observe that they are not randomly distributed but follow some definite laws. During initial and last stages, it is showing an exponential behavior. In the second and third stages, it follows a power series while it is logarithmic in the fourth stage.

There are two ways of calculation of tuning coefficient \( \alpha \) and that one way of doing is treating the total observations \( N \) and the time step \( t \) separately for each stage. Since the distribution of tsunami heights are different for each stage, the scale and shape parameters of the model is also calculated for each stage as shown in Table 1.

Another way of doing is that by considering total observations \( N = 20,400 \) and time step \( t \) (total \( t = 5 \) hrs and 40 mins) from the source of disturbance and not break then down into various stages. The distribution of tuning coefficient \( \alpha \) based on this approach also shows exponential behavior during initial and final stages and whilst, it is logarithmic.

**Figure 8.** Schematic representation showing small offshore slope, tsunami wave height at zero meter depth, horizontal distance from 1 m depth to zero meter depth, and beach angle.

**Figure 9.** Schematic representation showing large offshore slope and horizontal distance between 1 m depth to zero depth.
Table 1
Calibration coefficient (N) and parameters of Weibull distribution

<table>
<thead>
<tr>
<th>Stage</th>
<th>N</th>
<th>Scale parameter a</th>
<th>Shape parameter b</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>840</td>
<td>24.14</td>
<td>2.31</td>
</tr>
<tr>
<td>II</td>
<td>2160</td>
<td>2.34</td>
<td>5.66</td>
</tr>
<tr>
<td>III</td>
<td>7500</td>
<td>1.20</td>
<td>5.53</td>
</tr>
<tr>
<td>IV</td>
<td>8400</td>
<td>1.70</td>
<td>1.63</td>
</tr>
<tr>
<td>V</td>
<td>540</td>
<td>12.25</td>
<td>2.35</td>
</tr>
</tbody>
</table>

Calibration Coefficient (N-total number of observations) and model parameters (a-scale, b-shape) of modified Weibull distribution estimated by the method of maximum likelihood at each stage.

for other stages (Table 2). The goodness of fit by the functional curves is given by the coefficients of determinations ($R^2$). The agreement of the fit is more than 97% in almost all cases.

The expressions derived here for beach run-up heights by tsunamis are first used to simulate the run-up heights estimated by numerical experiments by Marchuk et al. 2001, for different slope angles (the offshore and beach slopes are considered to be equal) (Table 3). The wave height was taken to be 5.9 m near the coast.

The time tuning coefficients (t in secs) computed by us (Table 3) is the time required by the tsunami wave (5.9 m) to travel from 1 m depth to 0 m depth.

The time tuning coefficients (t) decrease as the off shore slope increase. This seems to be reasonable since the horizontal distance from d1 m depth (small depth) to 0 m depth is more in the case of small offshore slopes compared to large slopes (Figures 8 and 9). That is the terminal speed will be comparatively small and hence time tuning coefficients large (Table 3) for small offshore slopes.

The time tuning coefficient and the terminal speed is independent of the slope on land and depends only on offshore slope.

Table 2
Functional relationship between tuning coefficient ($\alpha$), time step (t) and depth (d)

<table>
<thead>
<tr>
<th>Stage</th>
<th>$\alpha$</th>
<th>Coefficient of Determination ($R^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$1.0661e^{0.1567t}$</td>
<td>0.9852</td>
</tr>
<tr>
<td>II</td>
<td>0.5231 ln(t)-1.1318</td>
<td>0.9947</td>
</tr>
<tr>
<td>III</td>
<td>0.5341 ln(t)-1.8123</td>
<td>0.9957</td>
</tr>
<tr>
<td>IV</td>
<td>0.5921 ln(d)-1.24</td>
<td>0.9977</td>
</tr>
<tr>
<td>V</td>
<td>$1.897e^{0.1024d}$</td>
<td>0.9832</td>
</tr>
</tbody>
</table>

Functional relationship between tuning coefficient ($\alpha$), time step (t-from source of disturbance in mins) and depth (d-in meters) and coefficients of determinations at various stages of tsunami propagation.
Beach run-up heights and time tuning coefficients for different beach and offshore angles $\theta^\circ$

<table>
<thead>
<tr>
<th>$\tan(\theta)$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beach and Offshore angle $\theta^\circ$</td>
<td>5.71</td>
<td>11.31</td>
<td>16.70</td>
<td>21.80</td>
<td>26.57</td>
<td>30.96</td>
<td>34.99</td>
<td>38.66</td>
<td>41.99</td>
<td>45.00</td>
</tr>
<tr>
<td>Run-up height by numerical experiments (m)</td>
<td>14.01</td>
<td>14.04</td>
<td>13.92</td>
<td>13.49</td>
<td>13.18</td>
<td>12.88</td>
<td>12.56</td>
<td>12.57</td>
<td>12.59</td>
<td>12.60</td>
</tr>
<tr>
<td>Time tuning coef (secs) (t)</td>
<td>7.6</td>
<td>3.80</td>
<td>2.50</td>
<td>1.82</td>
<td>1.43</td>
<td>1.16</td>
<td>0.97</td>
<td>0.85</td>
<td>0.76</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Beach run-up heights at different beach angles and time tuning coefficient (t) to travel from 1 m depth to 0 m depth (both offshore and beach angles are equal).

The time tuning coefficient (t) and the offshore slope ($\tan(\theta_1)$) exhibits a significant power series relationship (Figure 10) as

$$t = 0.6791 \cdot [\tan(\theta_1)]^{-1.0606}$$

Numerical modeling of tsunami wave run-up heights was carried out for real shore profile near Minehama village in Akita prefecture, Japan (Marchuk et al. 2001). Run-up height by the tsunami of May 26, 1983, was 14.08 m. To reach such a height, the tsunami wave height should be 8 m near the coastline.

This complicated beach is having 11 different slopes on land. Hence the one-third significant beach angle is computed to be 51°. From the expression derived from the work-energy theorem (5), the tsunami wave height at 0 m depth will be 8 m if the offshore slope is 55° and time tuning coefficient is 0.46 secs (6). Usually a 10 m tsunami wave near shore could have run-up height inland which may be much higher than 10 m (and even as much as double) because of the topography and the momentum of greater terminal speed.

The expression is derived for a plane beach slope with no obstacles. Hence the run-up height estimated by this expression gives the maximum vertical height limit from the mean sea level to which a particular tsunami wave height will climb.

**Figure 10.** Distribution and functional relationship of time tuning coefficient (t-time required to travel from 1 m depth to 0 m depth) with respect to offshore slope.
In short, the height to which a tsunami will reach on land will depend on the tsunami height at 0 m depth, on the offshore slope over which the tsunami travels (as that determines its terminal velocity), the slope on land and the land topography (trees, buildings, obstacles which may create friction).

Conclusion

Time elapsed since the generation of the tsunami is the main controlling factor for height depletion in stages one to three and water depth for height increase in stages four and five (USGS website, http://www.seed.slb.com/en/scictr/watch/living_planet/tsunami.htm). The tuning coefficient, $\alpha$ is also functionally related to time elapsed (t) in the first three stages and water depth (d) in the last two stages. Hence, estimation of $\alpha$ at various stages is an important factor in simulating the tsunami heights. Since it follows definite mathematical laws at different stages of propagation, it reveals the characteristics of the distribution of tsunami heights. The expression derived from the work-energy theorem for tsunami beach run-up heights seems to predict with reliable accuracy. The decrease of the time tuning coefficient $t$ for the tsunami wave to travel from 1 m depth to 0 m depth, with increase in offshore slopes is in agreement with reality as the horizontal distance between these depths decreases. For different estimated tsunami heights at 0 m depth, beach angle and time tuning coefficients for known offshore slopes, beach run-up height tables could be constructed for various coastal locations which are vulnerable to tsunami wave attack.

References