Preliminary Constituent Proportioning for Central Plant Hotmix Asphalt Recycling

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Abstract: Different methods for preliminary constituent proportioning of the recycled mix are proposed in various guidelines/studies for central plant hotmix asphalt recycling. These methods are sequential in nature, and the process requires at least one of the parameters (for example, quantity of reclaimed material or viscosity of virgin asphalt binder) as fixed. An attempt has been made in the present work to develop a generalized formulation for the mix proportioning which does not require any such prefixing; however, the formulation is flexible enough to take care of such a constraint, if imposed. The proposed formulation serves certain overall objectives, like minimization of material cost, maximization of usage of reclaimed material etc., with the help of a linear programming tool. The suggested approach handles the proportioning problem as a whole, and is therefore not dependent on the sequence of mix proportion calculations.

DOI: 10.1061/(ASCE)0899-1561(2007)19:9(740)

CE Database subject headings: Asphalt mixes; Asphalt pavements; Recycling; Viscosity.

Introduction

In central plant recycling of hotmix asphalt, calculated quantity of reclaimed asphalt pavement (RAP), virgin asphalt binder, and fresh aggregates are added and mixed to produce the recycled mix. Sometimes, commercially available rejuvenators are also added to the mix. Various guidelines/studies provide recommendations for preliminary constituent proportioning of the mix, which are subsequently finalized through some systematic mix design testing process. The recommended formulas for preliminary quantity estimation of the constituents rather require assumption of quantity (or property) of certain constituent(s) as prefixed. For example, Asphalt Institute (1981, 1997) guidelines assume the percentage of RAP as prefixed, and the quantities of the other constituents are obtained sequentially.

The present paper tries to tackle the problem of constituent proportioning with the help of a generalized formulation that does not require any prefixing of the quantity (or property) of any of the constituents. A linear programming approach has been proposed, where a mix designer can achieve objectives like minimization of the total material cost, or maximization of the total RAP quantity to be used in the recycled mix. This approach looks into the constituent proportioning problem as a whole and is, therefore, not dependent on the sequence of mix proportion calculations.

The paper contains six sections of which this is the first. The next section reviews various recycled mix design procedures and states the scope of the present study. The third section presents the proposed formulation. A numerical example has been solved in the fourth section. The last two sections contain the discussion and conclusions of the study.

Background

In a recycled mix design method proposed by Kari et al. (1979), aggregates and aged asphalt binder in RAP are tested for their gradation and penetration, respectively. Then, the virgin asphalt binder is selected. Subsequently, the proportions between the virgin and aged asphalt binders are fixed by using a suggested binder blending chart. The gradation of new aggregates is found by considering the target gradation, and some possible blends between old aggregates (in RAP) and new aggregates. Further, by the centrifuge kerosene equivalent (CKE) test, the approximate amount of asphalt binder demand of the recycled mix (based on the concept of minimum binder film thickness required to cover the total surface area of the aggregates) is estimated. The percentage of virgin asphalt binder is calculated by subtracting the RAP asphalt binder content from the total asphalt binder demand. Thus, the preliminary mix proportions are obtained. In this method, the virgin binder grade is fixed at the initial stage and the constituent proportions among RAP, fresh aggregates, and virgin binder are obtained subsequently. A similar approach, based on binder viscosity, has been suggested in Austroads guidelines (Austroads 2000).

In some other mix design studies/guidelines (Epps et al. 1980a,b; Asphalt Institute 1981, 1997; Kandhal and Mallick 1997), the amount of RAP to be used is fixed and accordingly the gradation of the new aggregates to be added to conform to the recommended gradation is estimated. In the next step, the approximate demand of asphalt binder (to cover the total surface area of the aggregates) of the recycled mix is estimated. From aged asphalt binder viscosity and the proportion between aged and virgin asphalt binder, the viscosity of the virgin asphalt binder...
is decided. Thus, the preliminary constituent proportions are obtained, which is finalized after conducting tests by either the Marshall or Hveem method. In this method the quantity of RAP is prefixed. Also, the quantity of virgin asphalt binder to be added is decided first and its viscosity is finalized subsequently.

Various guidelines/studies propose the use of binder penetration (Kari et al. 1979; Indian Roads Congress 1987; Ikeda and Kimura 1997) or, viscosity (Epps et al. 1980a,b; Asphalt Institute 1981, 1997; Kandhal and Mallick 1997; Austroads 2000) or, complex shear modulus (Kennedy et al. 1993; McDaniel et al. 2000; McDaniel and Anderson 2001) as the criterion to estimate asphalt binder proportion. Roberts et al. (1996) suggested that there should be an upper limit of the RAP percentage that can be used in a recycled mix.

By fixing initially the RAP percentage to be used, as is done in some methods (Epps et al. 1980a,b; Asphalt Institute 1981, 1997, Kandhal and Mallick 1997), one loses the opportunity to decide to what extent the RAP could have been utilized. Hence, utilization of RAP to the possible extent may not have been achieved in these methods. Also, some of the recycled mix design procedures (Asphalt Institute 1981, 1997; Kennedy et al. 1993; Kandhal and Mallick 1997) finally recommend the viscosity (or shear modulus) of the virgin asphalt binder to be used for recycling purposes. This may give rise to a disadvantageous situation, if the particular grade (viscosity) of asphalt binder is not available in the commercial market—especially with reference to those countries where only limited varieties of asphalt grades are available.

This has prompted the present investigators to develop an approach which tries to find an optimal solution of the recycled mix design problem, and does not require prefixing of any of the constituent proportion (or property) beforehand, yet flexible enough, that any such restrictions can be imposed later, if required.

**Proposed Formulation**

The purpose of the asphalt mix recycling is to modify the properties of the RAP, such that it tends to perform as good as a fresh mix. As evident from the above discussion, the requirements of the recycled mix can be summarized as follows:

1. The quantity of old aggregates and new aggregates are to be adjusted in such a way that the resultant gradation of aggregates conforms to the specified gradation;
2. The quantity of the aged asphalt binder, virgin asphalt binder, and the rejuvenator, if any, are to be adjusted in such a way that the resultant viscosity becomes equal to the desirable viscosity at operating temperature. Some other parameter (for example complex shear modulus, etc.) may be chosen instead of viscosity;
3. The total quantity of asphalt binder should be adjusted in such a way that it satisfies the desired asphalt binder quantity of the target mix; and
4. The other mix design parameters, for example, volumetric, strength, and performance related parameters should also be satisfied.

A three-component mixture constituted with RAP, new aggregates, and virgin asphalt binder is assumed in the present study. Viscosity is chosen as the basis of binder proportioning and rejuvenator has not been considered. The formulation for mix proportioning has been developed in two stages: first by the direct method and the same has been extended by using linear programming. Developments of these are presented in the following sections.

**Constituent Proportioning by Direct Method**

Consider a binary mix of aged and virgin asphalt binder. If the proportions (as fractions) of the aged and virgin asphalt binders are denoted as $p_{\text{ab}}$ and $p_{\text{vb}}$, respectively, then one can write $p_{\text{ab}} + p_{\text{vb}} = 1$. Using the viscosity mixing rule (at any given temperature) as, \[ \ln(\eta_v) = p_{\text{ab}} \ln(\eta_{\text{ab}}) + p_{\text{vb}} \ln(\eta_{\text{vb}}) \] (Asphalt Institute 1981, 1997), where the target viscosity of asphalt binder in recycled mix is $\eta_v$, the viscosity of aged asphalt binder is $\eta_{\text{ab}}$, and the viscosity of virgin asphalt binder is $\eta_{\text{vb}}$, one obtains

$$p_{\text{ab}} = \frac{\ln(\eta_v) - \ln(\eta_{\text{vb}})}{\ln(\eta_{\text{ab}}) - \ln(\eta_{\text{vb}})} \quad (1)$$

and

$$p_{\text{vb}} = \frac{\ln(\eta_v) - \ln(\eta_{\text{vb}})}{\ln(\eta_{\text{vb}}) - \ln(\eta_{\text{ab}})} \quad (2)$$

It may be noted that all these viscosities are measured with reference to a specified operating temperature. If the asphalt binder demand [or equivalently, optimum binder content (OBC)] in the recycled mix is assumed as $P_b^R$, the percentage of virgin asphalt binder ($p_{\text{vb}}^R$) in the recycled mix can be written as

$$p_{\text{vb}}^R = p_{\text{vb}} \times P_b^R \quad (3)$$

Similarly, the percentage of aged asphalt binder ($p_{\text{ab}}^R$) in the recycled mix is

$$p_{\text{ab}}^R = p_{\text{ab}} \times P_{\text{ab}}^R \quad (4)$$

If $P_{\text{RAP}}^R$ is denoted as the percentage of RAP used in the recycled mix, the aged asphalt binder present in the recycled mix is given by

$$P_{\text{ab}}^R = \left( \frac{P_{\text{RAP}}^R}{P_b^R} \right) \times P_{\text{ab}}^R \quad (5)$$

where $P_b^R =$ percentage of asphalt binder (aged) present in the RAP. Equating Eq. (5) with Eq. (4), one gets or

$$P_{\text{RAP}}^R = \left( \frac{p_{\text{ab}} \times P_b^R \times 100}{p_{\text{vb}}^R} \right) \quad (6)$$

Therefore, the percentage of new aggregates in the recycled mix ($p_{\text{na}}^R$) is

$$p_{\text{na}}^R = 100 - (P_{\text{RAP}}^R + P_{\text{ab}}^R) \quad (7)$$

Thus, the percentages of virgin asphalt binder, RAP and new aggregates can be obtained from Eqs. (3), (6), and (7), respectively. It should be noted that all these constituent percentages are expressed with reference to the total weight of mix. It should also be noted that $p_{\text{ab}}$ and $p_{\text{vb}}$ have been expressed as fractions, whereas the rest of the quantities have been expressed as percentages.

**Comparison with Asphalt Institute Method**

The Asphalt Institute (1997) provides certain formulas for calculating the percentages of different constituents. The notations used by the Asphalt Institute and those in the present work are different; hence they are compared in Table 1. It should be noted that the notation used in the present work is of a generalized form, where $P_i^y$ means the percentage of constituent “$x$” in mix “$y$.” The formulas for constituent proportioning by the Asphalt Institute and the proposed work are compared in Table 2. Although the
expressions look different, it is verified, by using suitable substitution, that the proposed formulas can be derived from the Asphalt Institute formulas and vice versa. It can be observed that the formulas developed in the present work have simpler expressions.

**Constituent Proportioning by Linear Programming Method**

In the above formulation (as given in Table 2), the OBC (i.e., $P^b_b$) value has been assumed to be known beforehand. However, at the constituent proportioning stage, one is expected to know only an approximate value of the OBC of the recycled mix. The OBC value is subsequently finalized from volumetric, strength, and other performance-related tests conducted on various trial mixes. Thus, it would be logical to assume a range of asphalt binder content ($P^b_b$) in the formulation within which the OBC of the recycled mix is expected to lie. The upper and lower limits of this range are given as $(P^b_b)^u$ and $(P^b_b)^l$, respectively.

Also, as a general situation, the target viscosity ($\eta_t$) at a specified operating temperature can be assumed as a range ($\eta_t^u$ and $\eta_t^l$ as lower and upper limits of $\eta_t$, respectively), instead of having a fixed value. This is because the mix at a particular working temperature is workable within a range of binder viscosity, rather than only at a particular viscosity value.

Thus, a generalized formulation of the mix design problem can be developed and can be expressed in the form of a linear programming (LP) problem. The constraints and objective function(s) are presented in the following subsection.

**Table 1.** Comparison of Notations Used in Asphalt Institute and Present Work

<table>
<thead>
<tr>
<th>Component</th>
<th>Asphalt Institute (1997)</th>
<th>Notations used in present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of RAP in recycled mix</td>
<td>$P^R_{ab}$</td>
<td>$P^R_{ab}$</td>
</tr>
<tr>
<td>Percentage of total asphalt binder in recycled mix</td>
<td>$P_b$</td>
<td>$P_b$</td>
</tr>
<tr>
<td>Percentage of asphalt binder in old mix</td>
<td>$P^b_{nb}$</td>
<td>$P^R_{nb}$</td>
</tr>
<tr>
<td>Percentage of virgin asphalt binder in recycled mix</td>
<td>$P^R_{ab}$</td>
<td>$P^R_{ab}$</td>
</tr>
</tbody>
</table>

*aWith respect to the total weight of the mix.

**Constraints**

Constraints regarding target viscosity at a specified temperature can be developed from the viscosity mixing rule and are given in Eqs. (8) and (9).

$$
\left( \ln \eta_t \times \frac{P^R_{RAP}}{100} + \ln P^R_{nb} \right) + \ln \eta_t^u \times \frac{P^R_{nb}}{100} \leq \ln \eta_t^u
$$

$$
\left( \ln \eta_t \times \frac{P^R_{RAP}}{100} + \ln P^R_{nb} \right) + \ln \eta_t^l \times \frac{P^R_{nb}}{100} \geq \ln \eta_t^l
$$

Eqs. (10) and (11) put constraints on the total bitumen content in the recycled mix. Thus

$$
\left( \frac{P^R_{RAP}}{100} \times P^R_{RAP} \right) + \left( P^R_{nb} \right) \leq \left( P^R_{nb} \right)^u
$$

$$
\left( \frac{P^R_{RAP}}{100} \times P^R_{RAP} \right) + \left( P^R_{nb} \right) \geq \left( P^R_{nb} \right)^l
$$

It is obvious that the sum of the percentage of all individual components ($P^R_{RAP}$, $P^R_{nb}$, $P^R_{na}$) in the recycled mix should be equal to 100. Thus the equality constraint is given by Eq. (12).

$$
P^R_{RAP} + P^R_{nb} + P^R_{na} = 100
$$

Some quantity of RAP is definitely being used in the recycling process, thus

$$
P^R_{RAP} > 0
$$

Since it is implicit that the old asphalt binder has higher viscosity compared to what is desirable, some virgin asphalt binder needs to be added to adjust the viscosity, thus

$$
P^R_{nb} > 0
$$

Also, in the recycling process some quantity of new aggregates is added. However, under certain special situations, if the quantity of virgin asphalt binder (from viscosity consideration) plus the old asphalt binder fulfills the OBC requirement of the recycled mix, and also the existing RAP aggregate gradation con-

**Table 2.** Comparison of Formulas for Calculating Recycled Asphalt Mix Constituent Percentages by Asphalt Institute and Present Work

<table>
<thead>
<tr>
<th>Percentage of component in recycled mix</th>
<th>Formulae derived in present work</th>
<th>Formulae given by Asphalt Institute (1997)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virgin asphalt binder ($P^R_{nb}$)</td>
<td>$P_{nb} \times P^R_{nb}$</td>
<td>$\frac{100(100-r)P^R_{ab}}{100-P^R_{ab}}$</td>
</tr>
<tr>
<td>RAP ($P^R_{RAP}$)</td>
<td>$\frac{(P_{nb} \times P^R_{nb} \times 100)}{P^R_{RAP}}$</td>
<td>$\frac{100(100-r)P^R_{nb}}{100-P^R_{ab}}$</td>
</tr>
<tr>
<td>New aggregates ($P^R_{na}$)</td>
<td>$100 - (P^R_{RAP} + P^R_{nb})$</td>
<td>$r \frac{P^R_{nb}}{100}$</td>
</tr>
</tbody>
</table>

*aWith respect to the total weight of the mix.

*b=r=percentage of new aggregates with respect to the total weight of aggregates in the recycled mix.
draws the feasible zone within which the solution is expected to be found.

1. Maximization of RAP: One may be interested in maximizing the quantity of RAP so that more material can be recycled. Hence, the objective function is

\[ \text{Max}(P_\text{RAP}) \]  

2. Minimization of the total material cost: One may be interested in minimizing the total material cost. Thus, the objective function can be written as

\[ \text{Min}((C_\text{RAP} \times P_\text{RAP}) + (C_\text{nb} \times P_\text{nb}) + (C_\text{na} \times P_\text{na})) \]  

where \( C_\text{RAP} \) = cost of RAP per unit quantity; \( C_\text{nb} \) = cost of new aggregates per unit quantity; and \( C_\text{na} \) = cost of virgin asphalt binder per unit quantity. Costs are assumed here as a linear function of the respective quantities.

It should be noted that this formulation does not require prefixing the quantity of any of the constituents. However, such constraints, if necessary, can be imposed easily. It would appear as another constraint equation in the LP formulation, and can be solved accordingly.

Example Problem

In order to illustrate the formulation developed in the previous section, an example problem has been presented in the following subsection. Also, three possible solutions are discussed as Cases 1, 2, and 3.

Problem Statement

It is proposed to recycle RAP collected from a road stretch within Kanpur City, India. Asphalt binder content present in the RAP is found as 5.21\% and viscosity of aged asphalt binder is measured as 6,240 mPa s at a reference temperature of 100°C (MORTH 2001). Viscosity of the virgin asphalt binder intended to be used is 1,140 mPa s at the same reference temperature. The viscosity of asphalt binder in the recycled mix is found to be satisfactory over a range of 1,800–2,300 mPa s (at the same reference temperature). The OBC of the recycled mix is expected to be within the range of 5.0–5.6\%. Estimate the proportion of RAP, new aggregates and virgin asphalt binder are in the recycled mix.

Case 1: Solution by Direct Method

From the details given, \( P_\text{RAP} = 5.21\% \), \( \eta_\text{na} = 6,240 \text{ mPa s} \), \( \eta_\text{nb} = 1,140 \text{ mPa s} \), \( \eta_\text{v} = 2,300 \text{ mPa s} \), and \( \eta_\text{a} = 1,800 \text{ mPa s} \). The direct method requires specific values of \( \eta_\text{a} \) and OBC. Therefore, in the present case, \( \eta_\text{a} \) is chosen as 2,050 mPa s (average of 1,800 and 2,300) and OBC is chosen as 5.3\% (average of 5.0 and 5.6). The proportion of aged asphalt binder in the mixture of aged and virgin asphalt binder is obtained using Eq. (1), i.e.,

\[ p_\text{nb} = \frac{\ln(2,050) - \ln(1,140)}{\ln(6,250) - \ln(1,140)} = 0.345 \]

Thus, the proportion of virgin asphalt binder in the mixture of the two binders is \( 1 - 0.345 - 0.345 = 0.355 \). From Eq. (3), the percentage of virgin asphalt binder in the recycled mix is \( p_\text{nb} = 0.355 \times 5.3 = 3.471\% \). From Eq. (6), the percentage of RAP in recycled mix is \( p_\text{RAP} = 0.345 \times 5.3 = 18.095\% \). From Eq. (7), the percentage of new aggregates in recycled mix is \( p_\text{na} = 100 - (35.095 + 3.471) = 61.434\% \). Thus, proportions of virgin asphalt, RAP, and new aggregates are obtained as 3.471, 35.095, and 61.434\%, respectively.

Case 2: Solution Using LP Approach for RAP Maximization

From Eq. (8), we have

\[ \left( \frac{\ln 6,240 \times 5.21}{100} \times P_\text{RAP} \right) + (\ln 1,140 \times P_\text{nb}) \leq \ln 2,300 \]

or

\[ (0.052 \times P_\text{RAP}) - (0.7019 \times P_\text{nb}) = 0 \]  

From Eq. (9) we have
Table 3. Constituent Proportions Obtained from Proposed Formulation

<table>
<thead>
<tr>
<th>Case method</th>
<th>( P^R_{\text{RAP}} ) (%)</th>
<th>( P^R_{\text{ab}} ) (%)</th>
<th>( P^R_{\text{nsa}} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Direct method</td>
<td>35.095</td>
<td>3.471</td>
<td>61.434</td>
</tr>
<tr>
<td>2: RAP maximization</td>
<td>44.379</td>
<td>3.287</td>
<td>52.334</td>
</tr>
<tr>
<td>3: Cost minimization</td>
<td>39.624</td>
<td>2.935</td>
<td>57.441</td>
</tr>
</tbody>
</table>

From Eq. (10) we have

\[
\left( \frac{5.21}{100} \times P^R_{\text{RAP}} \right) + \left( P^R_{\text{nsa}} \right) \leq 5.6
\]

or

\[
0.0647 \times P^R_{\text{RAP}} - 0.4568 \times P^R_{\text{nsa}} \geq 0 \tag{20}
\]

From Eq. (11) we have

\[
\left( \frac{5.21}{100} \times P^R_{\text{RAP}} \right) + \left( P^R_{\text{ab}} \right) \geq 5.0
\]

or

\[
0.0521 \times P^R_{\text{RAP}} + \left( P^R_{\text{ab}} \right) \geq 5.0 \tag{22}
\]

With the objective of RAP maximization [Eq. (17)] and using the above constraints [Eqs. (19)–(21)] and other constraints stated earlier [Eqs. (12)–(15)], the problem is solved using LP software. Thus, the proportions of RAP, new aggregates, and virgin asphalt binder are obtained as 44.379, 52.332, and 3.287%, respectively.

Case 3: Solution Using LP Approach for Cost Minimization

The cost minimization as objective [Eq. (18)], in this case, will require costs of individual constituents per unit quantity (i.e., the values of \( C_{\text{RAP}}, C_{\text{nsa}}, \) and \( C_{\text{ab}} \)). For a typical Indian project, the values of \( C_{\text{RAP}}, C_{\text{nsa}}, \) and \( C_{\text{ab}} \) can be calculated as INR 105, INR 385, and INR 15950, respectively (CPWD 2002; P. Joshi, personal communication, February 10, 2006) per ton of material. INR denotes Indian currency as Indian rupees. The problem is solved using LP software and the proportions of RAP, new aggregates, and virgin asphalt binder are obtained as 39.624, 57.441, and 2.935%, respectively. This solution, in fact, represents the point \( 'D' \) in Fig. 1.

All the results have been tabulated in Table 3. For different relative costs of the constituents, the solution could correspond to any of the corner points (A, B, C, or D) of the feasible zone identified in Fig. 1.

Discussion

1. A specific viscosity mixing rule has been used in the study (Asphalt Institute 1981; 1997) and accordingly \( P_{\text{ab}} \) and \( P_{\text{ab}} \) fractions have been calculated; choosing a different viscosity mixing rule (Chaffin et al. 1995) will not change the suggested formulation. Also, in the present work, viscosity values for one specified temperature have been used, and this formulation can be suitably upgraded for viscosity constraints at different temperatures.

2. Different researchers/guidelines suggest various RAP quantities to be used in the recycled mix (Sullivan 1996; Austroads 2000; McDaniel et al. 2000). While up to 80% of RAP has been used in some hotmix asphalt pavements (Kennedy et al. 1998), 20–50% of RAP is typically used (Solaimanian and Tahmoresi 1996; McDaniel et al. 2002). Maximizing the RAP quantity is not always desirable, even though it may enhance the rut resistance, but fatigue resistance may fall (Kandhal et al. 1995; Sullivan 1996; McDaniel et al. 2000). As mentioned earlier, if there is any practical limitation on the RAP quantity, it can be incorporated into the formulation; and

3. Superpave guidelines (McDaniel et al. 2000; McDaniel and Anderson 2001) recommend two methods for preliminary constituent estimation: one with a fixed RAP quantity and the other with a fixed binder grade. In these methods constituent proportions are estimated step by step, satisfying the high, intermediate, and low critical temperature requirements. The approach presented in this paper can be extended to Superpave recycled mix designs so as to optimize the total cost and/or RAP quantity while obtaining the mix constituent proportions directly.

Conclusions

The contributions made by the present paper can be summarized as follows:

1. Fresh formulation of preliminary constituent proportioning (for central plant hotmix recycling) by the direct method has been developed. The proposed formulas have simpler expressions than the Asphalt Institute formulae (1997);
2. The formulation has been extended to a generalized framework that can optimize the mix proportion toward objectives like overall material cost minimization or RAP quantity maximization using a simple linear programming tool;
3. The generalized formulation accepts some of the mix design parameter values in the form of its range, rather than a fixed value. This is a desirable situation, because some parameter values may not be precisely known at the preliminary constituent proportioning stage; and
4. The proposed approach handles the proportioning problem as a whole, and does not require prefixing any of the parameters beforehand. However, prefixing of any parameter value, if required, can be incorporated just as another constraint to the formulation.

Acknowledgments

The senior writer wishes to gratefully acknowledge the financial support provided by the All India Council for Technical Education (AICTE), Government of India, under the scheme of “Career Award for Young Teachers (2002–03).”
Notation

The following symbols are used in this paper:

\[ C_{na} \] = cost of new aggregates per unit quantity;
\[ C_{nb} \] = cost of virgin asphalt per unit quantity;
\[ C_{RAP} \] = cost of RAP per unit quantity;
\[ P_{ob}^{R} \] = asphalt binder demand of recycled mix (equivalently, optimum binder content, OBC);
\[ P_{b}^{R} \] = percentage of aged asphalt binder in RAP;
\[ P_{nb}^{R} \] = percentage of new aggregates in recycled mix;
\[ P_{ob}^{R} \] = percentage of virgin asphalt binder in recycled mix;
\[ P_{R}^{Pb} \] = percentage of aged asphalt binder in recycled mix;
\[ (P_{b}^{R})_{l} \] = lower limit on \( P_{b}^{R} \);
\[ (P_{b}^{R})_{u} \] = upper limit on \( P_{b}^{R} \);
\[ P_{ob} \] = proportion (as fraction) of virgin asphalt binder in binary mix of aged and virgin asphalt binder;
\[ P_{ob} \] = proportion (as fraction) of aged asphalt binder in binary mix of aged and virgin asphalt binder;
\[ \eta_{b} \] = viscosity of virgin asphalt binder;
\[ \eta_{b} \] = target viscosity of binary mixture of aged asphalt binder and virgin asphalt binder;
\[ \eta_{b}^{f} \] = lower limit on \( \eta_{b} \);
\[ \eta_{b}^{u} \] = upper limit on \( \eta_{b} \); and
\[ \eta_{0} \] = viscosity of aged asphalt binder.

References