Experimental determination of two-point Stokes parameters for a partially coherent broadband light beam

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The generalized Stokes parameters, which are two-point extensions of conventional Stokes parameters, are determined for a pair of points in the cross-section of a partially coherent broadband light beam. For this purpose, using a two-mirror and two-beam-splitter assembly, unpolarized, linearly polarized and partially polarized electromagnetic beams are generated. This simple experimental method for determining generalized Stokes parameters establishes an analogy with the experimental scheme of determining conventional Stokes parameters, which in succession contributes to appreciable reduction in uncertainty in the measurements.

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1. Introduction

In the last decade, research on the studies of vectorial optical fields, especially on the characterization of polarization using the Stokes parameters, has attracted much attention. In this connection, apart from the conventional Stokes parameter representations [1,2], which are linear combinations of intensities due to different orthogonal components of light, the two-point Stokes parameter representation has also been put forward [3,4]. These two-point (generalized) Stokes parameters contain the polarization properties, along with the spatial coherence properties at a pair of points in the cross-section of the electromagnetic beam [5]. This special feature equips them as a strong tool for investigating changes in the polarization properties of light during propagation.

Of late, various methods were put forward to determine the two-point polarization properties of the optical fields [6–8]. However, these methods were used for laser (highly monochromatic and directional) fields and were requiring highly sophisticated and lengthy procedures. In one of the experiments these parameters were determined using a modified version of Young’s interferometer [6,7]. The complexity in the alignment of this interferometer restricts its utilization for partially coherent broadband light. Another method relies on the electromagnetic spectral interference law giving a direct measure of these parameters from the conventional Stokes parameters [8,9]. This method is still a two step process and one needs to determine the one-point Stokes parameters and subsequently the two-point Stokes parameters are determined. Recently it has been proposed that the generalized Stokes parameters have physical interpretation as linear combinations of cross-spectral density functions of specific electric field components [10]. This is fully analogous to the classical intensity based interpretation of the conventional Stokes parameters. This analogy of these two kinds of Stokes parameters advocates for an experimental approach for determining the two-point Stokes parameters similar to the one used for determining the single-point (conventional) Stokes parameters.

The present paper elucidates an experimental measurement scheme for the generalized Stokes parameters for a pair of points in the cross-section of an electromagnetic beam. A two-mirror, two-beam-splitter assembly is used to control the polarization state of partially coherent broadband light beam. Using a classic Young’s double-slit interferometer with a set of polarizer and compensator, the diagonal elements of the electric cross-spectral density matrix are determined [11]. Using the algebraic relation between these elements, the generalized Stokes parameters at a pair of points for unpolarized, linearly polarized and partially polarized light beams are determined. This method exhibits a close resemblance with the existing method of determining the conventional Stokes parameters for a light beam [2,10]. However, instead of intensity (spectral density) measurements, the correlation measurements are performed to obtain the two-point Stokes parameters.

2. Theoretical background

The conventional Stokes parameters at a point \( \mathbf{r} \) in an electromagnetic beam can be expressed and are determined using the intensities (spectral) in space-frequency representation as [2,8]

\[
S_0(\mathbf{r}, \omega) = I_x(\mathbf{r}, \omega) + I_y(\mathbf{r}, \omega) = \varphi^0(0^0, 0^0) + \varphi(90^0, 0^0),
\]

(1a)
\[ S_i(\mathbf{r}, \omega) = I_i(\mathbf{r}, \omega) - I_j(\mathbf{r}, \omega) = \psi(0^\circ, 0^\circ) - \psi(0^\circ, 90^\circ), \] (1b)
\[ S_j(\mathbf{r}, \omega) = I_i(\mathbf{r}, \omega) - I_j(\mathbf{r}, \omega) = \psi(45^\circ, 0^\circ) - \psi(135^\circ, 0^\circ), \] (1c)
\[ S_j(\mathbf{r}, \omega) = I_i(\mathbf{r}, \omega) - I_j(\mathbf{r}, \omega) = \psi(45^\circ, 45^\circ) - \psi(135^\circ, 45^\circ), \] (1d)

where \((x, y), (\alpha, \beta), (r, l)\) are the basis vectors of different coordinate systems, i.e., linear, 45° linear and circular polarization, respectively. \(\psi(\theta, \phi)\) represents the spectral density at point \(\mathbf{r}\) for the polarizer making angle \(\theta\) with \(x\)-axis and the quarter wave plate having \(\phi\) orientation of the optic axis with the polarizer axis. The first Stokes parameter given in Eq. (1a) represents total intensity of the beam at point \(\mathbf{r}\), while the last three Stokes parameters given by Eqs. (1b)–(1d) physically indicate the dominance of one kind of polarization over the other in the respective coordinate system.

The generalized Stokes parameters, on the other hand, are expressible in correspondence with the conventional Stokes parameters.

\[ \eta_{ij}(\mathbf{r}, \omega) = \psi(\mathbf{r}, \omega) \times \left( \frac{\text{Re}(\psi_{ij}(\mathbf{r}, \omega))}{\text{Im}(\psi_{ij}(\mathbf{r}, \omega))} \right)^2 \] (5)

where \((i, j) = (x, y)\) are the polarization indices. This shows that a measurement has to be performed for each configuration of the polarizations. In Eq. (5), the spectral visibility \(\psi(\mathbf{r})\) of the interference fringes for each set of polarizations is obtained by the relation

\[ \psi(\mathbf{r}) = |\psi(\mathbf{r}, 1, 2, \omega)| = \frac{\psi^\text{max}(\mathbf{r}) - \psi^\text{min}(\mathbf{r})}{\psi^\text{max}(\mathbf{r}) + \psi^\text{min}(\mathbf{r})}, \] (6)

where \(\psi^\text{max}(\mathbf{r})\) and \(\psi^\text{min}(\mathbf{r})\) are the respective maximum and the minimum values of spectral density around the central fringe when both the slits are open [8,9]. In Eq. (5), the real part of the spectral degree of coherence for each set of polarizations is given by [7]

\[ \text{Re}(\psi_{ij}(\mathbf{r}, \omega)) = \frac{\psi(\mathbf{r}, \omega) - 2\psi^\text{max}(\mathbf{r})}{2\psi^\text{max}(\mathbf{r})}. \] (7)

It has been shown recently [see Eq. (9) of Ref. [10]] that using proper basis of unitary matrices, the generalized Stokes parameters are expressible in correspondence with the conventional Stokes parameters. In case the spectral densities at both the slits for same polarized components of the electromagnetic field are the same, we can write \(\psi_{ij}(\mathbf{r}, \omega) = \psi(\mathbf{r}, \omega)\) for \(i = 1, 2\) and also the spectral degree of coherence \(\eta(\mathbf{r}, 1, 2, \omega)\) for different polarized components of light as [6]

\[ \psi_{ij}(\mathbf{r}, \omega) = \psi(\mathbf{r}, \omega) \eta_{ij}(\mathbf{r}, \omega), \quad \text{for}(i, j) = (x, y). \] (3)

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3. Experimental method and results

To realize the proposed method experimentally, the schematic diagram is shown in Fig. 1. A tungsten-halogen lamp \(L\) (Mazda, colour temperature = 3200 K) was used as a continuous spectrum source. It was operated at 600 W using a regulated dc power supply (Heinzinger, stability 1 part of 10⁶). The outer glass jacket of the lamp was diffused (D) to obtain uniform illumination. The polychromatic light from the source was filtered using a glass filter \(F\) (peak \(\lambda=580\,\text{nm},\ FWHM=90\,\text{nm}\)) giving out broadband optical field. To obtain higher visibility of the interference fringes, the partial coherence of the light beam was enhanced by first passing the beam through a narrow single-slit (SS) having a width of 400 μm. The radiation from SS was then made incident on a double-slit DS (slit width = 250 μm, slit separation = 450 μm) placed at a distance of 100 cm from the single-slit, symmetric to the optical axis (see Fig. 1). The double-slit is used here to probe the beam at two different points in its cross-section. The diameter of the region on the double-slit plane outside which there is complete incoherence is given by the van-Cittert Zernike theorem and is equal to \(\frac{\lambda R}{\rho}\) for a rectangular

\[ \text{Fig. 1. Schematic of the experimental setup. Notations: L tungsten-halogen lamp, D diffuser, F broadband filter, SS single-slit, PC polarization controller consisting of BS beam-splitter, M mirror, P polarizer and N neutral density filter; Q quarter wave plate, DS double-slit, T stopper, O observation plane, SM spectrometer and DP data processor. Inset shows the direction of coordinate axes.} \]
slit, where $\lambda$ is the central wavelength, $\rho$ is the slit width and $R$ is the distance of the observation plane from SS [2,12]. The active area of the double-slit (700 $\mu$m) was well occupied within the partial coherence region developed at DS (using the van-Cittert Zernike theorem with SS), which was calculated to be $\approx 1.4$ mm for present geometry. The interference fringes are shown in Fig. 2(a).

The state of polarization of light beam was controlled before DS by passing the beam through a polarization controller PC, as shown in Fig. 1. Inside the polarization controller, to facilitate the insertion of polarizers ($P_1$, $P_2$) and neutral density filter N in the beam paths, the beam was first divided into two beams of nearly equal intensity using a 50:50 beam-splitter BS$_1$. After reflecting through the mirrors M and travelling equal paths, these two beams were recombined using another beam-splitter BS$_2$, which actually worked as beam combiner. A set of polarizer $P_3$ and quarter wave plate Q was used to choose proper basis of the coordinate systems. A fiber-coupled spectrometer SM (Photon Control, SPM002) connected with a personal computer DP was used for spectral measurements. The spectral densities of the beams passing through individual slits (of DS) alternately, were found nearly same when measured at the axial point P, as shown in Fig. 2(a) and (b). Making spectral measurements at the observation plane O and using Eq. (6), the visibility of the interference fringes was found approximately 29%.

The unpolarized light can be characterized as a combination of two equal-intensity orthogonally polarized light beams [1]. To make the light beam unpolarized, $P_1$ and $P_2$ were put in different beams (in PC) making angles 0° and 90° with $x$-axis respectively, as shown in Fig. 1. The recombined beam after BS$_2$ consists of two identical orthogonal electric field vibrations, hence unpolarized in nature. To verify this fact, the beam was passed through another polarizer ($P_3$) and the output intensity when plotted against the angle of the polarizer (or analyser) showed a flat response at all angles (Fig. 3a) showing the unpolarized nature of light beam.

The polarizer $P_3$ passing the $x$ component of the electric field was put before DS and the maximum and the minimum spectral densities around point P (for the central fringe) were measured. For the present geometry, the separation between the two minima in either side of the central maximum was found to be 2.7 mm. Using Eq. (6), the spectral visibility $\nu_{xx}(\tau)$ of the fringes for $x$ polarization components was obtained. The spectral density at each slit was measured by putting fiber tip on the respective slit. The spectral densities at point P for the individual beams were obtained by closing one of the slits using the stopper T (see Fig. 1) and also for the combined beam. These values were put in Eq. (7) and the real part of the spectral degree of coherence was obtained. This value along with the spectral visibility was put in Eq. (5) giving the complex value of the spectral degree of coherence $\eta_{xx}(\tau_1, \tau_2, \omega)$.

Similarly the spectral degree of coherence $\eta_{yy}(\tau_1, \tau_2, \omega)$ and the spectral density at the slit were measured for the $y$-axis orientation of $P_3$. These values in Eqs. (4a) and (4b) gave the first and the second generalized Stokes parameter $S_1(\tau_1, \tau_2, \omega)$, $i = 0, 1$; for the unpolarized light.

To determine the third two-point Stokes parameter $S_2(\tau_1, \tau_2, \omega)$, the spectral density at one of the slits and the spectral degree of coherence for +45° and −45° orientation of $P_3$ were measured, following the above mentioned procedure. These values were put in Eq. (4c) and the third generalized Stokes parameter was obtained. For the fourth parameter $S_3(\tau_1, \tau_2, \omega)$, $P_3$ at 45° was put following a quarter-wave plate Q having optic axis at angle 45° with the polarizer axis. The spectral density at one of the slits and the spectral degree of coherence were obtained for right handed and left handed circular polarizations. Using Eq. (4d) the fourth generalized Stokes parameter was obtained. The four two-point Stokes parameters determined for the unpolarized partially coherent broadband light using the above said procedure are shown in Table 1. The uncertainty in the measurements was evaluated by repeating the experiment four times and is shown with the two-point Stokes parameter values.

To produce linearly polarized light beam, both $P_1$ and $P_2$ were put with their axis in $x$-direction. This passed only the $x$-components of the electric field for both the beams and the recombined beam was linearly polarized (in $x$-axis). For this beam, the output intensity (passing the analyser) when plotted, followed the Malus law [2], as shown in Fig. 3(b). This shows the linearly polarized nature of the light beam. As mentioned above, for this beam, spectral measurements were made at the observation plane for different combinations of $P_1$ and $Q$, and using Eqs. (4a)–(7), the four two-point Stokes parameters for linearly polarized light were determined. These generalized Stokes parameters are shown in Table 1.

To form partially polarized state of light beam, $P_1$ was replaced by a neutral density filter N (transmittance 50%) in PC (see Fig. 1). Not only does this allow the unpolarized light to pass through but also puts its transmitted intensity at par with the transmitted intensity of $P_2$. As a result, the recombined beam consists of linearly polarized and unpolarized light with equal proportion of intensity, giving a state.
of partial polarization. This partially polarized nature of light beam was confirmed when the output intensity was plotted as a function of the angle of analyser for (a) unpolarized light, (b) linearly polarized light in x-direction and (c) partially polarized light. The error bars show uncertainty in the mean value calculated at a 95% confidence level.

![Fig. 3](image)

**Fig. 3.** Behaviour of normalized output intensity as a function of the angle of analyser for (a) unpolarized light, (b) linearly polarized light in x-direction and (c) partially polarized light. The error bars show uncertainty in the mean value calculated at a 95% confidence level.

<table>
<thead>
<tr>
<th>Unpolarized light</th>
<th>Linearly polarized light</th>
<th>Partially polarized light</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0(r_1, r_2, \omega) )</td>
<td>( (134.24 \pm 0.28) + (703.55 \pm 0.58)i )</td>
<td>( (136.48 \pm 0.25) + (661.10 \pm 0.33)i )</td>
</tr>
<tr>
<td>( S_1(r_1, r_2, \omega) )</td>
<td>( (1.90 \pm 0.28) + (155.56 \pm 0.25)i )</td>
<td>( (1.90 \pm 0.26) + (149.72 \pm 0.23)i )</td>
</tr>
<tr>
<td>( S_2(r_1, r_2, \omega) )</td>
<td>( (-0.41 \pm 0.01) - (12.68 \pm 0.11)i )</td>
<td>( (-1.46 \pm 0.02) - (2.99 \pm 0.03)i )</td>
</tr>
<tr>
<td>( S_3(r_1, r_2, \omega) )</td>
<td>( (-1.96 \pm 0.02) - (57.61 \pm 0.17)i )</td>
<td>( (2.46 \pm 0.02) + (7.38 \pm 0.04)i )</td>
</tr>
</tbody>
</table>

Experimentally determined values of the four two-point Stokes parameters for different polarization states of the light beam.

4. Discussion

The generalized Stokes parameters presented in Table 1 explicitly characterize the correlations (thus complex quantities) for different states of polarization of light at the two slits. To simplify the interpretation, the polarization state of light was set nearly the same for both the slits, in each of the three cases. In the first case, the last three generalized Stokes parameters \( S_n(r_1, r_2, \omega) \) for \( n = 1, 2, 3 \), show a little correlation between different polarization components. This signifies the unpolarized nature of the light beam. For the second case, on the other hand, the total correlation given by the first generalized Stokes parameter \( S_0(r_1, r_2, \omega) \) is the sole contribution of the linear polarization (second two-point Stokes parameter) of light at the double-slit. The positive value of this quantity (real part only) indicates the dominance of \( x \)-axis linear polarization over the \( y \)-axis linear polarization. For the third case, the major contribution in total correlation is furnished by the \( x \)-axis linear polarization, as obvious by the positive value of the second two-point Stokes parameter. However, the remaining contribution is offered by the unpolarized part of the light beam. This infers that the probed beam is having partially polarized nature. The unique feature of this experimental method is minimum uncertainty in the measurements due to minimum optical components and minimum time taken.

Thus we find that both these kinds of Stokes parameters characterize the polarization properties of the electromagnetic beam. However, due to two-point dependency, the generalized Stokes parameters contain additional information about the spatial coherence property of the optical field for different polarized components of light. The former are linear combinations of intensities and thus are real quantities while the latter are sum and differences of correlation functions and are complex quantities, however, both these kinds of parameters could be determined using the analogous treatments, as is evident from this study.

5. Conclusion

In summary, using a set of basis vector of different coordinate systems \((x, y), (\alpha, \beta), (r, \theta)\), which respectively correspond to linear, 45° linear and circular polarizations, we have experimentally determined the two-point Stokes parameters respectively for the unpolarized, linearly polarized and partially polarized broadband light beams. This method, owing to its explicit analogy with its counterpart method used to determine the conventional Stokes parameters, provide a
subtle means for determining the two-point parameters with more ease and less uncertainty. This scheme is expected to be applicable in polarization metrology especially for exploring the propagation dependent changes on polarization properties of electromagnetic fields, irrespective of the nature of the light source.

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