Dynamic instability of layered anisotropic composite plates on elastic foundations

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Abstract

The dynamic instability of laminated composite plates supported on elastic foundations, subjected to periodic in-plane loads, is investigated using C 1 eight-noded shear-flexible plate element. The element is based on a new kind of kinematics which allows one to exactly ensure the continuity conditions for displacements and stresses at the interfaces between the layers of the laminate, and also the boundary conditions at the top and bottom surfaces of the laminate. The present model accounts for in-plane and rotary inertia effects. The boundaries of the principal instability region obtained here are conveniently represented in the non-dimensional excitation frequency—non-dimensional load amplitude plane. The influences of various parameters such as orthotropicity, ply-angle, static load factor, thickness and aspect ratios, and elastic foundation stiffness on dynamic stability are brought out. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Dynamic instability; Composite; Finite element; Elastic foundation

1. Introduction

The ever-increasing use of fibre-reinforced composites in structural components has resulted in several studies of their dynamic behaviour. Components like plates/shells supported on an elastic medium often find applications in the construction of aerospace, mechanical, nuclear and offshore structures. They are, in general, subjected to various types of dynamic loads. Knowledge pertaining to the stability characteristics of such plates/shells for different types of system parameters is essential for optimal design and assessment of the structural failures.

Dynamic instability analysis of isotropic beams and columns supported on elastic foundation, subjected to periodic in-plane loads, has received considerable attention [1–5], whereas the study related to isotropic plates resting on elastic medium has been sparsely treated in the literature [6]. The elastic medium, considered in Ref. [6], is of Winkler foundation type. Furthermore, several investigators [7–10] have dealt with the dynamic instability of composite plates/shells due to in-plane periodic loads, without considering the effect of elastic foundation. But, the influences of elastic foundation stiffness on the free vibration and buckling studies of laminated composite plates are examined in Refs. [11–14]. However, to the authors’ knowledge, no work related to the dynamic instability of laminated anisotropic plates/shells resting on elastic medium has been reported in the literature. In this paper, an attempt is made to study the dynamic instability of laminated composite plates supported on elastic medium, like Winkler and Pasternak foundations, subjected to periodic in-plane loads.

Here, an eight-noded shear-flexible plate element, developed recently based on C 1 continuity requirement for transverse displacement as outlined in Refs. [15,16], is extended to analyse parametric resonance pertaining to laminated composite plates, supported on elastic foundation, with periodic loading. The present model includes the effects of in-plane and rotary inertia. Since the transverse shear deformation is represented by cosine functions, which is of a higher order, no shear correction factor is introduced in the analysis. A detailed parametric study to bring out the influences of orthotropicity, ply-angles, aspect and thickness ratios, and elastic foun-
2. Formulation

A laminated composite plate supported on an elastic medium is considered with the co-ordinates $x$, $y$ along the in-plane directions and $z$ along the thickness direction, as shown in Fig. 1. Using formulation based on shear flexible theory, the displacements in the $k$th layer, $u^{(k)}$, $v^{(k)}$ and $w^{(k)}$ at a point $(x, y, z)$ from the median surface are expressed as functions of mid-plane displacement $u$, $v$, $w$ and independent rotation $\theta_x$ and $\theta_y$ of normal in $xz$ and $yz$ planes, respectively, as,

$$
\begin{align*}
    u^{(k)}(x, y, z, t) &= u(x, y, t) - z\partial w/\partial x + [f_1(z) + g_1^{(k)}(z)]\{\partial w/\partial x + \theta_x\} \\
    v^{(k)}(x, y, z, t) &= v(x, y, t) - z\partial w/\partial y + [f_2(z) + g_2^{(k)}(z)]\{\partial w/\partial y + \theta_y\} \\
    w^{(k)}(x, y, z, t) &= w(x, y, t)
\end{align*}
$$

where $t$ is the time. The functions involved in Eq. (1) for defining the kinematics are as follows:

$$
\begin{align*}
    f_1(z) &= h/\pi \sin(\pi z/h) - h/\pi b_{sz} \cos(\pi z/h) \\
    f_2(z) &= h/\pi \sin(\pi z/h) - h/\pi b_{sz} \cos(\pi z/h) \\
    g^{(k)}_i &= a_i^{(k)} z + d_i^{(k)}, i = 1, 2, 3, 4; k = 1, 2, 3, \ldots, N
\end{align*}
$$

where, $N$ is the number of layers of the multi-layered structure, $h$ is the total thickness of the laminate, $\pi$ is equal to 3.141592, and $b_{sz}, b_{sz}, a_i^{(k)}, d_i^{(k)}$ are coefficients to be determined from contact conditions for displacements and stresses between the layers and from the boundary conditions on the top and bottom surfaces of the plate. The details of the derivations of these coefficients can be found from Refs. [15,16].

The linear strains in terms of mid-plane deformation can be written as

$$
\{\epsilon\} = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \chi \\ \omega \\ \gamma^o \end{bmatrix}
$$

The mid-plane strains $\{\epsilon\}$, bending strains (due to lower and higher order terms involved in defining the kinematics, Eq. (1)), $\{\chi\}, \{\omega\}$ and shear strains $\{\gamma^o\}$ in Eq. (3) are written as

$$
\begin{align*}
    \{\epsilon\} &= \begin{bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{bmatrix} \\
    \{\chi\} &= -\begin{bmatrix} \partial^2 w/\partial x^2 \\ \partial^2 w/\partial y^2 \\ 2\partial^2 w/\partial x\partial y \end{bmatrix} \\
    \{\omega\} &= \begin{bmatrix} \partial \gamma^o/\partial x \\ \partial \gamma^o/\partial y \\ \partial \gamma^o/\partial y \\ \partial \gamma^o/\partial x \end{bmatrix} \\
    \{\gamma^o\} &= \begin{bmatrix} \gamma^o_x \\ \gamma^o_y \end{bmatrix} = \begin{bmatrix} \partial w/\partial x + \theta_x \\ \partial w/\partial y + \theta_y \end{bmatrix}
\end{align*}
$$

If $\{N\}$ represents the membrane stress resultants ($N_{sx}, N_{sy}, N_{sz}$) and $\{M\}, \{\tilde{M}\}$, represent the bending stress resultants due to lower and higher order terms involved in defining the kinematics ($M_{sx}, M_{sy}, M_{sz}$), ($\tilde{M}_{sx}, \tilde{M}_{sy}, \tilde{M}_{sz}$), one can relate these to membrane strains $\{\epsilon\}$ and bending strains $\{\chi\}, \{\omega\}$ through the constitutive relations as,

$$
\begin{align*}
    \{N\} &= [A]\{\epsilon\} + [B]\{\chi\} + [E]\{\omega\} \\
    \{M\} &= [B]^T\{\epsilon\} + [D]\{\chi\} + [\tilde{B}]\{\omega\} \\
    \{\tilde{M}\} &= [E]^T\{\epsilon\} + [B]^T\{\chi\} + [\tilde{D}]\{\omega\}
\end{align*}
$$

Similarly, the transverse shear stress resultants $\{Q\}$ representing the quantities $Q_{xz}, Q_{yz}$ are related to the transverse strains $\{\gamma\}$ through the constitutive relation as
\[ \{ Q \} = [\bar{A}]\{ \gamma^0 \} \] (6)

The different matrices involved in Eq. (5a), (5b), (5c) and (6) are defined as follows [16]:

\[ [A] = \int_{-h/2}^{h/2} [Q_o] dz \quad [B] = \int_{-h/2}^{h/2} z[Q_p] dz \]
\[ [E] = \int_{-h/2}^{h/2} [Z(z)]^T [Q_p] dz \quad [D] = \int_{-h/2}^{h/2} \bar{z}^2 [Q_p] dz \]
\[ [\bar{B}] = \int_{-h/2}^{h/2} z[Z(z)][Q_p] dz \]
\[ [\bar{D}] = \int_{-h/2}^{h/2} [Z(z)]^T [Q_p][Z(z)] dz \]
\[ [\bar{A}] = \int_{-h/2}^{h/2} [Y(z)]^T [Q_i][Y(z)] dz \]

The matrices \( Y(z) \) and \( Z(z) \) are given as:

\[ Y(z) = \begin{bmatrix} \partial f_1/\partial z + \partial g_1^{(i)}/\partial z & \partial g_1^{(i)}/\partial z \\ \partial g_2^{(i)}/\partial z & \partial f_2/\partial z + \partial g_2^{(i)}/\partial z \end{bmatrix} \]

\[ Z(z) = \begin{bmatrix} f_1 + g_1^{(k)} & 0 & 0 & g_2^{(k)} \\ 0 & f_2 + g_4^{(k)} & g_3^{(k)} & 0 \\ g_3^{(k)} & g_2^{(k)} & f_1 + g_1^{(k)} & f_2 + g_4^{(k)} \end{bmatrix} \] (7a)

For a composite laminate of thickness \( h \), consisting of \( N \) layers with stacking angle \( \theta_k \) \((k = 1, 2, 3, \ldots, N)\), the layer thickness \( h_k \), the necessary expressions for computing the reduced stiffness coefficients of \( ([Q_o], [Q_i]) \) available in the literature [17], are used here.

The total strain energy functional \( U \) consisting of energy stored in the plate and in the foundation springs, is given by,

\[ U(\delta) = \frac{1}{2} \int \{N\}^T \{\epsilon^o\} + [M]^T \{ \chi \} + [\bar{M}]^T\{ \omega \} + [Q]^T\{ \gamma^0 \}) dA + \frac{1}{2} \int (w^T F_1 w) + (\partial w/\partial x)T F_2 (\partial w/\partial x) + (\partial w/\partial y)T F_3 (\partial w/\partial y) dA \] (8)

where, \( \delta \) is the vector of the degrees of freedom associated to the displacement field in a finite element discretisation. \( F_1 \) and \( F_2 \) are the foundation stiffnesses.

The kinetic energy of the plate is written as

\[ T(\delta) = \frac{1}{2} \int (\bar{\rho} [\dot{u}^{(k)}]^T [\dot{u}^{(k)}]) dA \] (9)

where the dot over the variable denotes the partial derivative with respect to time and \( \bar{\rho} \) is the mass density.

The potential energy due to external in-plane force, \( N^x_o \) in the \( x \)-direction, is written as,

\[ W(\delta) = \frac{1}{2} \int N^x_o (\partial w/\partial x)^2 dA \] (10)

Substituting Eqs. (8)–(10) in Lagrange’s equation of motion, one obtains the governing equation for the free vibration of plate as,

\[ \{M\}\{\ddot{\delta} \} + ([K] + [K_G])\{ \delta \} = \{0\} \] (11)

where \([M]\) is the consistent mass matrix, \([K]\) and \([K_G]\) are the structural stiffness and the geometric stiffness matrices, respectively.

The coefficient in the stiffness and mass matrices can be rewritten as the product of term having thickness co-ordinate \( z \) alone and the term containing \( x \) and \( y \). In the present study, while performing the integration for the evaluation of the stiffness and mass coefficients, terms having thickness co-ordinate \( z \) are explicitly integrated whereas the terms containing \( x \) and \( y \) are evaluated using full integration with \( 4 \times 4 \) points Gauss integration rule.

### 3. Parametric instability analysis

The state of periodic load is the uniform pulsating axial compressive force \( N^c_r \), which may be defined as

\[ N^c_r = (N_o + N_1 \cos \bar{\omega} t) = (\alpha + \beta \cos \bar{\omega} t)N^c_o \] (12)

where \( \alpha = N_o/N^c_o, \beta = N_1/N^c_o, \bar{\omega}, \ddot{\omega} \) are static buckling load of the plate and the frequency of the dynamic in-plane load, respectively. From Eqs. (11) and (12), we have the governing equation of the form

\[ \{M\}\{\ddot{\delta} \} + ([K] + (\alpha N^c_o + \beta N^c_o \cos \bar{\omega} t)[K_G])\{ \delta \} = \{0\} \] (13)

Eq. (13) represents the dynamic stability problem of a system subjected to a periodic in-plane axial force. The dynamic instability boundary is determined using the method suggested in the literature [18]. To obtain points on the boundaries of the instability region, the components \( \{ \delta \} \) are written in the Fourier series as

\[ \{ \delta \} = \frac{1}{2} \{ b \}_0 + \sum_{i = 2.4.6} [-a, \sin (i\bar{\omega} t/2) + \{ b \}_i \cos (i\bar{\omega} t/2)] \] (14)
with period \( T \), where \( T = 2\pi/\omega \), or

\[
\{\delta\} = \sum_{j = 1, 3, 5, \ldots} \left[ \{a\}_j \sin \left( \frac{i\omega t}{2} \right) + \{b\}_j \cos \left( \frac{i\omega t}{2} \right) \right]
\]

(15)

with period \( 2T \). These expressions are substituted into Eq. (13) and the coefficients of each sine and cosine terms are set equal to zero, as well as the sum of the constant terms. For nontrivial solutions, the determinants of the coefficients of these groups of linear homogeneous equations are equal to zero. The problem is now reduced to that of finding the eigenvalues of the systems. Using the standard eigenvalue extraction scheme, for the given value of \( \alpha \), the variation of the eigenvalues \( \tilde{\omega} \) with respect to \( \beta \) can be found out. The plot of such variation in the \( \beta - \tilde{\omega} \) plane shows the instability regions associated with the given plate subjected to harmonically excited in-plane load.

4. Element description

The eight-noded element used here is based on Hermite cubic function for transverse displacement, \( w \) according to the \( C^1 \) continuity requirement, Serendipity quadratic function for the in-plane displacements \( u, v \) and rotations \( \theta_\alpha, \theta_\gamma \). Further, the element needs eight nodal degrees of freedom \( (u, v, w, \partial w/\partial x, \partial w/\partial y, \partial^2 w/\partial x\partial y, \theta_\alpha, \theta_\gamma) \) for all corner nodes and four degrees of freedom \( (u, v, \theta_\alpha, \theta_\gamma) \) for the mid-node of the all four sides. The element is developed based on new kinematics as given in Eq. (1), which accounts for interlayer continuity for displacements and transverse shear stresses of the laminate. The element behaves very well for both thick and thin situations. It has no spurious mode and is represented by correct rigid body modes.

5. Results and discussion

In this section, we use the above formulation to investigate the effect of parameters like orthotropicity, ply-angle and elastic foundation stiffness etc. on the dynamic instability of composite laminates, supported on elastic medium, subjected to periodic loads. Based on progressive mesh refinement, \( 8 \times 8 \) mesh idealisation is found to be adequate to model the full plate for the present analysis. Before proceeding for the detailed analysis, the formulation developed herein is validated by considering the free vibration and buckling analyses of isotropic, orthotropic, and laminated plate with/without supported on elastic foundation. Table 1 shows the natural frequencies for fairly thick isotropic as well as orthotropic plates and are compared with the exact solutions of three-dimensional elasticity theory [18,19]. For anti-symmetric cross-ply laminates, the fundamental frequencies and critical buckling loads obtained are given in Table 2 along with the exact analytical results based on third-order shear deformation theory [20]. Table 3 compares the present results, concerning the effect of elastic support on vibration and buckling load of the isotropic plate, with those given in Ref. [6]. It is observed from these tables that the present results agree very well with the existing literature. Here, in view of the computational time involved for the parametric study, one term solution of Eq. (13) is employed, which furnishes accurate results for the low values of load amplitude. Furthermore, the analysis is focused mainly on the determination of boundaries of the primary instability region that occurs in the vicinity of simple resonance of first order, \( 2\omega_\alpha (\omega_\alpha \text{ is lth the natural frequency where } l = 1, 2, 3, \ldots) \) which is by far the largest one compared to the neighbourhoods of combination resonance of first order, \( (\omega_i \pm \omega_j) \). This is the most dangerous zone and has the greatest practical importance [7,8,21]. The material properties used in the present analysis are,

\[
\frac{E_L}{E_T} = 25.0, \quad G_{LT}/E_T = 0.5, \quad G_{TT}/E_T = 0.2, \quad \gamma_{LT} = 0.25
\]

(16)

where \( E, G \) and \( \gamma \) are Young’s moduli, shear modulus and Poisson’s ratio. L and T are the longitudinal and transverse directions, respectively, with respect to fibres. All the layers are of equal thickness and the ply-angle is measured with respect to the \( x \)-axis. The simply supported boundary conditions, considered here, are:

\[
\begin{align*}
\nu &= w = \theta_\gamma = \partial w/\partial y = 0 \text{ at } x = 0, a \\
u &= w = \theta_\alpha = \partial w/\partial x = 0 \text{ at } y = 0, b
\end{align*}
\]

(17)

Next, a dynamic instability analysis of isotropic/orthotropic/laminated square plates \( (a/b = 1, \ a/h = 100) \) is studied and the results are plotted in Figs. 2–5 for different values of foundation stiffness \( (K_1 = F_1a^4/D_{11}; \ K_2 = F_2a^4/D_{11}, \text{ where } D_{11} = E_i h^3/[12(1 - \nu_{i,T}^2\nu_{TT})]\). Here, the plot of primary instability region in

<table>
<thead>
<tr>
<th>Mode ((m, n))</th>
<th>Isotropic ([\omega_{mn} = \omega_{mn}h(\rho/GL^2)])</th>
<th>Orthotropic ([\omega_{mn}^* = \omega_{mn}h(\rho/C_{11})^{1/2}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode ((m, n))</td>
<td>Present Ref. [18]</td>
<td>Present Ref. [19]</td>
</tr>
<tr>
<td>(1, 1)</td>
<td>0.09303 0.0932</td>
<td>0.0474 0.0474</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>0.22201 0.0226</td>
<td>0.10325 0.1033</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>0.34074 0.3421</td>
<td>0.11880 0.1188</td>
</tr>
</tbody>
</table>

Table 1: Comparison of natural frequencies of isotropic and orthotropic square plates \( (a/h = 10, \text{ all sides are simply supported, orthotropic material properties of Aragonite crystal, Ref. [19]}) \)


Table 2
Comparison of frequency and buckling load (due to $N_y$ in the $y$ direction) of anti-symmetric cross-ply square plates (all sides are simply supported, material properties: $E_L/E_T = 40.0$, $G_{LT}/E_T = 0.6$, $G_{TT}/E_T = 0.5$, $\nu_{LT} = 0.25$)

<table>
<thead>
<tr>
<th>$a/h$</th>
<th>No. of layers</th>
<th>Fundamental frequency $[\Omega^* = (\alpha a^2/h)(\rho E_T)^{1/2}]$</th>
<th>Critical buckling load $[N_{cr}^* = (N_y a^2/(E_T h))^ {1/2}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FSDT</td>
<td>Present</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>10.473</td>
<td>10.568</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>15.779</td>
<td>15.771</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

$^*$FSDT using eight-node C$^0$ serendipity element.

Table 3
Comparison of natural frequency of plate with initial compressive load $N_x(0.2N_{cr}^*)$ and buckling load of plates with elastic foundation ($K_2 = 0$) (square isotropic plate ($a/h = 100$) with all sides simply supported)

<table>
<thead>
<tr>
<th>$K_1$</th>
<th>$\Omega^{**} = (\alpha a^2)(\rho h D_{11})^{1/2}$</th>
<th>$\lambda_{cr} = (N_y a^2)/(D_{11} \pi^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>27.2557</td>
<td>27.2584</td>
</tr>
<tr>
<td>2500</td>
<td>51.4088</td>
<td>51.4072</td>
</tr>
</tbody>
</table>

the neighbourhood of $2\omega_1$ in terms of non-dimensional excitation frequencies, $\Omega = \bar{\omega}(\rho a^2 h)/(D_{11})$ versus the dynamic in-plane load $\beta$ are depicted. The width of primary instability $\Delta\bar{\omega}$ is the separation of the boundaries of the primary instability region for the given plate. This can be used as an instability measure to study the influence of other parameters.

The variations of dynamic instability region with respect to amplitude of the load are seen in Figs. 2 and 3 for isotropic and single-layered orthotropic plates, respectively. The effect of foundation stiffness is to enhance the natural frequencies and buckling load. It is observed from Fig. 2 that the origin of primary instability region shifts to higher excitation frequencies with the increase in the value of Winkler foundation stiffness. The effect of two-parameter foundation (Pasternak) is to enhance the dynamic instability region and shifts the occurrence of primary instability zone to higher excitation frequencies, compared to those of Winkler foundation case. It is further noticed that, for the given non-dimensional dynamic load, the dynamic instability region increases with an increase in the value of foundation stiffness. Although the dynamic instability behaviour of the orthotropic plate is quite similar to that of the isotropic one, as brought out in Fig. 3, dynamic stability strength for the given dynamic load parameter, is more

Fig. 2. Instability regions versus dynamic in-plane load $\beta$ for isotropic plate with different elastic foundation parameter, ($a/b = 1$, $a/h = 100$, $\alpha = 0$).
for orthotropic case than that of isotropic plates. However, the introduction of orthotropicity i.e. increase in $E_1/E_T$, is to set the instability at lower frequencies in comparison with those of isotropic case. Also, it is revealed from Fig. 3 that the instability region corresponding to the vicinity of simple resonance of first order associated with higher frequencies, $2\omega_1$, overlap with an increase in the value of foundation stiffness. Hence, the plate is unstable in large intervals of frequency as the amplitude of the load increases. It can be concluded from Figs. 2 and 3 that the effect of foundation stiffness on dynamic instability region, in general, to postpone the occurrence of instability to higher frequencies, $2\omega_1$, irrespective of orthotropicity. Also, it can be inferred from these figures that the system resting on the two-parameter foundation is stable for a wide range of frequencies for the given load while the operating frequencies are in between the vicinity of $2\omega_1$ and $2\omega_2$.

A similar study was carried out for laminated composite plates. The laminates considered for such analysis ($a/b = 1$, $a/h = 100$) are cross-ply ($0^\circ/90^\circ$) and five different angle-ply ($\pm 15^\circ$, $\pm 30^\circ$, $\pm 45^\circ$, $\pm 60^\circ$ and $\pm 75^\circ$) plates. The dynamic stability characteristics are described in Figs. 4 and 5. It is opined from Figs. 3 and 4 that the cross-ply laminates are susceptible to dynamic instability at a lower value of forcing frequency in comparison with those of single-layered orthotropic plates. One can also draw the conclusion that, for the chosen dynamic load, the dynamic stability strength in the neighbourhood of simple resonance of first order, $2\omega_1$ is more for cross-ply laminates compared to those of the orthotropic case. Furthermore, for a higher load, the cross-ply plate is unstable for a wide range of excitation frequencies, due to the overlapping of different instability regions associated with higher frequencies. Fig. 5 shows the effect of angle-ply on the dynamic instability behaviour of the plate. It is shown from Fig. 5 that the variation in the frequency, among the angle-ply cases considered here for on-set off dynamic instability, is much less and the instability region is clustered for the case of excitation frequency in the vicinity of $2\omega_1$ compared to the case of higher frequency. From the instability region point of view, among all the angle-ply cases chosen for the present study, the laminate with $-15^\circ/15^\circ$ is more vulnerable when the excitation fre-
frequency is around $2\omega_1$, whereas the $-60^\circ/60^\circ$ case is significant for the value of frequency, $2\omega_2$. The origin of instability zone and its strength, in general, highly depends on ply-angle and foundation stiffness. The effect of foundation stiffness is to cluster the different instability regions and make the system unstable for a wide range of frequencies, even though the origin of instability is shifted to higher values. It can also be concluded from Figs. 4 and 5 that the origins of dynamic instability regions in the neighbourhood of $2\omega_1$ pertaining to angle-ply cases, considered here, occur at higher values of frequency than those of the cross-ply case.

The influence of thickness and static load factor on the stability boundaries of a square laminate are investigated by considering cross-ply as well as angle-ply laminates ($0^\circ/90^\circ$, $45^\circ/-45^\circ$, $a/b = 1$, $a/h = 10$, $\alpha = 0$, 0.2) and highlighted in Fig. 6. It is inferred from Figs. 4–6 that an increase in the value of the thickness of the laminates results in an increase in the dynamic stability strength. Fig. 7 describes the effect of aspect ratio of the plate with foundation. With an increase in the value of the aspect ratio, the dynamic instability region is shifted further to higher frequencies with the inclusion of foundation and an increase in the instability width can be seen for the given dynamic load.
6. Conclusions

A parametric instability study of laminated plates on elastic foundations, subjected to periodic in-plane loads, is examined by considering an eight-noded plate element based on shear flexible theory. Numerical results have been obtained for isotropic/orthotropic/laminated plates. From the detailed study, the following observations can be made:

1. The origin of the primary dynamic instability region shifts to higher excitation frequencies with the increase in the value of foundation stiffness of the plates, irrespective of orthotropicity, ply-angles and types of foundation.
2. The influence of the shear part of the stiffness parameter pertaining to the two-parameter foundation is to increase the instability region, for the given dynamic load.
3. The increase in the value of orthotropicity is to enhance the dynamic stability strength and shift the occurrence of instability to lower values of excitation frequencies, compared to those of the isotropic case.
4. With the introduction of foundation stiffness, the dynamic stability strength is less for excitation frequencies around the vicinity of higher resonance frequencies of the system i.e. the width of the instability region increases for the given non-dimensional dynamic load factor.
5. The dynamic stability strength as well as the value of frequency corresponding to the initiation of instability zone is high for the angle-ply laminate in comparison with those of cross-ply case.
6. In general, the stability strength and occurrence of the instability region for the given laminate depends on ply-angles and foundation stiffness.
7. The effect of thickness and aspect ratio is to increase the origin of the occurrence of dynamic instability to higher frequencies.

References