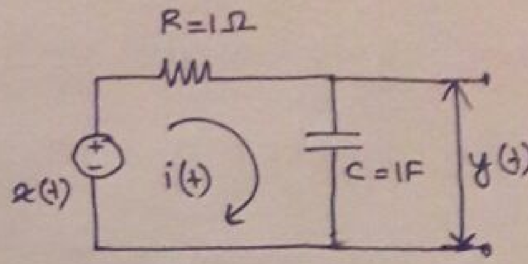
Q.2.61.b) Consider current $i(t)$ flowing through the circuit.



Applying KVL we get,

$$x(t) = Ri(t) + \frac{1}{C}\int i\,dt.$$

Differentiating both sides with respect to t,

$$\frac{dx(t)}{dt} = R\frac{di(t)}{dt} + \frac{1}{C}i(t) \quad — (1)$$

Natural response of the system is obtained by setting $x(t)=0$ i.e. no input given to the system.

$$\therefore \frac{dx(t)}{dt} = 0$$

$$\Rightarrow R\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0$$

$$\Rightarrow R\frac{di(t)}{dt} = -\frac{1}{C}i(t)$$

$$\Rightarrow \frac{di(t)}{dt} = -\frac{1}{RC}i(t) \quad — (2)$$

Integrating both sides,

$$\Rightarrow i(t) = e^{-1/RC\,t}$$

$$\therefore Ke^{-at} = e^{-1/RC\cdot t}$$

$$\therefore K=1 \text{ and } a = \frac{1}{RC} = \frac{1}{1.1} \quad [\because R=1; C=1F] \Rightarrow a=1.$$

Q.17i) $y[n] = \text{Real}\{x[n]\}$

Suppose $y = a + j\omega$ $\quad \therefore \text{Real}[y] = a$

For input $x_1[n]$, output $y_1[n] = \text{Real}\{x_1[n]\}$

For input $x_2[n]$, output $y_2[n] = \text{Real}\{x_2[n]\}$

For input $x_1[n] + x_2[n]$,

$$y[n] = \text{Real}\{x_1[n] + x_2[n]\}$$
$$= \text{Real}\{x_1[n]\} + \text{Real}\{x_2[n]\}$$
$$= y_1[n] + y_2[n]$$

$\therefore$ The system is additive.

For input $kx[n]$, the output

$$y_k[n] = \text{Real}\{kx[n]\} = k\,\text{Real}\{x[n]\} = ky[n]$$

$\therefore$ The system is homogeneous.

Q.17 ii) $y(t) = \frac{1}{x(t)} \left[\frac{dx(t)}{dt} \right]^2$

For input $x_1(t)$, the output $y_1(t) = \frac{1}{x_1(t)} \left[\frac{dx_1(t)}{dt} \right]^2$

For input $x_2(t)$, the output $y_2(t) = \frac{1}{x_2(t)} \left[\frac{dx_2(t)}{dt} \right]^2$

For input $x_1(t) + x_2(t)$, the output,

$$y(t) = \frac{1}{x_1(t) + x_2(t)} \left[\frac{d}{dt} (x_1(t) + x_2(t)) \right]^2$$

$$\neq y_1(t) + y_2(t)$$

$\therefore$ The system is not additive.

For input $kx(t)$, the output

$$y_n(t) = \frac{1}{kx(t)} \left[\frac{d(kx(t))}{dt} \right]^2$$

$$= k \cdot \frac{1}{x(t)} \left[\frac{dx(t)}{dt} \right]^2$$

$$= k\, y(t)$$

$\therefore$ The system is homogeneous.

Q.17. iii) $y[n] = \dfrac{x[n]\,x[n-2]}{x[n-1]}$ $\quad x[n-1] \neq 0$

$= 0 \quad x[n-1] = 0$

For input $x_1[n]$, output $y_1[n] = \dfrac{x_1[n]\,x_1[n-2]}{x_1[n-1]}$

For input $x_2[n]$, output $y_2[n] = \dfrac{x_2[n]\,x_2[n-2]}{x_2[n-1]}$

For input $x_1[n] + x_2[n]$, output

$$y[n] = \frac{(x_1[n] + x_2[n])\,(x_1[n-2] + x_2[n-2])}{(x_1[n-1] + x_2[n-1])}$$

$$\neq y_1[n] + y_2[n]$$

$\therefore$ The system is not additive.

For input $kx[n]$, the output

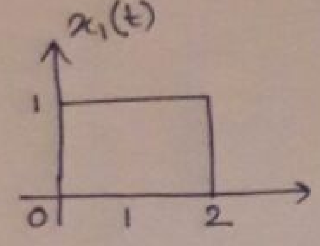
$$y_k[n] = \frac{kx[n]\,.\,kx[n-2]}{kx[n-1]}$$

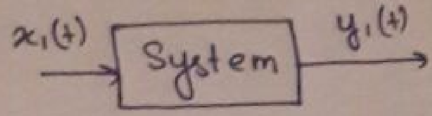
$$= k.\left[\frac{x[n]\,x[n-2]}{x[n-1]}\right]$$

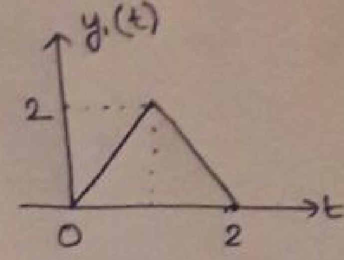
$$= ky[n]$$

$\therefore$ The system is homogeneous.

Q.20. a)

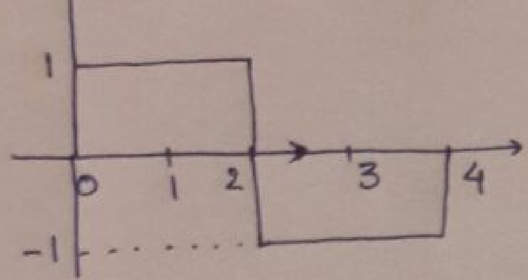


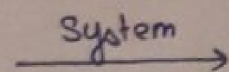


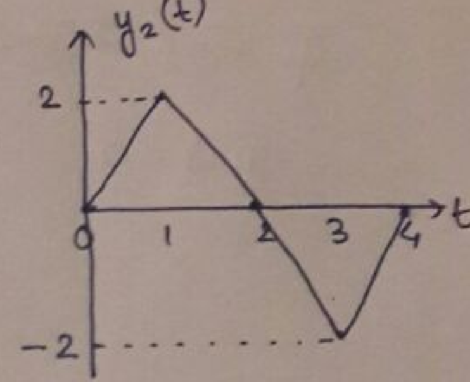


Input

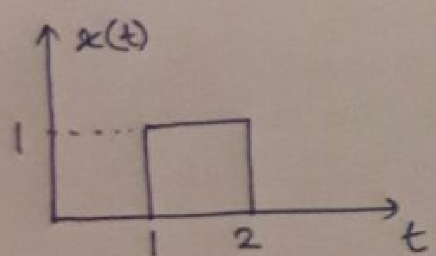
$x_2(t) = x_1(t) - x_1(t-2)$







b) If input $x(t) = u(t)$ then $y(t) = e^{-t}u(t) + u(-1-t)$

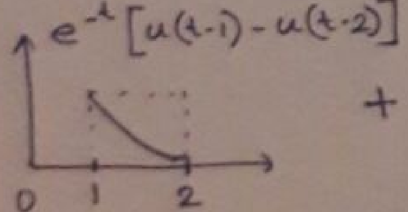


$\therefore \; x(t) = u(t-1) - u(t-2)$

For input $x(t) = u(t-1) - u(t-2)$

output $y(t) = e^{-t}[u(t-1) - u(t-2)] + [u(-1-(t-1)) - u(-1-(t-2))]$

$= e^{-t}[u(t-1) - u(t-2)] + [u(-t) - u(-t+1)]$



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