Assignment 2. ELL 700

Q. 1 Given $A$, $B$, $H$ matrices s.t.

$$\dot{x} = Ax + Bu$$
$$y = Hx$$

2

(i) Write a program (function) to find the largest Controlled-invariant Subspace $\mathcal{V}^* \subseteq \text{Ker } H$.

pseudocode:

$\mathcal{V}_0 := \text{Ker } H$

for $i = 0, 1, \ldots$

$\mathcal{V}_{i+1} = A^{-1}(\mathcal{V}_i + \text{im} B) \cap \text{Ker } H$.

Stop if $\mathcal{V}_{i+1} = \mathcal{V}_i$

Set $\mathcal{V}^* = \mathcal{V}_{i+1}$

(ii) Write a function that takes $\mathcal{V}^*$ as input and returns a Friend of $\mathcal{V}^*$ i.e a matrix $F$ s.t $(A + BF)\mathcal{V}^* \subset \mathcal{V}^*$

Q. 2 Now consider

$$\dot{x} = Ax + Bu + Ed$$

$$y = Hx$$

3

Given, A, B, H, E write a function that returns a feedback matrix F that solves the Disturbance decoupling problem. (DDP)

If DDP is not solvable the function must return a message saying DDP not solvable.

[Hint: This essentially requires you to ~~solve~~ check if $\operatorname{im} E \subset \mathcal{V}^*$ ].

Q. 3 [Bonus] (no marks!)

Let the largest ~~Find~~ Controllability subspace be $\mathcal{R}^* \subset \ker H$.

~~&~~ Write a program that computes basis for $\mathcal{R}^*$ given ~~basis for~~ A, B, H?