ELL 700 Linear Systems Theory.

Minor - 1	25%.
Minor - 2	25%.
Assignments/projects	20%.
Major	30%.

Books:

(i) Control Theory for Linear Systems, Harry L. Trentelman, A. A. Stoorvogel, M. Hautus

(ii) Linear Multivariable Control - A Geometric approach, Murray Wonham

(iii) Linear Systems, Thomas Kailath

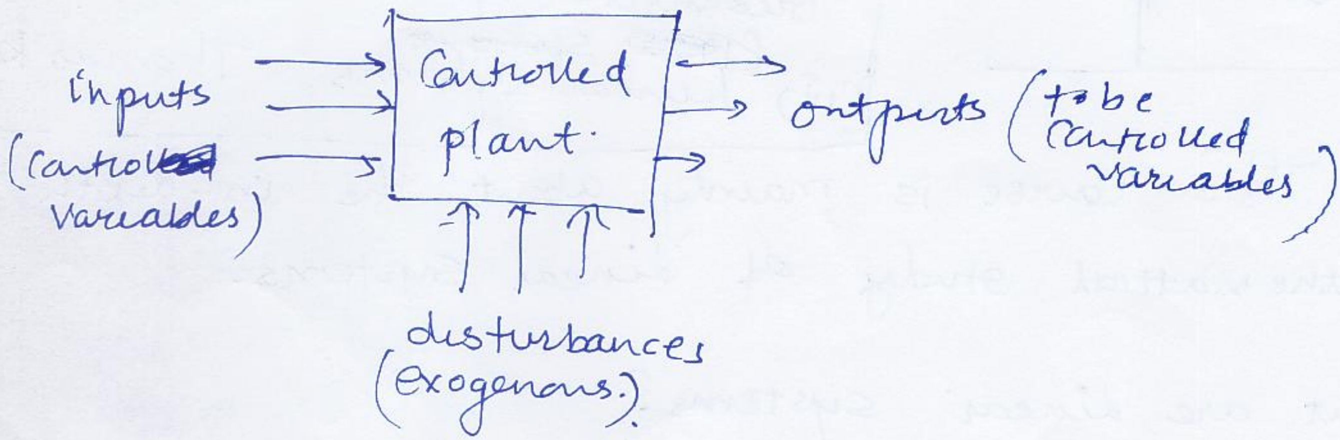
This Course is mainly about the in-depth Mathematical study of linear systems.

What are linear systems?

- System.

Typically a Control System Consists of a physical object or Collection of objects which are assembled to perform a desired task. There are design aspects & modelling aspects.

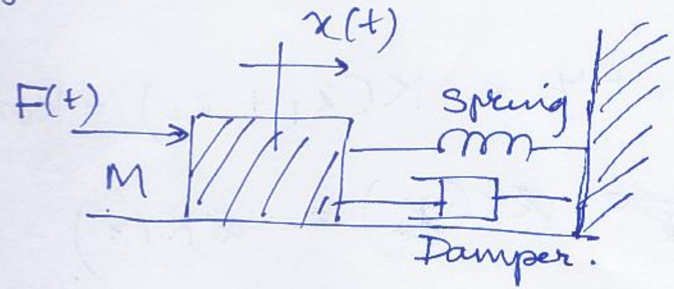
Usually first principle throws up a model with following structure familiar to us from Control Engineering Course.



Controlled plant is a model of a system which when provided with an input gives output.

Physical laws gives us some representation of this plant to be used to design inputs for a particular expectation of output.

For e.g:



Spring $k(x)$

Damper. $d(\dot{x})$

$$M\ddot{x}(t) + K(x(t)) + d(\dot{x}(t)) = F(t)$$

Usually we assume small displacements and over a range $K(x)$ is a <u>linear function</u>(?) $d(\dot{x})$ is a linear function. and we end up with.

$$M\ddot{x} + Kx + d\dot{x} = F$$

What do we mean when I say

$K(x)$ is a linear function.

Say.

$K(x) = y$

Let x_1 give $y_1 = K(x_1)$

and x_2 give $y_2 = K(x_2)$

then

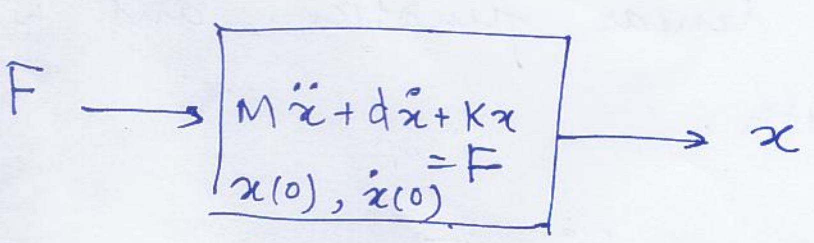
$x_1 + x_2$ gives $y_1 + y_2 = K(x_1 + x_2) = K(x_1) + K(x_2)$

and αx gives $\alpha y = \alpha K(x) = \alpha K(x) = \alpha y$.

This is linearity (loosely speaking!)
(linear system)

Is ~~What does~~ system $M\ddot{x}(t) + d\dot{x}(t) + Kx(t) = F(t)$

linear ?



Solve this system of equation for

$F(t) =$ [illegible] unit step function.

$$U(s) \longrightarrow \boxed{\dfrac{1}{s^2 M + ds + K}} \longrightarrow \sout{X(s)}\ Y(s)$$

The "so-called "transfer function" by using Laplace transforms.

or

$$u(t) \longrightarrow \boxed{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -K/M & -d/M\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1/M\end{bmatrix} u \qquad y = x_1} \longrightarrow y(t)$$

$\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ State-variables

$y \rightarrow$ Output

$u \rightarrow$ Control input.

the so-called State space system.

We will be studying these two systems

$$\dot{x}(t) = A x(t) + B u(t)$$

$$y(t) = C x(t) + D u(t)$$

$$x(0) = x_0.$$

$$G(s) = C(sI - A)^{-1} B + D.$$

"Zero initial conditions."

$$x_{k+1} = A x_k + B u_k$$

~~$y_{k+1} = C x_{k+1} + D u_k$~~

Time invariant system. (parameters, A, B, C, D are not time varying)

No explicit time dependence.

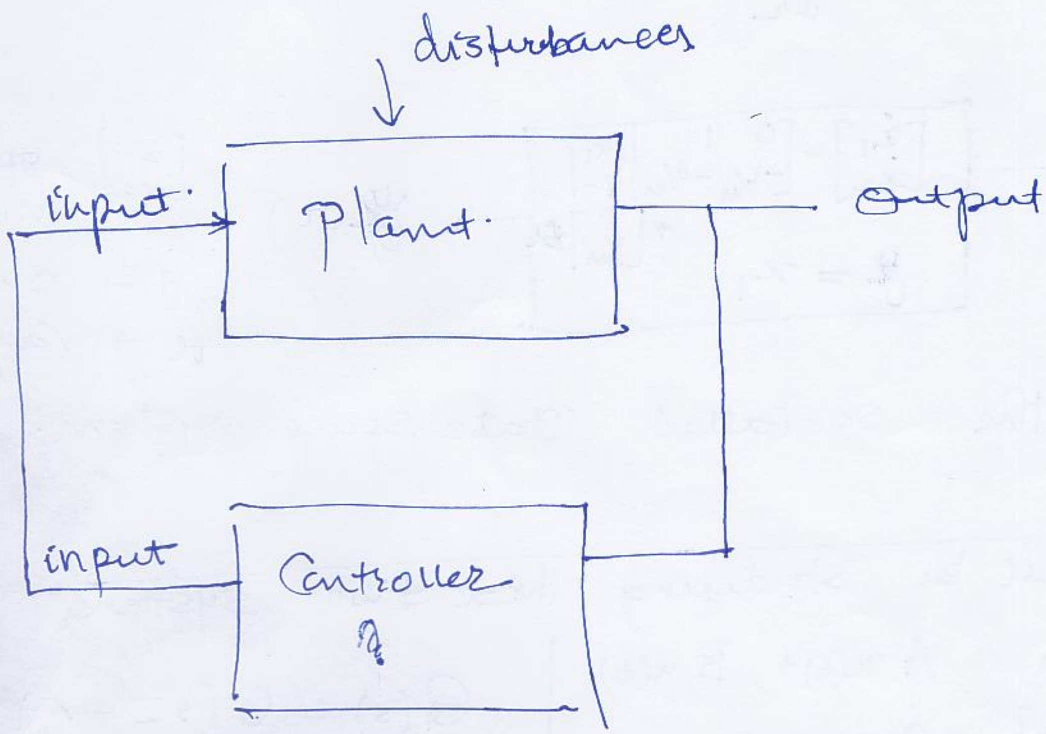
Control problem

~~Design~~ Effect of u ~~for~~ on ~~making~~ y ~~behave~~ in a ~~desired manner~~.

Observation problem:

From output y measurement & input u infer internal states x.

~~Then~~ Feedback Control problem.



Optimal Control problem.

LQR.

Designing a Controller minimizing a certain cost criterion and performing as per requirements.