Q. 1 Dead beat System. (Example from Trentelman)

$x(t+1) = A\,x(t)$ is dead-beat

if for any $x_0 \in \mathbb{R}^n$, $x(t) = 0$ for $t \geq n$

where n is the dimension of state-space.

Can we choose a feedback input $u = Fx$

s.t $x(t+1) = A\,x(t) + B\,u(t)$ becomes

a dead beat system?

Hint: Find F s.t $(A+BF)$ is

nilpotent.

Q. 2 Show that if (A, B) is controllable then $(A + BF, B)$ is also controllable.

Q. 3 $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & +11 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

Design an observer L and State feedback gain K s.t. the overall observer based state-feedback system is stable.

Also draw the block diagram of overall Observer-Controller configuration.

Q. 4 State whether following pair of matrices are (i) Controllable (ii) Stabilizable (iii) uncontrollable & not stabilizable

a) $A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ c) $A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

State, if following are
Q. 3 ~~that~~ a controlled invariant subspaces
for $\dot{x} = Ax + Bu$, $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(i) $\mathcal{V} = \text{span}\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ (iv) $\mathcal{V} = \text{span}\left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$.

(ii) $\mathcal{V} = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ (v) $\mathcal{V} = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

(iii) $\mathcal{V} = \text{Span}\left\{ \begin{bmatrix} -1 \\ +1 \end{bmatrix} \right\}$.

Also find friend for each of them if they space is controlled invariant.

Q. 4 For each controlled-invariant spaces given above ~~list~~ give one example ~~each~~ of E and H matrix s.t DDP is solvable.

Q. 5. Given $\dot{x} = Ax + Bu$, $J = \int_0^{\infty} u^T u \, dt$

Show that $u = B^T P^{-1} x$ minimizes J.

where $A^T P + PA - BB^T = 0$.