Q.1 Is it ~~always~~ possible to decompose "any" Matrix in lower and upper triangular matrices ~~?~~ i.e.,
given any $A \in \mathbb{R}^{n\times n}$, Does there exist $L \in \mathbb{R}^{n\times n}$ (lower triangular) and $U \in \mathbb{R}^{n\times n}$ (upper triangular) s.t $A = LU$?
If yes prove it or else give a counter example.

Q.2 If answer to above question is no, then ~~ca~~ is it possible to make adjustments to A s.t. L, U exists?

Q.3 For $A \in \mathbb{R}^{3\times 3}$, show that if leading 2×2 principle minor is zero then ~~pivoting~~ ~~will~~ row interchanges are required.

Q.4 $A = \begin{bmatrix} 2 & 1 & -1 \\ +1 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ Is it possible to find Cholesky factor? i.e Does there exist $R \in \mathbb{R}^{n\times n}$ (~~lower~~ upper triangular) s.t. $A = R^T R$.

Q.5 Is matrix A in Q.5 positive definite? Why?

Q. ~~6~~ Check if $A = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 2 \end{bmatrix}$ is positive definite.? obtain its Cholesky decomposition.

Q. 7 Show that for an orthogonal matrix $Q \in \mathbb{R}^{n \times n}$, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$.

(i) $\langle x, y \rangle = \langle z, w \rangle$ where $z = Qx$ and. $w = Qy$

(ii) $\|z\|_2 = \|x\|_2$

(iii) $\|z\|_p = \|x\|_p$ is true or false? why?

Q. 8 given $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, find an orthogonal matrix Q s.t $y = Qx = \begin{bmatrix} y_1 \\ 0 \end{bmatrix}$? what is the value of y_1 ?

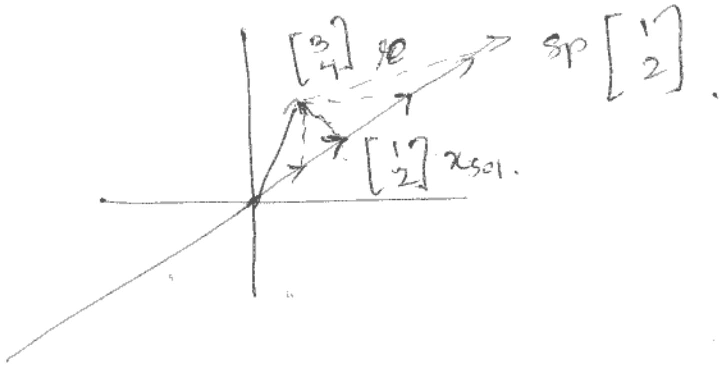
Q. 9 obtain QR decomposition for $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$?

Q. 10 Solve $Ax = b$ for $b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Does the solution ~~pos~~ exist?. If not find its least square solution. How many least square solutions are possible? Compute least norm solution if there are many.?

Q.11 Solve $\begin{bmatrix}1\\2\end{bmatrix}x = \begin{bmatrix}3\\4\end{bmatrix}$. ~~for least squares~~

~~solution~~. Solution will not exist. find least square solution.



Let x_{sol} be the least squares soln.

plot. the Residual $r = \begin{bmatrix}1\\2\end{bmatrix}x_{sol} - \begin{bmatrix}3\\4\end{bmatrix}$ in

~~a~~ $\mathbb{R}^2$. Plot residual for any other $x \in \mathbb{R}^2$ which is not equal to x_{sol}.

Compare lengths of both residuals.

Q.12 Obtain QR decomposition of

$A = \begin{bmatrix}1\\2\end{bmatrix}$. By using QR decomposition

compute least squares solution of

~~so~~ $Ax = \begin{bmatrix}3\\4\end{bmatrix}$

Q.13 $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$. Obtain QR decomposition.

Solve $Ax = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. Obtain least norm solution?

Q.14 Compute SVD of $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Q.15 Given $x = c_1^2 r_1^2 + c_2^2 r_2^2 + \cdots + c_n^2 r_n^2$

and $c_1^2 + c_2^2 + \cdots + c_n^2 = 1$.

show that $x \leq [\max\{r_1, \ldots, r_n\}]^2$

Using this and SVD of A prove that $\|A\|_2 = \sigma_1$.

Q.16 Given $A \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Compute nullspace of A and Image of A.

Q.17 Find a matrix B with rank 1 which is closest to A in induced two norm?

Q.18 Find a full rank matrix B near A. What is a distance between B and A in induced two norm

Q.19 Compute Pseudoinverse of matrix A in Q. 16. Compute least squares solution to $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ using this pseudoinverse.