

Q.1 $A = \begin{bmatrix} \sqrt{1} & 0 & 0 & 0 \\ e & 2 & 0 & -1 \\ \sqrt{2} & -3 & 2 & 2 \\ \sqrt{101} & 0 & -1 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$K_n(A, b)$ is Krylov Subspace of order n .

a) Generate $K_n(A, b)$ for $n=1, 2, 3, \dots$

b) Show that $K_1(A, b) \subseteq K_2(A, b) \subseteq K_3(A, b) \subseteq \dots$

c) Find $\eta := \min \{ n \mid K_n(A, b) = K_{n+1}(A, b) \}$

d) Find Orthonormal basis for $K_n(A, b)$

e) Express A in this Orthonormal bases. If needed use completion of basis.

Q.2 $A = \begin{bmatrix} 3 & -1 \\ 0 & 3 \end{bmatrix}$

a) Is A normal / simple?

by Find a matrix B s.t $\|A - B\|_2 \approx 0.001$

and B is diagonalizable.

Q.3 Let $A = \begin{bmatrix} 2 & -0.5 \\ -0.5 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Solve $Ax = b$ using

- a) Jacobi iteration
- b) Gauss Seidel iteration
- c) Steepest descent method
- d) Conjugate gradient method

Q.5 Verify if following directions are Conjugate w.r.t matrix

$$a) A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad b) A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$c) A_3 = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Directions: (i) $p_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad p_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(ii) $p_1 = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}, \quad p_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(iii) $p_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad p_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(iv) $p_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad p_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Q.6. Give ~~an~~ examples of ~~a~~ matrices for which power iteration converges in one iteration.

Q.7 Consider QR-iteration. ~~Show that~~ ^{with} $T_0 := A$ for $k = 1, 2, \dots$

$$T_{k-1} = Q_k R_k$$

$$T_k := R_k Q_k$$

Show that T_k is similar to A .