Q.1 $$T = \left[\begin{array}{c|cc} 1 & 1 & 2 \\ \hline 0 & 1 & 0 \\ 0 & 2 & 2 \end{array}\right] = \left[\begin{array}{c|c} T_{11} & T_{12} \\ \hline 0 & T_{22} \end{array}\right]$$

* Construct $\phi(X) := T_{11} X - X T_{22}$
* Check if $\phi : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ is non-singular
* Is it possible to construct Y s.t $Y^{-1} T Y = \left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 2 \end{array}\right]$? If yes then find such Y.

* Are there multiple such Y.? Construct them all in parametric form.

Q.2 $$A = \begin{bmatrix} 1 & 100 & 200 \\ 5 & 2 & 0.02 \\ 0.01 & 0.05 & 4 \end{bmatrix}$$

Use Gerschgorin's Disk theorem to estimate an eigenvalue of A. Also give a bound on estimated eigenvalue.

Q.3 Given $\|F\|_p < 1$, Show that $I - F$ is non-singular matrix. Also show that $\|(I-F)^{-1}\|_p \leq \frac{1}{1 - \|F\|_p}$ and $(I-F)^{-1} = \sum_{k=0}^{\infty} F^k$.

Q.4 Given $A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.001 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & \sqrt{3}/2 & 0 \end{bmatrix}$

(i) Determine $\|A\|_2$, $\|A\|_F$, $K_2(A)$, $\lambda(A^TA)$, $\lambda(AA^T)$, Eigenvectors of A^TA, and AA^T, ~~Range~~ Columnspace of A, Colspace(A^T), Nullspace(A), Nullspace(A^T).

(ii) Determine numerical rank if tol = 0.005. In which case also determine Nullspace(A), Nullspace(A^T).

Q.5 Given $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, Draw im(A) for all points $x \in \mathbb{R}^3$ s.t $\|x\|_\infty \leq 1$.

Q.6 Given $p(s) = s^2 + 3s + 1$. Compute its roots using eigenvalues of an appropriate matrix constructed using p(s).

Q.7 Given $A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(i) Determine exact rank of matrix A.

(ii) Find B s.t Rank(A) < Rank(B)

(iii) Find B s.t Rank(B) < Rank(A) and $\|A - B\|_2$ is minimized.

(iv) Given $b = [1\ 0\ 0]^T$. Find least square soln to $Ax = b$.