Department of Mathematics
MTL 725(Stochastic Processes and Applications))
Tutorial Sheet No. 3

1. A cable car starts off with n riders. The times between successive stops of the car are independent exponential random variables, each with rate λ. At each stop, one rider gets off. This takes no time and no additional riders get on. Let $X(t)$ denote the number of riders present in car at time t. Write down the Kolmogorov forward equations for the process $\{X(t), t \geq 0\}$. Find the mean and variance of the number of riders present in car at any time t.

2. A service center consists of two servers, each working at an exponential rate of two services per hour. If customers arrive at a Poisson rate of three per hour, then, assuming a system capacity of at most three customers,

   (a) what fraction of potential customers enter the system?

   (b) what would the value of part (a) be if there was only a single server, and his rate was twice as fast (that is, $\mu = 4$)?

3. Consider an automobile emission inspection station with three inspection stalls, each with room for only one car. It is reasonable to assume that cars wait in such a way that when a stall becomes vacant, the car at the head of the line pulls up to it. The station can accommodate at most four cars waiting (seven in the station) at one time. The arrival pattern is Poisson with a mean of one car every minute during the peak periods. The service time is exponential with mean 6 min. The chief inspector wishes to know the average number in the system during peak periods and the average wait (including service).

4. In a parking lot with N spaces the incoming traffic is according to Poisson process with rate λ, but only as long as empty spaces are available. The occupancy times have an exponential distribution with mean $1/\mu$. Let $X(t)$ be the number of occupied parking spaces at time t.

   (a) Determine rate matrix Λ and the forward Kolmogorov equations for the Markov process $\{X(t), t \geq 0\}$.

   (b) Determine the limiting equilibrium probability distribution of the process.

5. Consider an $M/M/1$ queue where the mean service rate depends on the state of the system. Suppose the server has two rates, the slow rate μ_1, and the fast rate μ_2. The server works at the slow rate till there are m customers in the system, after which it switches over to the fast rate.

   (a) Draw the state transition diagram for the system.

   (b) Find the steady state probabilities for the system.

6. Users arrive at IIT Central Library according to a Poisson process at a rate of 50 per hour. Each user in the library for an average of 30 minutes. Assume that the time spent by a user in the library is exponentially distributed and independent of the other users. How many users are there in the library on average in steady state?

7. Components arrive at a repair facility with a constant rate λ, and the service time is exponentially distributed with mean $1/\mu$. The last step in the repair process is a quality-control inspection, and with probability p, the repair is considered inadequate, in which case the component will go back into the queue for repeated service. Determine the steady state probability mass function of the number of components at the repair facility. Also, find the average response time to a repair request in steady state?

8. Patients visit a doctor in accordance with a Poisson process at the rate of 8 per hour, and the time doctor takes to examine any patient is exponential with mean 6 minutes. All arriving patients attended by the doctor.

   (a) Find the probability that the patient has to wait on arrival.

   (b) Find the expected total time spent (including the service time) by any patient who visits the doctor.