Department of Mathematics
MTL 725(Stochastic Processes and Applications))
Answers to Tutorial Sheet No. 3

1. P(No. of riders in the bus at time $t = k$)$={}^nC_k(e^{-\lambda t})^k(1-e^{-\lambda t})^{n-k}, \quad k = 0, 1, \ldots, n.$
Hence, mean and variance of number of riders at time t is : $ne^{-\lambda t}$ and $ne^{-\lambda t}(1-e^{-\lambda t})$ respectively.

2. $M/M/2/3$ queueing model with $\lambda = 3$ per hour and $\mu = 2$ per hour.

(a) $(1-\pi_3)$

(b) $(1-\pi_3)$ for $M/M/1/3$ queueing model with $\lambda = 3$ per hour and $\mu = 4$ per hour.

3. $M/M/3/7$ queueing model with $\lambda = 1$ per minute and $\mu = \frac{1}{6}$ per minute.

(a) $\sum_{k=1}^{7} k\pi_k$

(b) $\frac{L}{\lambda_{eff}} = \frac{L}{\lambda(1-\pi_7)}$

4. $\pi_0 = \left[\sum_{i=0}^{N} \frac{1}{i!}\frac{\lambda}{\mu}^i\right]^{-1}$, $\pi_k = \frac{1}{k!}\frac{\lambda}{\mu}^k[\sum_{i=0}^{N} \frac{1}{i!}\frac{\lambda}{\mu}^i]^{-1}$

5. (b) $\pi_0 = \left[\frac{1-(\lambda/\mu_1)^m}{1-(\lambda/\mu_1)} + (\frac{\lambda}{\mu_1})^{m-1}\frac{1}{1-(\lambda/\mu_2)}\right]^{-1}$, which exists if $\frac{\lambda}{\mu_2} < 1$.
Further: $\pi_m = \begin{cases} (\frac{\lambda}{\mu_1})^k\pi_0, & 1 \le k \le m-1 \\ \frac{\lambda^k}{\mu_1^{m-1}\mu_2^{k-m+1}}\pi_0, & k \ge m \end{cases}$

6. $\frac{\lambda}{\mu}$, $M/M/\infty$ model.

7. Birth and death process with $\lambda_i = \lambda,\ i = 0, 1, \ldots$ and $\mu_i = \mu,\ i = 1, 2, \ldots$.

(a) $\pi_n = \left(1 - \frac{\lambda}{\mu(1-p)}\right)\left(\frac{\lambda}{\mu(1-p)}\right)^n, \quad n = 0, 1, \ldots$

(b) Average waiting time $= \frac{1}{\mu(1-p)-\lambda} - \frac{1}{\mu(1-p)}$

8. $M/M/1$ queueing model with $\lambda = 8$ per hour and $\mu = 10$ per hour.

(a) $1 - \pi_0 = \frac{4}{5}$

(b) Average response time $= \frac{1}{\mu-\lambda} = \frac{1}{2}$