Department of Mathematics
MTL 725(Stochastic Processes and Applications))
Tutorial Sheet No. 6

1. For a branching process with offspring distribution given by $p_0 = \frac{1}{6}, p_1 = \frac{1}{2}, p_0 = \frac{1}{3}$, determine

   (a) Expected population at generation 5.

   (b) Probability of extinction.

2. Assume that the offspring distribution of a branching process is Poisson with parameter 1. Determine the expected combined population through generation 6.

3. The first generation of particles is the collection of offsprings of a given particle. The next generation is formed by the offsprings of these members. If the probability that a particle has k offsprings is p_k, where $p_0 = 0.4, p_1 = 0.3, p_2 = 0.3$, find the probability that there is no particle in second generation. Assume that particles act independently and identically irrespectively of the generation.

4. Consider bacteria reproduction by cell division. In any time t, a bacterium will either die(with probability 0.25), stay the same(with probability 0.25), or split into 2 parts(with probability 0.5). Assume they act independently and identically irrespective of time. Write down the expression for the generating function of the distribution of the size of the population at time $t = n$. Given that there are 1000 bacteria in the population at time $t = 50$, what is the expected number of bacteria at time $t = 51$.

5. Consider a branching process, denoted by Galton-Watson process, that model a population in which each individual in generation n produces some random number of individuals in generation $n+1$, according, in the simplest case, to a fixed probability distribution that does not vary from individual to individual. That is, the first generation of individuals is the collection of offsprings of a given individual. The next generation is formed by the offsprings of these individuals. Let X_n denote the number of individuals of the nth generation, starting with $X_0 = 1$ individual(size of zeroth generation). Let Y_i be the number of offspring of the ith individual of the nth generation. Suppose that, $\{Y_i, i = 1, 2, \ldots\}$ are non-negative integer valued i.i.d random variables with probability mass function $p_j = P(Y_i = j), j = 0, 1, \ldots$ and independent of the size of the generation. Then

$$X_n = \sum_{i=1}^{X_{n-1}} Y_i, \; n = 1, 2, \ldots$$

   and $\{X_n, n = 0, 1, \ldots\}$ is a discrete time Markov chain. Classify the states of the chain.

6. Consider a branching process with $p_0 = \frac{1}{4}, p_1 = \frac{1}{3}$ and $p_2 = \frac{5}{12}$. Find the probability of extinction.