Department of Mathematics
MAL 725 (Stochastic Processes and Applications)
Tutorial Sheet No. 1

1. Classify the following stochastic processes, mentioning the state space and the index set. Also, trace a sample path for the stated stochastic processes:

   (i) Number of customers at the airport at the end of each hour of a day.

   (ii) Number of printing jobs at a printer at any time during the day.

   (iii) The cumulative service time requirement of all jobs in a system at time t

   (iv) Waiting time of a customer.

2. Suppose that $\{X_n; n \geq 1\}$ is a sequence of independent and identically distributed random variables. Define a sequence $\{Y_n; n \geq 1\}$ as $Y_n = X_n + aX_{n-1}$ where a is a real constant. Then prove that $\{Y_n; n \geq 1\}$ is a strictly stationary process.

3. Show that an i.i.d. sequence of continuous random variables with common density function f is strictly stationary.

4. Let A be a positive random variable that is independent of a strictly stationary random process $\{X(t), t \geq 0\}$. Show that $Y(t) = AX(t)$ is also strictly stationary random process.

5. Let Z_1 and Z_2 be two independent normally distributed random variables, each having mean 0 and variance σ^2. Let $\lambda \in R$. Define: $X_t = Z_1 \cos \lambda t + Z_2 \sin \lambda t$. Then show that $\{X_t; t \in T\}$ is a second order stationary process.

6. In a communication system, a carrier signal at a receiver is modeled as a stochastic process $\{X(t) = \cos(2\pi f t + \Theta); t \geq 0\}$ where $\Theta \sim U[-\pi, \pi]$ and f is a constant. Find the mean function and correlation function of $X(t)$.

7. Let $\{N(t), t \geq 0\}$ be a Poisson process. Prove or disprove that, the process

$$X(t) = N(t + L) - N(t)$$

   where L is a +ve constant, is covariance or wide-sense stationary.

8. Let X amd Y be iid random variables each having uniform distribution on interval $[-\pi, \pi]$. Let $Z(t) = \sin\ (Xt + Y)\ (t \geq 0)$. Is $\{Z(t),\ t \geq 0\}$ covariance stationary? Justify your answer.

9. Show that every stochastic process $\{X(t), t = 0, 1, 2, \ldots\}$ with independent increments is a Markov process. Is the converse true?

10. Consider an urn containing 50 red balls and 50 blue balls. Balls are drawn one by one without replacement. Let X_n be the number of red balls remaining in the urn after the n^{th} ball is drawn. Then

    (i) does the sequence X_n possess the Markov property ?

    (ii) Compute $P(X_n + 1 = j | X_n = i)$

    (iii) Does the stochastic process X_n have stationary transition probabilities?

11. Let $Y_n = a_0 X_n + a_1 X_{n-1};\ n = 1, 2, \ldots$ where a_0, a_1 are constants and $X_0, X_1, \ldots$ are i.i.d random variables with mean 0 and variance σ^2.

    (a) Is $\{Y_n; n \geq 1\}$ covariance stationary?

    (b) Is it a Markov process?