

Department of Mathematics
MTL 390 (Sampling Distribution)
Tutorial Sheet No. 1
(Answers to Selected Problems)

27. If X denotes weight of the student, then
 $P(X < 2.42) = .1$ and $P(X > 2.5) = .15$
 $X \sim N(\mu, \sigma^2)$
 $\mu = 2.464, \sigma = 0.0346$

32. $Y \sim \chi_{(1)}^2$
 $Z \sim \chi_{(n)}^2$
 $f_Z(z) = \frac{z^{\frac{n}{2}-1} e^{-z/2}}{2^{\frac{n}{2}} \Gamma(\frac{n}{2})}, \text{ for } x \geq 0$

33. $X, Y \sim N(0, 1)$
 $U = X/Y, V = X/|Y|, W = |X|/|Y|$
 $f_{|X|}(x) = \frac{2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, x \geq 0$
 $f_U(u) = \frac{1}{(1+u^2)\pi}, u \in (-\infty, \infty)$
 $f_V(v) = \frac{1}{(1+v^2)\pi}, v \in (-\infty, \infty)$
 $f_W(w) = \frac{2}{(1+w^2)\pi}, w \in (0, \infty)$

35. Pdf of X is

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma x}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}}, x \in (0, \infty)$$

$$E(X) = e^{\mu + \frac{1}{2}\sigma^2}, \text{Var}(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$$

37. Distribution is student's t with degrees of freedom n .

38. Hint: $-2 \log P = \sum -2 \log X_i$

$$Y_i = -2 \log X_i$$

Find $M_{Y_i}(t)$, and then find $M_{\sum Y_i}(t)$ and show it to be equal to M.G.F of $\chi_{(2n)}^2$

39. $\frac{\frac{(X_1 - \mu_1)^2}{\sigma_1^2}}{\frac{(X_2 - \mu_2)^2}{\sigma_2^2}} \sim F(1, 1)$

$$\frac{\sigma_2^2}{\sigma_1^2} Z^2 \sim F(1, 1) = X$$

$$P(Z^2 < z) = P(X < \frac{\sigma_2^2}{\sigma_1^2} z) = F_X(\frac{\sigma_2^2}{\sigma_1^2} z)$$

$$f_Z(z) = \frac{\sigma_2^2}{\sigma_1^2} f_X(\frac{\sigma_2^2}{\sigma_1^2} z), x \in (0, \infty)$$

where $f_X(x)$ denotes pdf of $F(1, 1)$. Substitute the pdf of $F(1, 1)$ to compute the pdf of Z^2 .

45. 0.6681

47. $n \approx 163$