Department of Mathematics
MTL 390 (Sampling Distribution)
Tutorial Sheet No. 3
(Answers for Selected Problems)

1. $E(X) = 5.25, Var(X) = 0.1319$
   Population mean(μ) = 5.25, Population Variance(σ^2) = 1.1875

2. $P(\bar{X} < 75) = .3836$

3. $P(X < 48) = 0.01426$

4. He should keep at-least 12 cars.

5. (i) 0.27134 (ii) 0.05815

6. 0.07636

7. 0.03438

8. (a) 0.19146 (b) 0.07196 (c) 0.6321

9. 0.67448

10. 0.14457

11. $n \geq 15.36$

12.

13.

$$f(y) = \begin{cases} \frac{n^n}{\Gamma(n)} y^{n-1} (-\log y)^{n-1}, & 0 < y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

14. (a) 0.9952
    (b) $P(S > 41.7) = P(\chi^2_{(4)} > 2.76)$

15. (a) sum of independent poisson is poisson with parameter as sum of all parameters, $\sum X_i \sim Poiss(n\lambda)$
    So, pdf of $\bar{X}$ is

$$f(x) = \frac{ne^{-n\lambda}(n\lambda)^{nx}}{(nx)!}, \quad \text{for} \ \ x = 0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots$$

    (b) $\bar{X} \sim Gamma(\frac{nm}{2}, \frac{n}{2})$

16. $E(R) = 0$

17. $E(\bar{X}) = \mu, \ Var(\bar{X}) = \frac{\sigma^2}{n}$
    $E(S^2) = \sigma^2, \ Var(S^2) = \frac{2\sigma^4}{n-1}$

18. Hint: Prove that $\lim_{n\to\infty} \log M_{\frac{X-v}{\sqrt{2v}}}(t) = \frac{t^2}{2}$

19. Hint: $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$

20. (a) $\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$
    (b)(i) $\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sigma^2 \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0, 1)$
    (ii) $\chi^2_{(n_1+n_2-2)}$ prove using moment generating function.