Department of Mathematics
MTL 390 (Sampling Distribution)
Tutorial Sheet No. 3

1. If random samples of size three and drawn without replacement from the population consisting of four numbers $4, 5, 5, 7$. Find sample mean X for each sample and make sampling distribution of X. Calculate the mean and standard deviation of this sampling distribution. Compare your calculations with population parameters.

2. Assume that a school district has 10,000 6th graders. In this district, the average weight of a 6th grader is 80 pounds, with a standard deviation of 20 pounds. Suppose you draw a random sample of 50 students. What is the probability that the average weight of a sampled student will be less than 75 pounds?

3. Find the probability that of the next 120 births, no more than 40% will be boys. Assume equal probabilities for the births of boys and girls. Assume also that the number of births in the population (N) is very large, essentially infinite.

4. Suppose a Taxi Service receives on the average 8 requests per hour. How many car should he keep in order that 90% of the requests are met?

5. A Manufacturer claims that at most 10% of his products are defective. To test this claim 18 units are inspected, and the claim is accepted if at most 2 are defective.

   (i) What is the probability that his claim will be accepted if actually 20% products are defective.

   (ii) What is the probability that his claim will be rejected if actually 5% products are defective.

6. A normal population has a mean of 0.1 and standard deviation of 2.1. Find the probability that mean of a sample of size 900 will be negative?

7. In any lecture course, for boys the average number of absences is 15 with a standard deviation of 7; for girls, the average number of absences is 10 with a standard deviation of 6. In an institute wise survey, suppose 100 boys and 50 girls are sampled taken from normal population. What is the probability that the male sample mean will have at most three more days of absences than the female sample mean?

8. An electronic device has a life length T which is exponentially distributed with parameter $\alpha = 0.001$, that is, its pdf is $f(t) = e^{.0001}e^{-.0001t}$. Suppose that 100 such devices are tested yielding observed values $T_1, T_2, \ldots, T_{100}$.

   (a) What is the probability that $950 < \overline{T} < 1000$?

   (b) What is the probability that the largest observed values exceeds 7200 hours?

   (c) What is the probability that the shortest time to failure is less than 10 hours?

9. Independent samples of size 10 and 15 are taken from a normally distributed random variables with expectation 20 and variance 3. What is the probability that the mean of the two samples differ (in absolute value) by more than 0.3?

10. Independent samples of size $n_1 = 40$ and $n_2 = 50$ are taken from normal populations having the mean $\mu_1 = 68$ and $\mu_2 = 66.6$ and the variances $\sigma_1^2 = 120$ and $\sigma_2^2 = 150$. What is the probability that the mean of the first sample exceed that of the second sample by at least 4?

11. Let X be the live of electric coil used in heater have mean μ and s.d. 120 hours. If n of these coils are put on test till they fail, resulting in observations $X_1, \ldots, X_n$, find the minimum value of n so that the probability that X differs by μ by less than 60 hours is at least 0.95?

12. Let $X_1, X_2, \ldots, X_n$ be a random sample from $\mathcal{N}(\mu, \sigma^2)$. Let $\overline{X}$ be the sample mean and s^2 be the sample variance of $X_1, X_2, \ldots, X_n$. Find the sampling distribution of $\frac{(n-1)s^2}{\sigma^2}$.

13. Suppose that population distribution is an uniform distribution on the interval $(0, 1)$ and $X_1, X_2, \ldots, X_n$ is a random sample from this population. Find the sampling distribution of $\left(\prod_{i=1}^{n} X_i\right)^{1/n}$.

14. Calculate the probability that, for a random sample of 5 values taken from a $N(100, 252)$ population

    (a) $\overline{X}$ will be between 80 and 120

    (b) S will exceed 41.7

15. Let $X_1, X_2, \ldots, X_n$ be a random sample on X. Find sampling distribution of $\overline{X}$ if (a) $X \sim P(\lambda)$ (b) $X \sim \chi^2 - dist.$ with m degrees of freedom.

16. Let $(X_1, X_2, \ldots, X_n)$ be a random sample from uniform distribution on an interval (0,1). Find expected value of range of the sample $R = [max(X_i) - min(X_i)]$.

17. Let $X_1, X_2, \ldots, X_n$ be a random sample from normal population $N(\mu, \sigma^2)$. Find first two central moments of the sample mean $\overline{X}$ and sample variance s^2 .

18. Prove that if X is a random variable having a chi-square distribution with ν degree of freedom and ν is sufficiently large then the distribution of random variable $\frac{X-\nu}{\sqrt{2\nu}}$ can be approximated by the standard normal distribution.

19. Prove that the sampling distribution of S^2, for a random sample of size n from a normal population with variance σ^2, has mean σ^2 and variance $2\sigma^4/(n-1)$.

20. Independent random samples of size n_1 and n_2 are taken from the normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively

    (a) Write down the sampling distributions of $\overline{X_1}$ and $\overline{X_2}$ and hence determine the sampling distribution of $\overline{X_1} - \overline{X_2}$, the difference between the sample means.

    (b) Now assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2$,

        (i) Express the sampling distribution of $\overline{X_1} - \overline{X_2}$ in standard normal form.

        (ii) State the sampling distribution of $\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{\sigma^2}$.

        (iii) Using the $N(0, 1)$ distribution from (a) and the χ^2 distribution from t distribution to find the sampling distribution of $\overline{X_1} - \overline{X_2}$ when σ^2 is unknown.