

Department of Mathematics
MTL 390 (Hypothesis Testing)
Tutorial Sheet No. 5

1. Consider a queueing system describe the number of telephone ongoing calls in the telephone. The mean time of a queueing system is required to be at least 180 seconds. Past experience indicates that the standard deviation of the talk time is 5 seconds. Consider a sample of 10 customers who reported the following talk time

| | | | | | | | | | | |
|-----------|-----|-----|-----|-----|-----|----|-----|-----|-----|-----|
| Talk time | 210 | 195 | 191 | 202 | 152 | 70 | 105 | 175 | 120 | 150 |
|-----------|-----|-----|-----|-----|-----|----|-----|-----|-----|-----|

Would you conclude at the 5% level of significance that the system is unacceptable? What about at the 10% level of significance.

2. In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20 . A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of 0.02 . Suppose 10 independent measurements yielded the following pH values:

| | | | | | | | | | | |
|-----------|------|------|------|------|------|------|------|------|------|------|
| pH values | 8.18 | 8.17 | 8.16 | 8.15 | 8.17 | 8.21 | 8.22 | 8.16 | 8.19 | 8.18 |
|-----------|------|------|------|------|------|------|------|------|------|------|

- (a) What conclusion can be drawn at the $\alpha = 0.10$ level of significance?
 (b) What about at the $\alpha = 0.05$ level of significance?

3. An automobile manufacturer claims that the average mileage of its new two wheeler will be at least 40 km. To verify this claim 15 test runs were conducted independently under identical conditions and the mileage recorded (in km) as:

| | | | | | | | | | | | | | | | |
|---------|------|------|------|------|----|------|----|----|------|------|----|------|------|----|----|
| Mileage | 39.1 | 40.2 | 38.8 | 40.5 | 42 | 45.8 | 39 | 41 | 46.8 | 43.2 | 43 | 38.5 | 42.1 | 44 | 36 |
|---------|------|------|------|------|----|------|----|----|------|------|----|------|------|----|----|

Test the claim of the manufacturer at $\alpha = 0.05$ level of significance.

4. The life of a certain electrical equipment is normally distributed. A random sample of lives of twelve such equipments has a standard deviation of 1.3 years. Test the hypothesis that the standard deviation is more than 1.2 years at 10% level of significance.
5. Random samples of the yields from the usage of two different brands of fertilizers produced the following results : $n_1 = 10, X = 90.13, s_1^2 = 4.02; n_2 = 10, Y = 92.70, s_2^2 = 3.98$. Test at 1% and 5% level of significance whether the difference between the two sample means is significant.
6. Consider the strength of a synthetic fibre that is possible affected by the percentage of cotton in the fibre. Five levels of this percentage are considered with five observations at each level. The data is shown in the Table 1.

Table 1: Strength of a synthetic fibre for Exercise 6

| | | | | | |
|----|----|----|----|----|----|
| 15 | 7 | 7 | 15 | 11 | 9 |
| 20 | 12 | 17 | 12 | 18 | 18 |
| 25 | 14 | 18 | 18 | 19 | 19 |
| 30 | 19 | 25 | 22 | 19 | 23 |
| 35 | 7 | 10 | 11 | 15 | 11 |

Table 2: Data for Exercise 11

| Day | Sun | Mon | Tue | Wed | Thur | Fri | Sat |
|---------------------------------|-----|-----|-----|-----|------|-----|-----|
| Number of Earthquakes (f_i) | 156 | 144 | 170 | 158 | 172 | 148 | 152 |

Use the F -test, with $\alpha = 0.05$ to see if there are differences in the breaking strength due to the percentages of cotton used.

- It is desired to determine whether there is less variability in the marks of Probability and Statistics course by IITD students than in that by IITB students. If independent random samples of size 10 of the two IIT's yield $s_1 = 0.025$ and $s_2 = 0.045$, test the hypothesis at the 0.05 level of significance.
- Two analysts A and B each make positive determinations of percent of iron content in a batch of prepared ore from a certain deposit. The sample variances for A and B turned out to be 0.4322 and 0.5006 respectively. Can we say that analyst A is more accurate than B at 5% level of significance.
- Elongation measurements are made on 10 pieces on steel, 5 of which are treated with method A (aluminium only) and the remaining 5 are method B (aluminium plus calcium). The results obtained are as under:

| | | | | | |
|----------|----|----|----|----|----|
| Method A | 78 | 29 | 25 | 23 | 30 |
| Method B | 34 | 27 | 30 | 26 | 23 |

Test the hypothesis that

- $\sigma_A^2 = \sigma_B^2$.
- $\mu_B - \mu_A = 10\%$.

at 2% level of significance by choosing approximate alternatives.

- Suppose the weekly number of accidents over a 60-week period in Delhi is as follows:

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 3 | 4 | 0 | 2 | 1 | 4 | 2 | 2 | 0 | 0 | 5 | 2 | 1 | 3 | 0 | 1 |
| 1 | 8 | 0 | 2 | 0 | 1 | 9 | 3 | 3 | 5 | 1 | 3 | 2 | 0 | 7 | 0 | 0 | 0 | 1 | 3 |
| 3 | 3 | 1 | 6 | 3 | 0 | 1 | 2 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 3 | 0 | 0 | 2 |

Test the hypotheses that the number of accidents in a week has a Poisson distribution. Assume $\alpha = 0.05$. ($\chi_{3,0.05}^2 = 7.81$; $\chi_{4,0.05}^2 = 9.48$).

- A study was investigated to see if Southern California earthquakes of at least moderate size (having values of at least 4.4 on the Richter Scale) are more likely to occur on certain days of the week than on others. The catalogs yielded the following data on 1100 earthquakes: Test at the 5% level of significance, the hypotheses that an earthquake is equally likely to occur on any of the 7 days of the week.

Table 3: Data for Exercise 12

| | | | | |
|---------------------|----|----|----|----|
| Blood Type | O | A | B | AB |
| Frequency (f_i) | 87 | 59 | 20 | 4 |

Table 4: Data for Exercise 13

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 3 | 3 | 1 | 2 | 3 | 4 | 6 | 5 | 6 |
| 2 | 4 | 1 | 3 | 4 | 5 | 3 | 4 | 3 | 4 |
| 3 | 3 | 4 | 5 | 4 | 5 | 6 | 4 | 5 | 1 |
| 6 | 3 | 6 | 2 | 4 | 6 | 4 | 6 | 3 | 5 |
| 6 | 3 | 6 | 2 | 4 | 6 | 4 | 6 | 3 | 2 |
| 5 | 4 | 6 | 3 | 3 | 3 | 5 | 3 | 1 | 4 |

12. The proportions of blood types O, A, B and AB in the general population of a particular country are known to be in the ratio 49:38:9:4, respectively. A research team, investigating the a small isolated community in he country, obtained the following frequencies of blood type is shown in Table 3. Test the hypotheses that the proportions in this community do not differ significantly from those in the general population. Test at the 5% level of significance.
13. Consider the data of Table 4 that corresponds to 60 rolls of a die:
Test the hypotheses that the die is fair ($P_i = \frac{1}{6}$, $i = 1, \dots, 6$), at 0.5% level of significance.
14. A sample of 300 cars having cellular phones and one of 400 cars without phones are tracked for 1 year. The Table 5 gives the number of cars involved in accidents over that year.
Use the above to test the hypotheses that having a cellular phone in your car and being involved in an accident are independent. Use the 5 percent level of significance.
15. A randomly chosen group of 20,000 nonsmokers and one of 10,000 smokers were followed over a 10-year period. The following data of Table 6 relate the numbers of them that developed lung cancer during the period.
Test the hypotheses that smoking and lung cancer are independent. Use the 1% level of significance.
16. A politician claims that she will receive at least 60 % o the votes in an upcoming election. The results of a simple random sample of 100 voters showed that 58 of those sampled will vote for her. Test the politician's claim at the 5% level of significance.
17. Use the 10% level of significance to perform a hypotheses test to see if there is any evidence of a difference between the Channel A viewing area and Channel B viewing area in the proportion of residents who viewed a news telecast by both the channels. A simple random sample of 175 residents in the Channel A viewing area and 225 residents in the Channel B viewing area is selected. Each resident in the sample is asked whether or not he/she viewed the news telecast. In the Channel A telecast, 49 residents viewed the telecast, while 81 residents viewed the Channel B telecast.
18. Can it be concluded from the following sample data of Table 7 that the proportion of employees favouring a new pension plan is not the same for three different agencies. Use $\alpha = 0.05$.

Table 5: Data for Exercise 14

| | | |
|----------------|----------|-------------|
| | Accident | No Accident |
| Cellular Phone | 22 | 278 |
| No Phone | 26 | 374 |

Table 6: Data for Exercise 15

| | Smokers | Non Smokers |
|----------------|---------|-------------|
| Lung cancer | 62 | 14 |
| No Lung Cancer | 9938 | 19986 |

Table 7: Data for Exercise 18

| | Agency 1 | Agency 2 | Agency 3 |
|--------------------------|----------|----------|----------|
| For the pension plan | 67 | 84 | 109 |
| Against the pension plan | 33 | 66 | 41 |
| Total | 100 | 150 | 150 |

19. In a study of the effect of 2 treatments on the survival of patients with a certain disease, each of the 156 patients was equally likely to be given either one of the 2 treatments. The result of the above was that 39 of the 72 patients given the first treatment survived and 44 of the 84 patients given the second treatment survived. Test the null hypotheses that the 2 treatments are equally effective at $\alpha = 0.05$ level of significance.
20. Three kinds of lubricants are being prepared by a new process. Each lubricant is tested on a number of machines, and the result is then classified as acceptable or non-acceptable. The data in the Table 8 represents the outcome of such an experiment. Test the hypotheses that the probability p of a lubricant resulting in an acceptable outcome is the *same* for all three lubricants. Test at the 5% level of significance.
21. Twenty five men between the ages of 25 and 30, who were participating in a well-known heart study carried out in New Delhi, were randomly selected. Of these, 11 were smokers and 14 were not. The following data refers to readings of their systolic blood pressure.
- Use the data of Table 9 to test the hypothesis that the mean blood pressures of smokers and nonsmokers are the same at 5% level of significance.
22. Polychlorinated biphenyls(PCB), used in the production of large electrical transformers and capacitors, is extremely hazardous when released into the environment. Two methods have been suggested to monitor the levels of PCB in fish near a large plant. It is believed that each method will result in a normal random variable that depends on the method. Test the hypothesis at the $\alpha = 0.10$ level of significance that both methods have the same variance, if a given fish is checked 8 times by each method with the data (in parts per million) recorded given in Table 10.
23. An oil company claims that the sulphur content of it's diesel fuel is at most 0.15 percent. To check this claim, the sulphur contents of 40 randomly chosen samples were determined; the resulting sample mean and sample standard deviation were 0.162 and 0.040. Using the 5 percent level of significance, can we conclude that the company's claims are invalid?
24. Historical data indicate that 4% of the components produced at a certain manufacturing facility are defective. A particularly acrimonious labour dispute has recently been concluded, and management is curious about whether it will result in any change in this figure of 4%. If a random sample of 500 items indicated 16 defectives, is this significant evidence, at the 5% level of significance, to conclude that a change has occurred.

Table 8: Data for Exercise 20

| | Lubricant 1 | Lubricant 2 | Lubricant 3 |
|----------------|-------------|-------------|-------------|
| Acceptable | 144 | 152 | 140 |
| Non-acceptable | 56 | 48 | 60 |
| Total | 200 | 200 | 200 |

Table 9: Data for Exercise 21

| Smokers | Nonsmokers |
|---------|------------|
| 124 | 130 |
| 134 | 122 |
| 136 | 128 |
| 125 | 129 |
| 133 | 118 |
| 127 | 122 |
| 135 | 116 |
| 131 | 127 |
| 133 | 135 |
| 125 | 120 |
| 118 | 122 |
| | 120 |
| | 115 |
| | 123 |

Table 10: Data for Exercise 22

| | | | | | | | | |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|
| Method 1 | 6.2 | 5.8 | 5.7 | 6.3 | 5.9 | 6.1 | 6.2 | 5.7 |
| Method 2 | 6.3 | 5.7 | 5.9 | 6.4 | 5.8 | 6.2 | 6.3 | 5.5 |

25. An auto rental firm is using 15 identical motors that are adjusted to run at fixed speeds to test 3 different brands of gasoline. Each brand of gasoline is assigned to exactly 5 of the motors. Each motor runs on 10 gallons of gasoline until it is out of fuel. Table 11 gives the total mileage obtained by the different motors. Test the hypotheses that the average mileage obtained is not affected by the type of gas used. Use the 5% level of significance.
26. To examine the effects of pets and friends in stressful situations, researchers recruited 45 people to participate in an experiment and data is shown in Table 12. Fifteen of the subjects were randomly assigned to each of the 3 groups to perform a stressful task alone (Control Group), with a good friend present, or with their dog present. Each subjects mean heart rate during the task was recorded. Using ANOVA method, test the appropriate hypotheses at the $\alpha = 0.05$ level to decide if the mean heart rate differs between the groups.
27. A fisheries researcher wishes to conclude that there is a difference in the mean weights of 3 species of fish (A,B,C) caught in a large lake. The data is shown in Table 13. Using ANOVA method, test the hypotheses at $\alpha = 0.05$ level.

Table 11: Data for Exercise 25

| Gas 1 | Gas 2 | Gas 3 |
|-------|-------|-------|
| 220 | 244 | 252 |
| 251 | 235 | 272 |
| 226 | 232 | 250 |
| 246 | 242 | 238 |
| 260 | 225 | 256 |

Table 12: Data for Exercise 26

| | n | Mean | SD |
|---------|-----|--------|------|
| Control | 15 | 82.52 | 9.24 |
| Pets | 15 | 73.48 | 9.97 |
| Friends | 15 | 91.325 | 8.34 |

Table 13: Mean weights of 3 species of fish for Exercise 27

| A | B | C |
|-----|-----|-----|
| 1.5 | 1.5 | 6 |
| 4 | 1 | 4.5 |
| 4.5 | 4.5 | 4.5 |
| 3 | 2 | 5.5 |