Transient solution of a link with finite capacity supporting multiple streams

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Abstract

Most of the real-time applications involve connection establishment in point-to-point (unicast) communication in computer networks that can be modelled as a problem of resource allocation and resource sharing. These applications have different streams, such as video, voice, graphics, etc., each having different arrival rates. A stream (call) will be admitted into the network only if the network guarantees the ‘Quality of Service’ (QoS) parameters of the call such as bandwidth, delay, jitter and loss probability. Otherwise, the call is rejected. Hence, the study of call blocking probabilities in such communication networks is a problem of growing importance. Application of queueing theory to solve these problems has received considerable attention in the research community.

Due to the real-time nature of the applications, it is pertinent to study the time-dependent behaviour of such systems. In this paper, a real-time unicast communication has been modelled as a multi-dimensional Markov process to obtain the time-dependent blocking probabilities. The time-dependent system size probabilities, means, variances and correlation coefficients of different streams are also obtained. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Transient analysis; Tridiagonal determinants; Blocking probabilities

1. Introduction

Most of the real-time applications involve connection establishment that can be modelled as the problem of resource allocation and resource sharing. The real-time applications have streams like video, voice, graphics, etc., [5,19]. A real-time stream requires ‘Quality of Service’ (QoS) guarantees in terms of bandwidth, delay, jitter and loss probability. A real-time stream (call) is accepted by the system only if QoS guarantees of the call can be met; otherwise the call is rejected (i.e., the call is blocked). Performance is one of the key factors that influences the design, development, configuration and tuning of these networks. A means for allocating the resources in order to resolve these conflicting demands is one of the most important aspects of system operation which exemplify the successful application of queueing theory in performance evaluation studies [2,3,6,8,21].

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Of specific interest is the product form equilibrium call loss probabilities [4]. This may be due to the fact that the balance equations involved are simple and relatively straightforward techniques can be employed. However, stationary cell loss probabilities can only say little about this QoS parameters. A network connection is of finite duration and steady state might never be reached. Steady state system performance measures are inadequate and they are often inappropriate in situations when the time horizon of operations is finite. Due to the real-time nature of the traffic, it is pertinent to investigate the effect of control on the transient queueing behaviour.

Transient analysis is useful in obtaining optimal solutions leading to the control of the system. In many potential applications of queueing theory, the practitioner needs to know how the system will operate up to some specified time [22]. It is quite arduous to obtain the transient solution of even simple Markovian queueing systems with constant rates and the solutions involve infinite series of Bessel functions and/or their integrals [13–15,20]. As the complexity of the queueing system increases, one cannot obtain closed form expressions for the transient solution. Hence it is pertinent to develop numerical techniques to solve the resulting birth and death equations and to gain an insight into the behaviour of the various system characteristics [12]. In queueing networks theory there are only a few exact results on the transient solutions [1,9–11,16].

In this paper, the time-dependent system size probabilities are obtained for a real-time communication with a single link supporting multiple streams for finite bandwidth capacity ($N$ tokens). In this communication, a stream can materialise only if at least one token is available; otherwise the stream is lost. This leads to \[ \binom{c+N}{c} \] forward–Kolmogorov differential–difference equations. The transient probabilities are obtained by converting these multi-dimensional probabilities into one-dimensional case by using an elegant transformation. The well-known solution [17] of this one-dimensional equation involving the roots of Charlier polynomials is used to determine the time-dependent blocking probabilities, means, variances and their correlation coefficients. The numerical results of transient system size probabilities for specific values of parameters are presented in the form of tables and graphs.

2. Model description

A real-time communication can be modelled as a multi-dimensional Markov process, where the dimension of the process is equal to the number of different stream (call) types admitted into the system. The real-time application have video, voice, text and graphics streams (in general $c$ streams) whose arrival rates are different. In our model, we consider a single link of capacity $N$ ($N$ tokens) supporting $c$ different stream types, where $N$ is the maximum number of streams admitted in the link. A stream of type $k$ arrives according to a Poisson process with arrival rate $\lambda_k$ ($1 \leq k \leq c$) independent of the other arrival rates. The stream holding time is exponential with parameter $\mu$. A stream is admitted if there is a token. Otherwise, it is rejected. The state space of this multi-dimensional Markov process has \[ \binom{c+N}{c} \] states, given by \( \{(n_1, n_2, \ldots, n_c) | n_k \geq 0 \quad \forall k, \sum_{k=1}^c n_k \leq N\} \).

Let $X_k(t)$ denote the number of traffic streams of type $k$ at time $t$ and $X(t) = \sum_{k=1}^c X_k(t)$. Define $P_{n_1,n_2,\ldots,n_c}(t) = \Pr\{X_k(t) = n_k, 1 \leq k \leq c\}$ and $q_j(t) = \Pr\{X(t) = j\}$. We will prove later that a transformation helps us to find the set of all $P_{n_1,n_2,\ldots,n_c}(t)$ satisfying $n_1 + n_2 + \cdots + n_c = j$ simultaneously for $0 \leq j \leq N$. Accordingly with $u = \mu t$, $\rho_k = \lambda_k/\mu$ ($1 \leq k \leq c$) and $\rho = \sum_{k=1}^c \rho_k$, the forward
Kolmogorov equations for the system can be written as

$$P_{00\ldots 0}(u) = -\rho P_{00\ldots 0}(u) + P_{10\ldots 0}(u) + P_{01\ldots 0}(u) + \cdots + P_{00\ldots 1}(u)$$  \hspace{1cm} (2.1)$$

for $0 \leq n_k < N$ and $1 \leq n_1 + n_2 + \cdots + n_c (= j) < N,$

$$P'_{n_1n_2\ldots n_c}(u) = \rho_1 P_{n_1n_2\ldots n_c}(u) + \rho_2 P_{n_1n_2\ldots n_c-1}(u) + \cdots + \rho_c P_{n_1n_2\ldots n_c-1}(u)$$
$$-(\rho + j) P_{n_1n_2\ldots n_c}(u) + (n_1 + 1) P_{n_1+1n_2\ldots n_c}(u)$$
$$+(n_2 + 1) P_{n_1n_2+1\ldots n_c}(u) + \cdots + (n_c + 1) P_{n_1n_2\ldots n_c+1}(u)$$  \hspace{1cm} (2.2)$$

and for $0 \leq n_k \leq N$ and $n_1 + n_2 + \cdots + n_c = N,$

$$P'_{n_1n_2\ldots n_c}(u) = \rho_1 P_{n_1n_2\ldots n_c}(u) + \rho_2 P_{n_1n_2\ldots n_c-1}(u) + \cdots + \rho_c P_{n_1n_2\ldots n_c-1}(u) - NP_{n_1n_2\ldots n_c}(u).$$  \hspace{1cm} (2.3)$$

We assume that the total number of streams initially in the system is $m$. They are distributed at various traffic streams with multinomial probabilities given by

$$P_{n_1n_2\ldots n_c}(0) = \frac{m!\rho_1^{n_1}\rho_2^{n_2}\cdots \rho_c^{n_c}}{n_1!n_2!\cdots n_c!},$$  \hspace{1cm} (2.4)$$

so that

$$\sum_{n_1+n_2+\cdots+n_c=m} P_{n_1n_2\ldots n_c}(0) = 1.$$

Only with this initial condition it is possible to obtain the transient solution of this multimedia communication as this considerably simplifies the problem.

The identities of certain tridiagonal determinants are useful in determining the system size probabilities. The transient solution of an $M/M/N/N$ queue involves the roots of Charlier polynomials which cannot be expressed in the closed form [17]. These are also the roots of $|A(s)| = D_{N+1}(s)) = 0$ and they are evaluated numerically where

$$|A(s)| = \begin{vmatrix}
    s + \rho & -1 & 0 & . & . & . \\
    -\rho & s + \rho + 1 & -2 & . & . & . \\
    0 & -\rho & s + \rho + 2 & . & . & . \\
    . & . & . & . & . & . \\
    . & . & . & -\rho & s + \rho + N - 1 & -N \\
    . & . & . & . & 0 & -\rho & s + N \\
\end{vmatrix}_{N+1}$$

We obtained the subdeterminant $D_n(s)$ from $|A(s)|$, by taking the first $n$ rows and $n$ columns with $D_0(s) = 1$. These determinants help us to write

$$|A(s)| = sD_N(s + 1).$$  \hspace{1cm} (2.5)$$

It is well known that the roots of $|A(s)| = 0$ (i.e., the roots of the Charlier polynomials) are real, negative and distinct [23]. One of the roots is zero by Eq. (2.5).
3. Transient solution

In this section, we obtain the transient solution of system size probabilities at various traffic streams in the following theorem.

**Theorem 1.** The transient solution of system size probabilities for various traffic streams with
\[n_1 + n_2 + \cdots + n_c = j\] are given by
\[
P_{n_1n_2\ldots n_c}(t) = \frac{\rho_1^{n_1} \rho_2^{n_2} \cdots \rho_c^{n_c}}{n_1! n_2! \cdots n_c!} \left\{ \frac{1}{\sum_{k=0}^{j-1} N! \rho^{N-j}} \sum_{r=1}^{N} D_m(x_r) D_j(x_r) \exp(x_r \mu t) \right\}^{N-j} x_r D'_N(x_r + 1) D_N(x_r)
\]
for \(j = 0, 1, \ldots, N\) and \(x_r\) are the roots of the Charlier polynomial \(D_N(s + 1) = 0\).

**Proof.** We define the following transformation:
\[
P_{n_1n_2\ldots n_c}(u) = \frac{\rho_1^{n_1} \rho_2^{n_2} \cdots \rho_c^{n_c}}{n_1! n_2! \cdots n_c!} q_j(u), \quad n_1 + n_2 + \cdots + n_c = j.
\]
It is verified that the Eqs. (2.1)–(2.4) are satisfied if
\[
\begin{align*}
q_0'(u) &= -\rho q_0(u) + q_1(u), \\
q_j'(u) &= \rho q_{j-1}(u) + (\rho + j) q_j(u) + (j + 1) q_{j+1}(u), \quad 1 \leq j < N, \\
q_N'(u) &= \rho q_{N-1}(u) - N q_N(u)
\end{align*}
\]
and \(q_j(0) = \delta_{nj}\). The equations for \(q_j(u)\) correspond to those for the transient probabilities for an \(M/M/N/N\) queue. An explicit representation for \(q_j(u)\) was given by Riordan [17, p. 85, Eq. (10)]. Hence the theorem follows from (3.2).

The above transient system size probabilities determine the mean and variance at various traffic streams and their correlation coefficients between various streams. In our model the arrival rates for various streams are different and the traffic stream with higher arrival rate blocks the other streams. Thus, the correlation between any two traffic streams is due to the fact that both are correlated with the other streams. This consideration leads us to examine the correlations between stream types 1 and 2 when the remaining stream types are held constant, i.e., conditionally upon the remaining streams taking certain fixed values. This is called partial correlation denoted by \(r_{12,34\ldots c}\).

If one is interested in how the stream \(1 \leq i \leq c\) depend simultaneously on other streams, then the multiple correlation coefficient comes into play. The multiple correlation describes the target quantity in terms of other influence quantities, e.g., \(R_{1,23\ldots c}\) is the multiple correlation between the number of streams of type 1 and the ‘best fitting’ linear combination of other stream types. No other linear function of other stream types will have greater correlation with stream type 1 [7,18]. Specifically, if there are three stream types then partial correlation coefficient \(r_{12,3}\) between the number of stream types 1 and 2 for a fixed value of the number of stream type 3 is defined by
\[
r_{12,3} = \frac{r_{12} - r_{13} r_{23}}{(1 - r_{13}^2)(1 - r_{23}^2))^{1/2}}
\]
and the multiple correlation coefficient $R_{1,23}$ is defined by

$$R_{1,23} = \left( \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2} \right)^{1/2}.$$ 

Graphs of time-dependent correlation coefficients, partial and multiple correlation coefficients are presented for three stream types with capacity 5 in the next section.

**Remark 1.** The first term in the right-hand side of (3.1) represents the equilibrium probabilities which can also be obtained from Eqs. (2.1)–(2.3) by setting the left-hand sides to zero.

**Remark 2.** The time-dependent blocking probabilities at various streams for $n_1 + n_2 + \cdots + n_c = N$ are given by

$$P_{n_1n_2\ldots n_c}(t) = \frac{\rho_1^{n_1}\rho_2^{n_2}\cdots\rho_c^{n_c}}{n_1!n_2!\cdots n_c!} \left\{ \frac{1}{\sum_{k=0}^{N} \rho^k / k!} + N! \rho^m \sum_{r=1}^{N} \frac{D_m(x_r) \exp(x_r\mu t)}{x_r D_N(x_r + 1)} \right\}.$$

From the above equation, by summing over all $n_i$ and $n_1 + n_2 + \cdots + n_c = N$, we find the blocking probability (i.e., $q_N(t)$).

**Remark 3.** When there is one token ($N = 1$) in the system, the analytical results for the system size probabilities at various streams are obtained easily. When there are initially no streams in the entire system

$$P_{00\ldots 0}(t) = \frac{1}{1 + \rho} \{1 + \rho \exp(-\mu(1 + \rho)t)\}$$

and if 1 is in the $i$th place

$$P_{00\ldots 1\ldots 0}(t) = \frac{1}{\mu(1 + \rho)} \{ \lambda_i - \exp(-\mu(1 + \rho)t) \}.$$

When there is one stream of type $i$ initially in the system with

$$P_{00\ldots 1\ldots 0}(0) = \frac{\rho_i}{\rho}, \quad i = 1, 2, \ldots, c,$$

the system size probabilities are given by

$$P_{00\ldots 0}(t) = \frac{1}{1 + \rho} \{1 - \exp(-\mu(1 + \rho)t)\}$$

and if 1 is in the $i$th place

$$P_{00\ldots 1\ldots 0}(t) = \frac{\rho_i}{1 + \rho} \left\{ \frac{1}{\rho} + \frac{1}{\rho} \exp(-\mu(1 + \rho)t) \right\}.$$

**Remark 4.** When there are two tokens ($N = 2$) and the initial state is $m (= 0, 1$ and 2) in the system (see (2.4)), the time-dependent system size probabilities for various streams are given by

$$P_{n_1n_2\ldots n_c}(t) = \frac{\rho_1^{n_1}\rho_2^{n_2}\cdots\rho_c^{n_c}}{n_1!n_2!\cdots n_c!} \left\{ \frac{1}{1 + \rho + \rho^2/2} + \frac{\rho^{2-m}}{\sum_{r=1}^{2} \frac{D_m(x_r) D_j(x_r) \exp(x_r\mu t)}{x_r (x_r + 1 - \rho)}} \right\},$$

where $j$ is the number of streams with $x_r$ tokens.
where \( n_1 + n_2 + \cdots + n_c = j, \quad j = 0, 1 \) and 2,
\[
x_1, x_2 = -\frac{1}{2}(2\rho + 3 \pm (1 + 4\rho)^{1/2}),
\]
\( D_0(s) = 1, \quad D_1(s) = s + \rho \) and \( D_2(s) = s^2 + s(1 + 2\rho) + \rho^2 \).

4. Numerical results

We have shown in Section 2 that the transient solutions of system size probabilities involve the roots of Charlier polynomials which cannot be expressed in closed form. Hence numerical procedure helps us to visualise the nature of the solutions. In this section, the numerical results of transient system size probabilities for specific values of parameters are presented. For the sake of simplicity, a link with three streams and a maximum capacity of five tokens in the system is considered with the arrival rates \( \lambda_1 = 10, \quad \lambda_2 = 3, \quad \lambda_3 = 5 \) and the service rate \( \mu = 10 \). Initially there are four streams in the system distributed as in \((2.4)\). There are 56 possibilities distributed as follows:

<table>
<thead>
<tr>
<th>Total No. in the system</th>
<th>No. of possibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
</tr>
</tbody>
</table>

In Table 1, the system size probability values are tabulated for different values of time points. From the column corresponding to time zero, we observe that
\[
\sum_{n_1, n_2, \ldots, n_c: n_1 + n_2 + \cdots + n_c = 4} P_{n_1 n_2 \ldots n_c}(0) = 1.
\]

It is found that the system reaches the steady state around time 1.0 unit and the steady state probabilities are depicted in the last column (see Remark 1).

In Table 2, the time-dependent means and variances for three streams, correlation coefficients, partial correlation coefficients and multiple correlation coefficients are tabulated for different values of time points and the last column gives steady state values.

In Fig. 1, the time-dependent system size probabilities \( P_{000}(t), P_{100}(t), P_{110}(t), P_{111}(t), P_{301}(t) \) and \( P_{410}(t) \) are plotted at different time points. There are three types of behaviour. As time increases, probability curves corresponding to the initial state steadily decrease to the equilibrium, some curves increase to reach a maximum and then decrease to the steady state whereas the remaining increase to the steady state.

In Fig. 2, the time-dependent blocking probabilities for various streams are plotted against time. For the sake of clarity, the probability curves close to time axis are not plotted. We observe that initially the probability curves increase rapidly to certain extent till time 0.025 units and then decrease to reach the equilibrium values around 0.3 time units.
The time-dependent probability values for various streams

<table>
<thead>
<tr>
<th>Time</th>
<th>$P_{00}(t)$</th>
<th>$P_{01}(t)$</th>
<th>$P_{02}(t)$</th>
<th>$P_{03}(t)$</th>
<th>$P_{10}(t)$</th>
<th>$P_{11}(t)$</th>
<th>$P_{12}(t)$</th>
<th>$P_{20}(t)$</th>
<th>$P_{21}(t)$</th>
<th>$P_{22}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
<td>5.1968(2)</td>
</tr>
<tr>
<td>0.3</td>
<td>1.5083(1)</td>
<td>1.5083(1)</td>
<td>1.5083(1)</td>
<td>1.5083(1)</td>
<td>1.5083(1)</td>
<td>1.5083(1)</td>
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<td>1.5083(1)</td>
<td>1.5083(1)</td>
</tr>
<tr>
<td>0.5</td>
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<td>1.6515(1)</td>
<td>1.6515(1)</td>
<td>1.6515(1)</td>
<td>1.6515(1)</td>
<td>1.6515(1)</td>
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<td>1.6515(1)</td>
<td>1.6515(1)</td>
<td>1.6515(1)</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
<td>1.6696(1)</td>
</tr>
<tr>
<td>1.0</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
<td>1.6702(1)</td>
</tr>
</tbody>
</table>

Table 1
In Fig. 3, the time-dependent means for the stream types 1, 2 and 3 are plotted against time. The mean curve corresponds to stream type \( i \) is labelled by \( i \). In the equilibrium, the mean is large for the stream 1, when compared to the stream type 3 which in turn is large when compared to the stream type 2.

In Fig. 4, the time-dependent variances of all streams are plotted at different time points. The variance curve corresponds to stream type \( i \) is labelled by \( i \). We observe that as time increases the curve corresponding to stream type 1 increases initially and decreases to steady state whereas the curves corresponding to stream types 2 and 3 decrease to steady state. In the equilibrium, the variance is large

<table>
<thead>
<tr>
<th>Time</th>
<th>0.0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{131}(t) )</td>
<td>0.0000</td>
<td>9.8493(4)</td>
<td>4.3181(4)</td>
<td>3.8215(4)</td>
<td>3.7607(4)</td>
<td>3.7585(4)</td>
</tr>
<tr>
<td>( P_{122}(t) )</td>
<td>0.0000</td>
<td>2.4623(3)</td>
<td>1.0795(3)</td>
<td>9.5538(4)</td>
<td>9.4017(4)</td>
<td>9.3963(4)</td>
</tr>
<tr>
<td>( P_{113}(t) )</td>
<td>0.0000</td>
<td>2.7359(3)</td>
<td>1.1995(3)</td>
<td>1.0615(3)</td>
<td>1.0463(3)</td>
<td>1.0440(3)</td>
</tr>
<tr>
<td>( P_{104}(t) )</td>
<td>0.0000</td>
<td>1.1400(3)</td>
<td>4.9977(4)</td>
<td>4.4230(4)</td>
<td>4.3526(4)</td>
<td>4.3501(4)</td>
</tr>
<tr>
<td>( P_{050}(t) )</td>
<td>0.0000</td>
<td>8.8643(6)</td>
<td>3.8862(6)</td>
<td>3.4394(6)</td>
<td>3.3846(6)</td>
<td>3.3827(6)</td>
</tr>
<tr>
<td>( P_{041}(t) )</td>
<td>0.0000</td>
<td>7.3869(5)</td>
<td>3.2385(5)</td>
<td>2.8661(5)</td>
<td>2.8205(5)</td>
<td>2.8189(5)</td>
</tr>
<tr>
<td>( P_{032}(t) )</td>
<td>0.0000</td>
<td>2.4623(4)</td>
<td>1.0795(4)</td>
<td>9.5538(5)</td>
<td>9.4017(5)</td>
<td>9.3963(5)</td>
</tr>
<tr>
<td>( P_{023}(t) )</td>
<td>0.0000</td>
<td>4.1039(4)</td>
<td>1.7992(4)</td>
<td>1.5923(4)</td>
<td>1.5670(4)</td>
<td>1.5660(4)</td>
</tr>
<tr>
<td>( P_{014}(t) )</td>
<td>0.0000</td>
<td>3.4199(4)</td>
<td>1.4993(4)</td>
<td>1.3269(4)</td>
<td>1.3058(4)</td>
<td>1.3050(4)</td>
</tr>
<tr>
<td>( P_{005}(t) )</td>
<td>0.0000</td>
<td>1.1400(4)</td>
<td>4.9977(5)</td>
<td>4.4230(5)</td>
<td>4.3526(5)</td>
<td>4.3501(5)</td>
</tr>
</tbody>
</table>

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In Fig. 4, the time-dependent variances of all streams are plotted at different time points. The variance curve corresponds to stream type \( i \) is labelled by \( i \). We observe that as time increases the curve corresponding to stream type 1 increases initially and decreases to steady state whereas the curves corresponding to stream types 2 and 3 decrease to steady state. In the equilibrium, the variance is large

<table>
<thead>
<tr>
<th>Time</th>
<th>0.0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{12}(t) )</td>
<td>0.0000</td>
<td>9.8493(4)</td>
<td>4.3181(4)</td>
<td>3.8215(4)</td>
<td>3.7607(4)</td>
<td>3.7585(4)</td>
</tr>
<tr>
<td>( r_{13}(t) )</td>
<td>0.0000</td>
<td>2.4623(3)</td>
<td>1.0795(3)</td>
<td>9.5538(4)</td>
<td>9.4017(4)</td>
<td>9.3963(4)</td>
</tr>
<tr>
<td>( r_{23}(t) )</td>
<td>0.0000</td>
<td>2.7359(3)</td>
<td>1.1995(3)</td>
<td>1.0615(3)</td>
<td>1.0463(3)</td>
<td>1.0440(3)</td>
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<tr>
<td>( r_{123}(t) )</td>
<td>0.0000</td>
<td>1.1400(3)</td>
<td>4.9977(4)</td>
<td>4.4230(4)</td>
<td>4.3526(4)</td>
<td>4.3501(4)</td>
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<tr>
<td>( r_{132}(t) )</td>
<td>0.0000</td>
<td>8.8643(6)</td>
<td>3.8862(6)</td>
<td>3.4394(6)</td>
<td>3.3846(6)</td>
<td>3.3827(6)</td>
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<tr>
<td>( r_{231}(t) )</td>
<td>0.0000</td>
<td>7.3869(5)</td>
<td>3.2385(5)</td>
<td>2.8661(5)</td>
<td>2.8205(5)</td>
<td>2.8189(5)</td>
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<td>( R_{123}(t) )</td>
<td>0.0000</td>
<td>2.4623(4)</td>
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<td>9.5538(5)</td>
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<td>9.3963(5)</td>
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<tr>
<td>( R_{132}(t) )</td>
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<td>4.1039(4)</td>
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<td>1.5923(4)</td>
<td>1.5670(4)</td>
<td>1.5660(4)</td>
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<tr>
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<td>1.4993(4)</td>
<td>1.3269(4)</td>
<td>1.3058(4)</td>
<td>1.3050(4)</td>
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</table>

\( k \) denotes \( 10^{-k}, k = 0, 1, 2 \).
Fig. 1. The time-dependent system size probabilities for the case $\lambda_1 = 10, \lambda_2 = 3, \lambda_3 = 5, \mu = 10, m = 4, c = 3$ and $N = 5$.

Fig. 2. The time-dependent blocking probability values for the case $\lambda_1 = 10, \lambda_2 = 3, \lambda_3 = 5, \mu = 10, m = 4, c = 3$ and $N = 5$. 
Fig. 3. The time-dependent means for streams type 1, 2 and 3.

Fig. 4. The time-dependent variances for streams type 1, 2 and 3.
Fig. 5. The time-dependent correlation coefficients for the case \( \lambda_1 = 10, \lambda_2 = 3, \lambda_3 = 5, \mu = 10, m = 4, c = 3 \) and \( N = 5 \).

Fig. 6. The time-dependent partial correlation coefficients for the case \( \lambda_1 = 10, \lambda_2 = 3, \lambda_3 = 5, \mu = 10, m = 4, c = 3 \) and \( N = 5 \).
for the stream type 1, when compared to the stream type 3 which in turn is large when compared to the stream type 2.

In Fig. 5, the time-dependent correlation coefficients versus time are plotted. We observe that initially all curves are increasing steadily to equilibrium with time. We note that, $r_{23}(t) > r_{12}(t) > r_{13}(t)$ for all time points.

In Fig. 6, the time-dependent partial correlation coefficients versus time are plotted. We observe that initially all curves are starting from $-1$ and steadily increase to equilibrium with time. We note that, $r_{13,2}(t) > r_{12,3}(t) > r_{23,1}(t)$ for all time points.

In Fig. 7, the time-dependent multiple correlation coefficients versus time are plotted. We observe that initially all curves are starting from 1 and reach equilibrium around 0.3 time units. We note that, $R_{1,23}(t) > R_{2,31}(t) > R_{3,21}(t)$ for all time points.

5. Conclusions

In this paper, we have obtained the transient solution for unicast communication, in which a single link with finite capacity, supports multiple traffic streams having different arrival rates. From our study, we observe that the stream with less arrival rate experiences more blocking compared to streams having higher arrival rates. The study involving multiple traffic streams helps us to understand the correlation between streams as given in Fig. 5. In conclusion, the proposed model captures multiple arrival rates which is more realistic and complex than the single arrival rate model.
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