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Lecture 07
Finding Roots – Brent's Methods

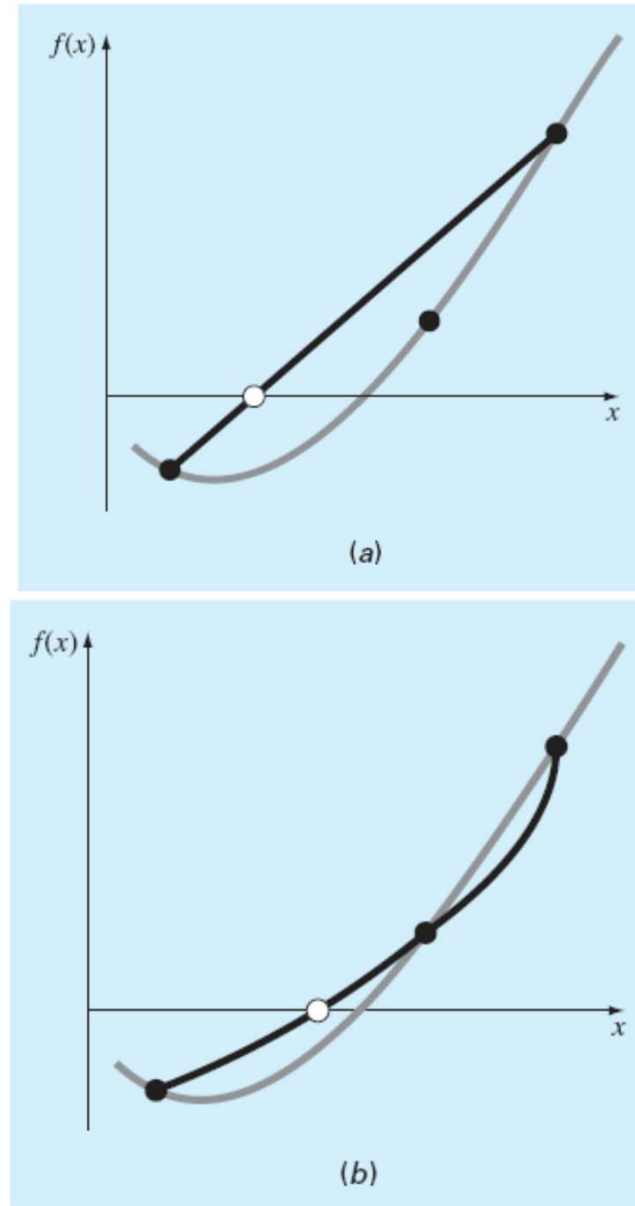
Open Methods

- Fixed Point Iteration and its convergence
- Newton-Raphson method
- Secant method and Modified Secant method
- Brent's method combining bracketing method with open method.
- Matlab fzero examples

Brent’s Method

It is a hybrid method which combines the reliability of bracketing method and the speed of open methods

- The approach was developed by Richard Brent (1973)
- a) The bracketing method used is the bisection method
- b)The open method counterpart is the secant method or the inverse quadratic interpolation



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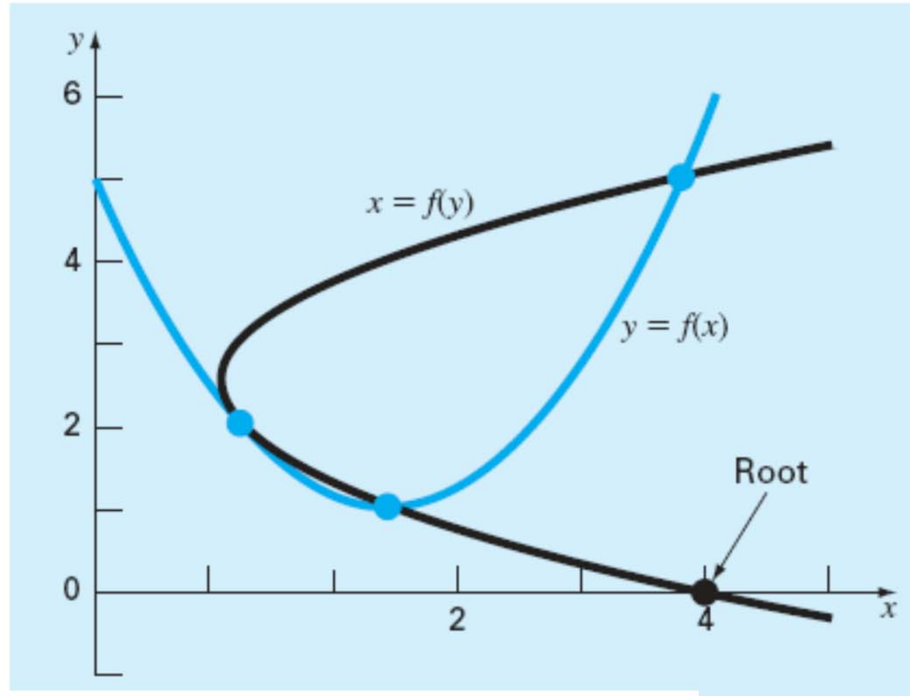
Inverse Quadratic Interpolation

In secant method we interpolate with a line. Here we interpolate with a parabola.

If the three points are designated as $(x_{i.2}, y_{i.2})$, $(x_{i.1}, y_{i.1})$, and (x_i, y_i), then the quadratic function is given as



$$g(y) = \frac{(y - y_{i-1})(y - y_i)}{(y_{i-2} - y_{i-1})(y_{i-2} - y_i)} x_{i-2} + \frac{(y - y_{i-2})(y - y_i)}{(y_{i-1} - y_{i-2})(y_{i-1} - y_i)} x_{i-1} + \frac{(y - y_{i-2})(y - y_{i-1})}{(y_i - y_{i-2})(y_i - y_{i-1})} x_i$$

OR

$$x = \frac{[y - f(x_1)][y - f(x_2)]}{[f(x_3) - f(x_1)][f(x_3) - f(x_2)]} + \frac{[y - f(x_2)][y - f(x_3)]}{[f(x_1) - f(x_2)][f(x_1) - f(x_3)]} + \frac{[y - f(x_3)][y - f(x_1)]}{[f(x_2) - f(x_3)][f(x_2) - f(x_1)]}$$

Brent's Method

- Now for y=0 the expression in the previous slide can be used to write the expression for the root as:

$$x_{i+1} = \frac{y_{i-1}y_i}{(y_{i-2}-y_{i-1})(y_{i-2}-y_i)}x_{i-2} + \frac{y_{i-2}y_i}{(y_{i-1}-y_{i-2})(y_{i-1}-y_i)}x_{i-1} + \frac{y_{i-2}y_{i-1}}{(y_i-y_{i-2})(y_i-y_{i-1})}x_i$$

- This polynomial is also known as Lagrange Polynomial

Brent's Method

- To find successive root estimates set y = 0, which gives

$$x = x_2 + \frac{P}{Q}$$

where

$$P = S[T(R - T)(x_3 - x_2) - (1 - R)(x_2 - x_1)]$$

$$Q = (T - 1)(R - 1)(S - 1)$$

$$R = \frac{f(x_2)}{f(x_3)} \qquad S = \frac{f(x_2)}{f(x_1)} \qquad T = \frac{f(x_1)}{f(x_3)}$$

- The point at which y = 0 becomes the new x_2 and the old x_2 becomes x_1, but if x_3 and the new x_2 no longer bound the root, discard x_3 and use x_1 instead.

Although there are many advantages of Brent's method, when used on exponential functions such as $f(x)=e^x$ or $f(x)=1/x$ it may not converge.