Section of Solid
Conic Sections: Curves appear on the surface of a cone when it is cut by some typical cutting planes.

- **ELLIPSE**: $\theta > \alpha; \theta < 90^\circ$
- **PARABOLA**: $\theta = \alpha$
- **HYPERBOLA**: $\theta < \alpha$

Section Plane Through Generators
- Plane $\perp$ axis. Circles ???
- Open/unbounded Section Plane Parallel to end generator.
Locus of point moving in a plane such that the ratio of its distances from a fixed point (focus) and a fixed line (directrix) always remains constant. Ratio is called ECCENTRICITY (E)

A) For Ellipse $E<1$
B) For Parabola $E=1$
C) For Hyperbola $E>1$

How do I identify ELLIPSE, PARABOLA, HYPERBOLA:

Assume
A moving point
F Fixed point
Line Fixed line

\[ \alpha \]
\[ \theta \]
**PROBLEM:** Point F is 50 mm from a line AB. A point P is moving in a plane such that the **ratio** of its distances from F and line AB remains constant and equals to \( \frac{3}{2} \). Draw locus of point P.

**STEPS:**
1. Draw a vertical line AB and point F 50 mm from it.
2. Divide 50 mm distance in 5 parts.
3. Name 2\(^{nd}\) part from F as V. It is 20 mm and 30 mm from line AB and point F resp. It is first point giving ratio of its distances from AB and F \( 2/3 \) i.e. 20/30.
4. Form more points giving same ratio such as 30/45, 40/60, 50/75 etc.
5. Taking distances 30, 40 and 50 mm from line AB, draw three vertical lines to the right side of it.
6. Now with 45, 60 and 75 mm distances in compass cut these lines above and below, with F as center.
7. Join these points through V in smooth curve.
This is required locus of P. It is an Hyperbola.
PROBLEM: Point F is 50 mm from a vertical straight line AB. Draw locus of point P, moving in a plane such that it always remains equidistant from point F and line AB.

SOLUTION STEPS:
1. Locate center of line (CF), perpendicular to AB. Bisect CF and find vertex V.
2. Mark 5 mm distance to right side of V, name those points 1, 2, 3, 4 and from those draw lines parallel to AB.
3. Take C-1 distance as radius and F as center draw an arc cutting first parallel line to AB. Name upper point P1 and lower point P2.
4. Similarly repeat this process by taking again 5 mm to right and locate P3, P4.
5. Join all these points in smooth curve. It will be the locus of P equidistance from line AB and fixed point F.
**PROBLEM:-** POINT F is 50 mm from a LINE AB. A POINT P is MOVING in a PLANE SUCH THAT RATIO of IT’S DISTANCES (E) FROM F and LINE AB REMAINS CONSTANT and EQUALS TO 2/3. DRAW LOCUS OF POINT P.

**STEPS:**
1. Draw a vertical line AB and point F 50 mm from it.
2. Divide 50 mm distance in 5 parts.
3. Name 2nd part from F as V. It is 20mm and 30mm from F and AB line resp. It is first point giving ratio of it’s distances from F and AB 2/3 i.e 20/30
4. Form more points giving same ratio such as 30/45, 40/60, 50/75 etc.
5. Taking 45, 60 and 75mm distances from line AB, draw three vertical lines to the right side of it.
6. Now with 30, 40 and 50mm distances in compass cut these lines above and below, with F as center.
7. Join these points through V in smooth curve.
This is required locus of P.
Complete ELLIPSE: Locus of a point (A) moving in a plane such that the SUM of its distances from TWO fixed points (FOCUS 1 & FOCUS 2) always remains constant. This *sum equals* to the length of *major axis*.

\[ A3 + A4 = \text{Length of major axis.} \]
• Draw the line AB

• With P as center and any convenient radius, draw an arc cutting AB at C (shown blue)

• With the same radius cut 2 equal divisions CD and DE (shown red)

• With same radius and centers D and E, draw arcs (green and brown) intersecting at Q

• PQ is the required perpendicular
To draw a normal and a tangent to an arc or circle at a point P on it

- With centre P and any convenient radius, mark off two arcs cutting the arc/circle at C and D.

- Obtain QR, the perpendicular bisector of arc CD. QR is the required normal.

- Draw the perpendicular ST to QR for the required tangent.
Tangent to a given arc AB (or a circle) from a point P outside it.

- Join the centre O with P and locate the midpoint M of OP.
- With M as a centre and radius = MO, mark an arc cutting the circle at Q.
- Join P with Q. PQ is the required tangent.
- Another tangent PQ’ can be drawn in a similar way.
Tangent to two circles

Radius of red filled circle is \( R_2 - R_1 \)

Draw tangent from center A to circle (red colored circle)

External tangent

Internal tangent

Radius of red filled circle is \( R_2 - R_1 \)
Drawing Arc between two straight lines
Drawing Arc ($R_1$) between Line & Arc ($R_2$)
Arc 3

Drawing Arc (R) between two Arcs (RA & RB)
Drawing Arc (R) between two Arc ($R_A$ & $R_B$)
Arc/Circle passing through 3 points
Q No. 1:- Construct an equilateral triangle, regular hexagon and regular heptagon on a common base of 40 mm side (all in one figure). (1.5 MARKS)

Q No. 2:- A heptagon prism with a base side of 45mm and height 90mm has its axis perpendicular to the ground. One of the sides of the base is inclined at 30° to the vertical plane. A section plane inclined at 70° to the ground and perpendicular to the vertical plane and passing through the midpoint of the axis cuts the prism. Draw TV, FV and the side view of the sectioned prism. (2.5 MARKS)

Q No. 3:- Two fixed points are 100mm apart. A point P moves in such a way that the sum of its distances from the two fixed points is always constant and equal to 150mm. Trace the path of the point and name the curve. (1.0 MARK)

Q No. 4:- A cone with a base diameter of 70mm and a height of 80mm is placed coaxially on a circular disc with a diameter of 120 mm and thickness of 35mm. An auxiliary plane inclined to the ground cuts the cone and the disc. Trace the views of the sectioned solids.
Section of Solids

Orthographic Projections
Projection of solids
Section (Hatching)
True Shape / Auxiliary view
Example:

For TV

For True Shape

SECTION PLANE

Apparent Shape of section

SECTIONAL T.V.

TRUE SHAPE OF SECTION

SECTION LINES (45° to XY)
Section more than one component in the same drawing (e.g. concentric cylinders)

Section lines of adjacent components are drawn in different directions

Section lines for alternate components can be drawn in the same direction but with different spacing between section lines
A cube of 65 mm long edges has its vertical faces equally inclined to the FP. It is cut by a section plane, perpendicular to the FP so that the true shape of the section is a regular hexagon. Determine the inclination of the cutting plane with the HP and draw the sectional top view and true shape of the section.
## Location of Section Planes

<table>
<thead>
<tr>
<th>Horizontal Plane</th>
<th>Inclined Plane to H.P. (A.I.P.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T</strong></td>
<td><strong>T</strong></td>
</tr>
<tr>
<td><strong>F</strong></td>
<td><strong>F</strong></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Vertical Plane</th>
<th>Auxiliary V. P.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T</strong></td>
<td><strong>T</strong></td>
</tr>
<tr>
<td><strong>F</strong></td>
<td><strong>F</strong></td>
</tr>
</tbody>
</table>

P.P. cutting plane ??
Oblique cutting plane
A cone, base 75 mm diameter and height 100 mm, has its base on the HP. A section plane parallel to one of the end generators and perpendicular to the FP cuts the cone intersecting the axis at a point 75 mm from the base. Draw the sectional Top View and the true shape of the section.
A cone base 75 mm diameter and axis 100 m long, has its base on the HP. A section plane parallel to one of the end generators and perpendicular to the FP cuts the cone intersecting the axis at a point 75 mm from the base. Draw the sectional Top View and the true shape of the section.
A sphere of 75 mm diameter is cut by a section plane, perpendicular to the FP and inclined at 30° to the HP in such a way that the True Shape of the section is a circle of 50 mm dia. Draw its front view and sectional top view.
Sphere $\phi 75\, 30^\circ$
Section of Pyramid

Pentagonal pyramid base 30 mm and height 65 mm.
A pentagonal pyramid (side of base = 50 mm and height = 100 mm) is resting on its base on the ground with axis parallel to frontal plane and perpendicular to the top plane. One of the sides of the base is closer and parallel to the frontal plane. A vertical section plane cuts the pyramid at a distance of 15 mm from the axis with section plane making an angle of 50° with FP. Draw the remaining part of the pyramid and the true shape of the cut section.
True shape of the section → Auxiliary view to the top view with the reference line parallel to the section plane.
Given pyramid is cut by plane, \( \perp \) to the frontal plane and inclined at \( 70^\circ \) to the top plane. The cutting plane cuts the axis of the pyramid at 15mm from the apex. Draw the projections of the remaining part of the pyramid and the true shape of the cut section.

Since the section plane is perpendicular to the frontal plane, the section line is drawn in the front view.
How to locate the point “l”

Draw an imaginary horizontal line from the axis to the edge oc intersecting at z.

Project the point z into the Top view (oz is TL here).

With o as center and oz as radius draw an arc cutting od at l.

This can also be done by projecting onto ob at y and rotating.

Basically the imaginary line with length oz = oy is rotating inside the pyramid from one edge to another.

This can also be obtained by drawing a line from z in the Top view parallel to dc (as dc is TL here).
How to locate the point “l”
Section of Cylinder

- Cylinder dia = 40 mm.
- Height = 60 mm.
- Axis is vertical.
- Section plane perpendicular to VP, but inclined to 45 degree to the HP and intersecting the axis 32 mm above the base.
A cylinder, diameter of base 30 mm is standing on its base on ground and positioned in third quadrant. The position of center of upper base is O₁ (25, 30, 25) and the center of the lower base is O₂ (25, 30, 85).

Points A (0, 60, 45), B(15, 5, 80) and C(65, 35, 35) lie on a plane that cuts the cylinder in two parts. Draw the two orthographic views of the cut portion of the cylinder.
When cutting plane is oblique
Problem: A square (side 40 mm) pyramid (height 70mm) stands on its base on H.P. and all the base sides are equally inclined to the V.P. A section plane (⊥ to V.P. and inclined at 45° to H.P.) bisects the axis of pyramid. Draw sectional top and sectional side views.
Problem: A sphere of 60 mm diameter is cut by a section plane perpendicular to the V.P., inclined at 45° to H.P. and at a distance of 12 mm from its center. Draw sectional top view.
Ex: A square pyramid of 50 mm side of base and 80 mm length of axis is resting on its base on the H.P., having a side of base $\perp$ to V.P. It is cut by 2-cutting planes. One plane is parallel to its extreme right face and 10 mm away from it, while other is parallel to the extreme left face and intersects first cutting plane on the axis of pyramid. Draw FV and sectional TV.
Ex: A cylinder is cut by an auxiliary plane such that true shape of section is an ellipse of major and minor axes of length 100 mm and 60 mm respectively. The smallest generator of the truncated cylinder is 20 mm. Find inclination (with axis) of the section plane.

Ex: A cone, having base dia of 60 mm and height of 80 mm is resting on its base on HP. It is cut by a section plane such that the true shape of the section in front view is a rectangular hyperbola with a base of 40 mm. Find front and top views.
Cylinder is cut by plane such that true section shape of section is an ellipse (100 mm by 60 mm). Smallest generator of the truncated cylinder is 20 mm.
Ex: A cone, having base dia of 60 mm and height of 80 mm is resting on its base on HP. It is cut by a section plane such that the true shape of the section in front view is a rectangular hyperbola with a base of 40mm. Find front and top views.
Section plane $\perp$ to HP & VP

Section will not be visible either in TV or FV $\Rightarrow$ Side view
ENGINEERING CURVES

Point undergoing two types of displacements

INVOLUTE: Locus of a free end of a *string* when it is wound round a (circular) pole

CYCLOID: Locus of a *point* on the periphery of a circle which rolls on a straight line path.

SPIRAL: Locus of a *point* which revolves around a fixed point and at the same time moves towards it.

HELIX: Locus of a point which moves around the surface of a right circular cylinder / cone and at the same time advances in axial direction at a speed bearing a constant ratio to the speed of rotation.
• Problem:
  Draw Involute of a circle. String length is equal to the circumference of circle.
**Problem:** Draw Involute of a circle. String length is MORE than the circumference of circle.
**Problem:** Draw Involute of a circle.
String length is LESS than the circumference of circle.

**INVOLUTE OF A CIRCLE**
String length LESS than $\pi D$

---

In the diagram, the string length is illustrated to be less than $\pi D$, showing the relationship between the string length and the circumference of the circle for an Involute of a circle.
INVOLUTE OF A PENTAGON

Thread should be taut
**Problem:** A pole is of a shape of half hexagon (side 30 mm) and semicircle (diameter 60 mm). A string is to be wound having length equal to the pole perimeter draw path of free end \( P \) of string when wound completely.

Calculate perimeter length
DEFINITIONS

CYCLOID:
LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A STRAIGHT LINE PATH.

SUPERIOR TROCHOID:
If the point in the definition of cycloid is outside the circle

INFERIOR TROCHOID:
If it is inside the circle

EPI-CYCLOID
If the circle is rolling on another circle from outside

HYPO-CYCLOID
If the circle is rolling from inside the other circle,
PROBLEM: Draw locus (one cycle) of a point (P) on the periphery of a circle (diameter=50 mm) which rolls on straight line path.

Point C (zero radius) will not rotate and it will traverse on straight line.
PROBLEM: Draw locus of a point (P), 5 mm away from the periphery of a Circle (diameter=50 mm) which rolls on straight line path. **SUPERIOR TROCHOID**
**PROBLEM:** Draw locus of a point, 5 mm inside the periphery of a Circle which rolls on straight line path. *Take circle diameter as 50 mm*
EPI CYCLOID:

PROBLEM: Draw locus of a point on the periphery of a circle (dia=50mm) which rolls on a curved path (radius 75 mm).

Distance by smaller circle = Distance on larger circle

Solution Steps:
1. When smaller circle rolls on larger circle for one revolution it covers \( \pi D \) distance on arc and it will be decided by included arc angle \( \theta \).
2. Calculate \( \theta \) by formula \( \theta = \frac{r}{R} \times 360^\circ \).
3. Construct a sector with angle \( \theta \) and radius \( R \).
4. Divide this sector into 8 number of equal angular parts.
EPI-CYCLOID

If the circle is rolling on another circle from outside

\[ r = \frac{r}{R} \times 360^0 \]
**PROBLEM:** Draw locus of a point on the periphery of a circle which rolls from the inside of a curved path. *Take diameter of Rolling circle 50 mm and radius of directing circle (curved path) 75 mm.*

\[ \bigodot = \frac{r}{R} \times 360^0 \]

- \( OC = R \) (Radius of Directing Circle)
- \( CP = r \) (Radius of Generating Circle)
CYCLOID

SUPERIOR TROCHOID

INFERIOR TROCHOID
Interpenetration of Solids / Intersection of Surfaces / Lines & Curves of Intersection

More points common to both the solids

Basic required knowledge:
~ Projections of Solid
~ Section of Solid
For TV

SECTION LINES
(45° to XY)

SECTIONAL T.V.

Intersection of Surfaces

Intersection of Solids

Lines & Curves of Intersection

Apparent Shape of section

For TV

SECTIONAL T.V.

Intersection of Surfaces

Intersection of Solids

Lines & Curves of Intersection

Apparent Shape of section
Square Pipes. Circular Pipes.

Minimum Surface Contact. (Point Contact) (Maximum Surface Contact)

Lines of Intersections. Curves of Intersections.

Two plane surfaces (e.g. faces of prisms and pyramids) intersect in a straight line.

The line of intersection between two curved surfaces (e.g. of cylinders and cones) or between a plane surface and a curved surface is a curve.

More points common to both the solids.
How to find Lines/ Curves of Intersection

Generator line Method

Cutting Plane Method
Guidelines

• Interpenetration of solids produce closed loops which may be made straight lines or curves.
  – Two lines intersect at a point common to both the lines.
  – Two surfaces intersect along a line/curve common to both surfaces.

• Interpenetration of solids containing plane surfaces (prism with prism, pyramid with pyramid, prism with pyramid) results in a polygon.

• Solids having curved surfaces results in closed curve.
What is expected?

- Projection of solid 1.
- Projection of solid 2 with given position w.r.t. solid.
- Finding common points on solid 1 and solid 2.
- Joining common points in proper sequence to get desired line/curve of intersection.
- Correcting/finalizing the orthographic projections.
When one solid completely penetrates another, there will be two curves of intersection.

Problem: Find intersection curve.

Draw convenient number of lines on the surface of one of the solids. Transfer point of intersection to their corresponding positions in other views. When one solid completely penetrates another, there will be two curves of intersection.
Problem: CYLINDER (50mm dia. and 70mm axis) STANDING & SQ. PRISM (25 mm sides and 70 mm axis) PENETRATING. Both axes intersect & bisect each other. All faces of prism are equally inclined to Hp. Draw projections showing curves of intersections. (I-angle)
Problem. SQ.PRISM (30 mm base sides and 70mm axis) STANDING & SQ.PRISM (25 mm sides and 70 mm axis) PENETRATING. Both axes intersects & bisect each other. All faces of prisms are equally inclined to Vp. Draw projections showing curves of intersections.
Problem: A vertical cone, base diameter 75 mm and axis 100 mm long, is completely penetrated by a cylinder of 45 mm diameter and axis 100 mm long. The axis of the cylinder is parallel to Hp and Vp and intersects axis of the cone at a point 28 mm above the base. Draw projections showing curves of intersection in FV & TV. in I angle projection system
**Problem:** Vertical cylinder (80 mm diameter & 100 mm height) is completely penetrated by a horizontal cone (80 mm diameter and 120 mm height). Both axes intersect & bisect each other. Draw FV & TV projections showing curve of intersections in I angle projection system.
Problem. SQ.PRISM (30 mm base sides and 70mm axis; faces equally inclined to VP) STANDING & SQ.PRISM (25 mm sides and 70 mm axis) PENETRATING. Both axes Intersect & bisect each other. Two faces of penetrating prism are 30° inclined to Hp. Draw projections showing curves of intersections in I angle projection system.

Other possible arrangements ????
**Problem:** A vertical cylinder 50mm dia. and 70mm axis is completely penetrated by a horizontal triangular prism of 45 mm sides and 70 mm axis. One flat face of prism is parallel to Vp and contains axis of cylinder. Draw projections showing curves of intersections in I angle projection system.
**Problem:** Cone (cone 70 mm base diameter and 90 mm axis) standing & sq. prism penetrating (both axes vertical)

Axis of prism is // to cone’s axis and 5 mm away from it. A vertical plane containing both axes is parallel to Vp. Take all faces of sq. prism equally inclined to Vp. Base side of prism is 30 mm and axis is 100 mm long. Draw projections showing curves of intersections.
Intersection of two cylinders oblique to each other – Use PAV

III angle projection
Intersection of two cylinders oblique to each other – Use PAV
Intersection of two cylinders oblique to each other – Use AV
Intersection of Cone and Oblique cylinder using PAV
Intersection of irregular Prism & offset Cylinder

T

F

III angle projection
Intersection of irregular Prism & offset Cylinder
Intersection of irregular Prism & offset Cylinder
Intersection of irregular Prism & offset Cylinder
DEVELOPMENT OF SURFACES OF SOLIDS

Solids are bounded by geometric surfaces:
- Plane \(\rightarrow\) Prism, Pyramid
- Single curved \(\rightarrow\) Cone, Cylinder
- Double curved \(\rightarrow\) sphere

LATERLAL SURFACE is the surface excluding the solid’s top & base.

Development \(\sim\) obtaining the area of the surfaces of a solid.
Surface Development of Hollow Solids

• Negligible thickness.
• Cutting hollow solid along any of its edge/generator and spreading it as sheet of paper.
  – All dimensions of the developed surface MUST be of TRUE LENGTH.
Cylinder cut by three planes
Methods to Develop Surfaces

1. **Parallel-line development**: Used for prisms (full or truncated), cylinders (full or truncated). Parallel lines are drawn along the surface and transferred to the development.

- **Cylinder**: A Rectangle
  - H = Height
  - D = base diameter

- **Prisms**: No. of Rectangles
  - H = Height
  - S = Edge of base
**Ex:**

Development by Faces:
- Front (Rear)
- Right (Left..Symmetry)
- Top (Bottom..Symmetry)

Complete development.
DOTTED LINES are never shown on development
Methods to Develop Surfaces

1. **Parallel-line development**

2. **Radial-line development**: Used for pyramids, cones etc. in which the true length of the slant edge or generator is used as radius

   - **Cone**: (Sector of circle)
     - \( R = \text{Base circle radius.} \)
     - \( L = \text{Slant height.} \)
     - \( \theta = \frac{R}{L} \times 360^\circ \)

   - **Pyramids**: (No.of triangles)
     - \( S = \text{Edge of base} \)

   \[
   \theta = R 
   \]
   \[
   L = \text{Slant edge.} 
   \]

---

Development of Lateral surface

Cone

Base

(Not necessary)
Parallel vs Radial line method

Parallel line method

Radial line method
**Important points.**

1. Development is a shape showing AREA, means it’s a 2-D plain drawing.
2. All dimensions of it must be TRUE dimensions.
3. As it is representing shape of an un-folded sheet, no edges can remain hidden and hence DOTTED LINES are never shown on development.
If the slant height of a cone is equal to its diameter of base then its development is a semicircle of radius equal to the slant height.

Development by Radial Method—Pyramids (full or Truncated) & Cones (full or Truncated).
Ex:
Complete development of cube cut by cutting plane (inclined to HP at 30 degrees and perpendicular to VP)
Intersection of Plane & Pyramid. Development of resulting lateral truncated Pyramid

Develop
1-D-A-2-1
2-A-B-3-2
3-B-C-4-3
1-D-C-4-1
Development of Oblique Objects

- Right regular objects – Axis of object perpendicular to base.
- Axis of any regular object (prism, pyramid, cylinder, cone, etc.) inclined at angle other than right angle – Oblique OBJECT. Use ARC method.
Oblique prism
Draw the development of an oblique circular cylinder with base diameter 30 mm and axis inclined at 75° with the base. Height of the cylinder is 50 mm.
Oblique Cone
Methods to Develop Surfaces

1. **Parallel-line development**: Prismatic objects (cylinder, prism)
2. **Radial-line development**: Non-prismatic objects (cone, pyramid).
   - Apex as center and slant edge as radius.
3. **Triangulation development**: Complex shapes are divided into a number of triangles and transferred into the development

**EXAMPLES**:
- Boiler Shells & chimneys,
- Pressure Vessels, Shovels, Trays,
- Boxes & Cartons, Feeding Hoppers, Large Pipe sections,
- Body & Parts of automotives,
- Ships, Aero planes.
Connect two hollow objects having different base.

Transition Pieces

Triangulation Method: Dividing a surface into a number of triangles and transfer them to the development.
Ex: In an air conditioning system, a square duct of 50mm by 50mm is connected to another square duct of 25mm by 25mm by using a connector (transition piece) of height 25mm. Draw development of lateral surface of the connector (Neglect thickness of connector).

Pyramids: (No. of triangles)

L = Slant edge.
S = Edge of base
Development of Transition Piece for Difference Shapes and Sizes

- Connect a square pipe with circular pipe.
- Ex: Imagine a transition piece (height = 25) to connect a chimney of square cross section 50mm * 50 mm to circular pipe of 30mm diameter. Draw the projections and develop the lateral surface of the transition piece.
1/8 of circumference

1 angle projection
Development of Sphere using Frustum of Cones: Zone Method

Zone 1: Cone
Zone 2: Frustum of cone
Zone 3: Frustum of cone
Zone 4: Frustum of cone

θ = \frac{R}{L} \times 360°
Development of Sphere/Hemisphere using Lune Method

- 25% circumference
- 50% circumference