Journal bearing design using multiobjective genetic algorithm and axiomatic design approaches

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Abstract
This paper describes the optimum design methodology for improving operating characteristics of fluid-film steadily loaded journal bearings. This methodology consists of (1) a simplified closed form solution to accelerate the computation, (2) finite difference mass conserving algorithm for accurate prediction of lubricant flow and power loss, (3) Pareto optimal concept to avoid subjective decision on priority of objective functions, (4) a genetic algorithm to deal with multimodal nature of hydrodynamic-bearing and develop a Pareto optimal front, (5) fitness sharing to maintain genetic diversity of the population used in genetic algorithm, and (6) axiomatic design to provide inside of objective functions and design variables. In the optimum design of journal bearings, the design variables such as radial clearance, length to diameter ratio, groove geometry, oil viscosity and supply pressure are used to simultaneously minimize oil flow and power loss. A step-by-step procedure, graphs and tables are presented to demonstrate the concept and effectiveness of suggested design methodology.

Keywords: Multiobjective optimization of journal bearing; Genetic algorithm for Pareto optimal front; Axiomatic design

1. Introduction
The self-acting journal bearings are well known for their exceptional damping, negligible friction and no wear environment. Such bearings are designed on hydrodynamic lubrication theory, which assumes separation of two surfaces in relative motion by a thin layer of viscous fluid. The surfaces are kept apart by pressure generated in the fluid film due to viscous forces. The bearing performance is governed by design variables such as radial clearance between rotor and bearing, bearing length, groove geometry and its location, and lubricant supply pressure and viscosity. A number of researchers [1–6] adopted various assumptions and simplifications of Reynolds hydrodynamic equation, different optimization techniques to design bearing clearance, and length, and decide the lubricant viscosity.

To optimize the journal bearing design variables at minimum computation efforts, Hashimoto [1] and Yang et al. [2] considered short bearing approximation. However, short bearing approximation overestimates the film-pressure, and thus results in higher oil flow and more power loss. To reduce computation time and increase accuracy, the present paper utilizes the hybrid scheme [7]. The hybrid scheme considers analytical pressure solution to find out eccentricity ratio and attitude angle. Mass conservation algorithm utilizes these values as an initial approximation for eccentricity ratio and attitude angle, and predicts reliable results for objective functions (power loss and oil flow) and design constraints (minimum film thickness, maximum pressure, and temperature rise).

multiobjective optimization of rotor bearing system. Hashimoto [8] minimized linear summation of temperature rise and supply lubricant quantity using three optimization methods: successive quadratic programming, genetic algorithm, and direct search methods. Hashimoto [8] concluded that the computation times needed to obtain the optimized solution by direct search and genetic algorithm were about 5 times and over 20 times, respectively, as the computation time by successive quadratic programming.

The current paper considers minimization of lubricant flow and minimization of power loss as the two objective functions. These objective functions are contradictory to each other. Increase in oil viscosity and bearing length reduces oil flow but increases the power loss. Similarly, increasing the bearing radial clearance decreases the loss in power, but increases the oil flow drastically. Hence, journal bearing problem has no single optimal solution that could optimize all objective functions simultaneously. There is an inevitable trade off between design objectives, which can be found using any multiobjective optimization algorithm [9]. Generally, multiobjective optimization methods are classified into two broad categories:

- **Priori articulation of preferences**: Combining individual objective functions using pre-decided weight factors into a single utility function [1–3, 6, 8] and solve problem as a single objective optimization problem. Weight factors express relative importance of objective function in the overall utility measure.
- **Posteriori articulation of preferences**: In this method optimizer first generates a number of non-inferior (a set of equally efficient) solutions and then final decision is made on any one solution. This approach is often referred to as Pareto optimal approach. As a set of many tradeoff solutions are already available with their cons and pros, Pareto optimal approach helps high-level qualitative decision.

The present paper uses the second approach and derives Pareto-optimal front for journal bearing. As genetic algorithm uses a population of solutions, it is easier to capture multiple non-inferior solutions (Pareto optimal set) in the final population. There are a number of advantages of genetic algorithms such as:

- easy to deal with discrete design variables (such as oil viscosity),
- useful in finding global solution of multimodal objective function, and
- effortless capturing of multiple non-inferior solutions.

Therefore, this paper utilizes a genetic algorithm to solve multiobjective optimization problem of journal bearing. A rank-based approach is used to allot the rank (which will help to decide the fitness) to the individual design solution. Further, fitness sharing [9] is used to maintain the diversity in the Pareto optimal set.

It is interesting to note that theoretical results for oil viscosity obtained by Hashimoto [1] and Hirani [6] are far from oil viscosity used in the real practice. Hirani [6] analyzed three established case studies and showed a significant reduction in friction using thinner viscosity oil. Similarly, results in Hashimoto [1] paper optimize the minimum level of oil viscosity. The difference in the practical and theoretical approach motivated us to utilize axiomatic design approach [10] to the journal bearing. As per axiomatic design there should be one-to-one matching between functions and design variables/components/subassembly. If there are two objective functions then for the best results, there should be only two design parameters. The present paper uses this approach for deeper understanding of journal bearing design.


The design of journal bearing is an inverse problem, where for a given load and speed, the eccentricity ratio and attitude angle are determined. Most of the available numerical methods provide reliable results for journal bearing work for the given eccentricity ratio and attitude angle. To solve bearing-inverse problem, initial guess of eccentricity ratio and attitude angle are made, and an iterative procedure is used to match the given load and speed. Such an iterative process takes lot of computation time, particularly for the optimization case where bearing problem is solved a number of times to obtain various design vectors. To accelerate the optimization process the current paper utilizes bearing design table (Table 1) suggested by Hirani et al. [11].

An initial guess of eccentricity ratio is made using following equation [1]

$$\varepsilon(x) = \exp(-2.236\sqrt{n_s\mu D^4/48C^2F})$$  \hspace{1cm} (1)

where \(C\) is radial clearance (m), \(F\) is fluid force, \(D\) is journal diameter (m), \(n_s\) is journal rotational speed (rps), and \(A\) is length to diameter ratio. This value of eccentricity ratio is updated to required eccentricity ratio using:

$$\varepsilon_{i+1} = \varepsilon_i + \left(\frac{F_{\text{given}}}{F_{\text{estimated}}} - 1\right)0.005$$  \hspace{1cm} (2)

Once eccentricity ratio and attitude angle are known, pressure distribution across the circumferential direction (\(\theta\)) and along with (Z) directions is determined using [11]:

$$P = \frac{12\mu UR}{C^2g_0} \frac{A^2(0.25 - (Z/L)^2)\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta)^3} \left[1 + \frac{2\varepsilon A^2}{g_0(1 + \varepsilon \cos \theta)(2\varepsilon + \cos \theta)}\right]$$  \hspace{1cm} (3)

Although eccentricity ratio, pressure distribution and attitude angle can be calculated with reasonable accuracy
viscosity of lubricant (Pa s). In the present paper, and 
and $P_{B21} = B_{21}(1 + \varepsilon)(2 + \varepsilon)$; $B_{22}$
$= B_{21}(1 - \varepsilon)(2 - \varepsilon)$

3. $B_{3j} = \sqrt{B_{2j} + 0.25}; B_{3j} = \frac{B_{2j} + 0.5}{B_{2j} - 0.5}; j = 1, 2$

4. $H = 1 + \varepsilon \cos \theta$; $B_{21} = H(1 + H)$; $B_{33} = \sqrt{B_{23} + 0.25} B_{43}$

5. $Q_{hi} = UCL\left[1 - \frac{e^2}{25(B_{21} - B_{43}) + B_{22}(1 - B_{42})}\right]$, $mf_{1} = (1 + \varepsilon \cos \theta_{1})^{3} + (1 + \varepsilon \cos \theta_{2})^{3}$.

$mf_{2} = [\theta + 3\varepsilon \sin \theta + e^{2}(1.5\theta + 0.75 \sin 2\theta) + e^{3}(\sin \theta - 0.333 \sin^{3} \theta)]^{1/2}$

$Q = Q_{h} + Q_{p} - 0.3V_{Q}\sqrt{Q_{1}Q_{p}}$

6. $I_{i} = 6ULR_{m}\sqrt{\frac{e}{C_{r1}}\left(1 + \frac{2 + \varepsilon}{(2 + \varepsilon)^{2}}\right) - B_{43}}$

7. $F_{i} = -\int_{0}^{\infty} I_{i}R \cos \theta \, d\theta$; $F_{p} = -\int_{0}^{\infty} I_{i}R \sin \theta \, d\theta$; $F = \sqrt{F_{i}^{2} + F_{p}^{2}}$

10. fric = $ULR_{m}\pi \frac{2 + \varepsilon}{C_{r1} - 2R} + \frac{C_{r}F_{p}}{C_{r1}}$; $W = \text{fric} \times U$

11. $\Delta T = \frac{\varepsilon W}{\rho C_{p}Q}$; $\mu = \mu_{0} e^{-\Delta T}$; $\phi = \tan^{-1}(F_{p}/F_{i})$

Here fric is friction force, $L$ is length (m), $\rho$ is oil density (kg/m$^3$), $C_{r}$ is specific heat of lubricant (J/kg K), $\theta$ is circumferential coordinate, $R$ is radius (m), $U$ is velocity (m/s), $\Delta T$ is temperature rise, $k$ is temperature coefficient (1/°C), $Q$ is oil flow (m$^3$/s), $W$ is power loss (W), $\phi$ is attitude angle (radians) and $\mu$ is average viscosity of lubricant (Pa s). In the present paper, $\rho = 860$ kg/m$^3$, $k = 0.029$ 1/°C and $C_{r} = 4190$ J/kg K are used.

using this approach, the predictions of oil flow and power will be erroneous [7]. Therefore this approach is used only for an initial approximation.

### 3. Finite difference mass conserving algorithm [7]

The eccentric position of journal in a hydrodynamic bearing forms a converging–diverging clearance space. Due to unavailability of sufficient liquid lubricant, liquid-streamers separated by gas/vapor space are formed in the divergent clearance space. This ‘cavitation’ phenomenon cannot be appropriately predicted using the Reynolds equation alone, or with Gumbel/Reynolds boundary conditions [7]. Errors in calculation of oil flow and power loss dominates, particularly when the oil groove is placed in the converging domain.

Therefore, the universal equation [7] that describes both the full film and cavitation regions of a bearing is used in the current paper. The equation under static load, is expressed as

$$
\frac{\partial}{\partial x} \left( \frac{hU}{2} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^{3}}{12\mu} f \frac{\partial}{\partial z} \right) = 0
$$

where $\beta$ is bulk modulus (Pa), $P_{c}$ is cavitation pressure (Pa), $g = \begin{cases} 1 & \text{in full region} \\ 0 & \text{in cavited region} \end{cases}$ and $P = P_{c} + g\beta \log \theta$.

The value of $g=1$ makes Eq. (4) an elliptic partial differential equation (PDE), while $g=0$ makes Eq. (4) a hyperbolic PDE. A robust convergent solution needs a central-finite-difference-scheme for elliptical PDE, and upwind-finite-difference scheme for hyperbolic PDE. This can be achieved by adopting a type difference scheme, such as [12]:

$$
\frac{\partial}{\partial x} \left( \frac{hU}{2} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^{3}}{12\mu} f \frac{\partial}{\partial z} \right) = 0
$$

(4)

$$
\frac{\partial}{\partial x} \left( \frac{hU}{2} - \beta h^{3} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^{3}}{12\mu} f \frac{\partial}{\partial z} \right) = 0
$$

(5)

$g = \begin{cases} 1 & \text{in full region} \\ 0 & \text{in cavited region} \end{cases}$ and $P = P_{c} + g\beta \log \theta$. The value of $g=1$ makes Eq. (4) an elliptic partial differential equation (PDE), while $g=0$ makes Eq. (4) a hyperbolic PDE. A robust convergent solution needs a central-finite-difference-scheme for elliptical PDE, and upwind-finite-difference scheme for hyperbolic PDE. This can be achieved by adopting a type difference scheme, such as [12]:

$$
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$$

(4)
Substituting Eqs. (5)–(7) in Eq. (4) provides Eq. (8) with \( \theta \) as unknown:

\[
f(\theta) = 0
\]

(8)

To evaluate the values of \( \theta \), the Newton iterative scheme [12] is used

\[
f(\theta^*) + \left( \frac{\partial f}{\partial \theta} \right)_{\theta=\theta^*} (\theta - \theta^*) = 0
\]

where \( \theta^* \) is the available value of \( \theta \). The Newton method provides rapid convergence if initial guess of the unknown is in the neighborhood of real solution. Here the analytical approach, discussed in previous section, helps to determine reasonably good initial approximation.

The oil leakage and power loss can be evaluated by using the following expressions:

\[
Q = \int \frac{h^2}{12\mu} \frac{\partial P}{\partial z} R \, d\theta
\]

(10)

\[
W = \int \left( \frac{\mu U}{h} + \frac{h}{2\, R} \frac{\partial P}{\partial \theta} \right) UR \, d\theta \, dz
\]

(11)

This mass conserving approach provides [7] reasonably accurate results of bearing performance parameter for a given set of inputs, such as bearing clearance, length, oil feed-groove geometry, lubricant supply pressure and lubricant viscosity. To optimize a journal bearing, various different values of design variables are supplied to mass conserving algorithm. Such values are provided using one of optimization algorithms. In the present case a genetic algorithm is used for this purpose.

4. Genetic algorithm

This section describes the details of the GA, tailored to solving bearing optimization problem. Five design variables: the radial clearance, \( C \) [in the range of 35–70 \( \mu \)m], slenderness ratio \( L \) [0.2 \( \leq L \leq 0.9 \)], oil viscosity \( \mu \) (1 \( \leq \mu \leq 16 \) mPa s), groove location (in the range of 20–90° from load line), and supply pressure \( P_t \) (in the range of 100–450 kPa) are selected to control the bearing performance. Binary coding GA with five digits for each variable is chosen. Total population of 50 members, with crossover probability of 0.9 and mutation probability of 0.01 are considered. Constraint on maximum pressure (from material strength point of view, \( P_{\text{max}} \leq 30 \) MPa), on minimum film thickness (to assure full hydrodynamic regime, \( h_{\text{min}} \geq 5 \mu \)m), and on maximum temperature rise (to limit the thermal thinning of lubricant, \( \Delta T \leq 10^\circ \)C) are imposed. To penalize the solutions that violate constraints, following normalization approach is employed:

\[
\text{Penalty} = 0.0
\]

If \( P_{\text{max}} > 30 \) MPa, then Penalty = Penalty + \( P_{\text{max}}/30.e6 \)

If \( \Delta T > 10^\circ \)C, then Penalty = Penalty + \( \Delta T/10 \)

If \( h_{\text{min}} < 5 \mu \)m, then Penalty = Penalty + 5/\( h_{\text{min}} \)

\[
W_i = \frac{W_i - W_{\text{min}}}{W_{\text{max}} - W_{\text{min}}} + \text{Penalty},
\]

\[
Q_i = \frac{Q_i - Q_{\text{min}}}{Q_{\text{max}} - Q_{\text{min}}} + \text{Penalty}
\]

(12)

Simultaneous minimizations of flow rate and power loss are targeted, as power loss and flow rate are conflicting and no single set of design variables can be considered as the best for both the objective functions. Therefore, exploration of a set of equally efficient, or non-inferior, alternative design vectors, known as ‘Pareto Optimal Set’ is aimed. A design vector \( \mathbf{x}_i \) is treated non-inferior if the following conditions are fulfilled:

- \( W_i \leq W_j \) and \( Q_i < Q_j \) or
- \( W_i < W_j \) and \( Q_i \leq Q_j \)

To the generated new population that requires fitness, a rank based fitness is employed. All non-dominated solutions are assigned rank 1 (highest fitness). Rank 2 is allocated to a solution which is dominated only by one member of population. Fitness, to a member is assigned as:

\[
\text{fitness}_i = 1 - \frac{\text{rank}_i}{\sum_{j=1}^{N} \text{rank}_j}
\]

(13)

To generate Pareto-optimal front, it is important to maintain diversity in non-inferior solution [9]. Losing diversity of the population is quite a possibility due to stochastic selection process used in genetic algorithm. To reduce this possibility, following sharing fitness function that determines the neighborhood and degree of sharing is used [9]:

\[
\text{Sh}(\text{dis}) = \begin{cases} 
1 - (\text{dis}/\sigma_{\text{share}}), & \text{if dis} \leq \sigma_{\text{share}} \\
0, & \text{otherwise}
\end{cases}
\]

(14)

This sharing function helps to reduce the number of copies of same function, by reducing its fitness in following manner:

\[
\text{fitness} = \frac{\text{fitness}}{\sum_{i=1}^{N} \text{Sh}(\text{dis})}
\]

(15)

The effectiveness of this sharing function and rank-based fitness is demonstrated in Fig. 1. Bearing optimization is performed for 20,000 N load and 50 rps rotational frequency. Fig. 1 illustrates the diversity in the design solution is maintained even in 1000 run of generation. However, this figure shows inferior and non-inferior solutions. This brings out the drawbacks of stochastic programming. It is difficult to maintain all good solutions obtained at every generation. Often elite preserving operator [9] is used to directly carry over elites (non-inferior solutions) to the next generation. The introduction of elitism ensures the good solution does not deteriorate.
To preserve non-inferior solutions and to maintain their active role in the genetic operation, a separate pool scheme is introduced. To understand this, let us assume that 10–20% of the solutions are non-inferior at every generation. This scheme preserves all non-inferior solutions in a separate pool, named as ‘elite pool’ and lets the genetic algorithm run as usual. After every 25 generations, filter out inferior solutions by comparing all elite (rank 1) solutions obtained from 25 generations. This approach helps faster convergence without impairing any property of evolutionary algorithm. Fig. 2, illustrates the results obtained using ‘a separate pool scheme’. Results of power loss and flow rate for optimized design variables, along with maximum pressure, minimum film thickness and temperature rise are listed in Table 2.

Results of Table 2 are interesting to study. Optimized radial clearance falls in the range of 35–40 μm, even though selected design range was 35–70 μm. Larger clearance reduces the power loss but increases the oil flow. However, lower limit is imposed by rise in the temperature, while upper limit is exerted by constraint on film thickness. Optimized results for length to diameter ratio, groove location and supply pressure do not explain any systematic behavior of these variables. However, there is a strong effect of oil viscosity. High value of viscosity shoots-up the power loss and lower value of the same helps in reducing the oil flow. Constraint on the minimum film thickness restricts the lower limit of viscosity, while constraint on temperature rise plays its part by limiting the upper value of viscosity.

Interestingly, Table 2 suggests lower value of viscosity, even for load 20,000 N and rotational speed of 50 rps. Similarly, low values of oil viscosity were computed by Hashimoto [1]. Hirani [7] analyzed three case studies, and suggested lower oil viscosity 2–4 mPa s in the place of 20–30 mPa s. This huge difference in the practical and theoretical approach motivates us to utilize axiomatic design approach [10] to the journal bearing.

5. Axiomatic design

Axiomatic design [10] provides a thinking process to create a new design and/or to improve the existing design. To improve a design, the axiomatic approach uses two axioms named as ‘independence axiom’ and ‘information...
axiom. The independence axiom examines the independence of functional requirements. This axiom is particularly useful for multiobjective optimization. For example, consider two objective functions that need to be minimized. As shown by the arrow in Fig. 3, the designers’ target needs to be absolute optimum of every objective function. Therefore, front 4 is the best among all Pareto optimal fronts. However, front 4 results are far away from absolute optima (\( f_1 = 30 \) and \( f_2 = 30 \)). In such a case independent axiom helps in the journey towards absolute minima. It tries to remove the conflicts between objective functions.

To understand function independence let us consider two cases:

- Hashimoto [1] assumed \( \Delta T = f_1 \) and \( Q = f_2 \) as two objective functions for bearing optimization. As \( \Delta T \) is function of \( W \) and \( Q \), which means \( f_1 = \text{func}(W, f_2) \). Therefore, Hashimoto’s [1] objective function does not satisfy axiomatic design.
- Hirani [7] considered \( W = f_1 \) and \( Q = f_2 \) as two objective functions for bearing optimization. Therefore Hirani’s [1] objective functions satisfy function independence.

Once independent functions are identified, next step is to choose suitable design parameters (DPs) that bring the solution near absolute optima.

In the present paper two objective functions (\( FR_1 = W \) and \( FR_2 = Q \)), and five design parameters (\( DP_1 = C \), \( DP_2 = A \), \( DP_3 = \) groove location, \( DP_4 = \) supply pressure, and \( DP_5 = \mu \)) are considered. This multi-FR (multiobjective) design problem can be represented by [10]

\[
\begin{align*}
\{ FR_1 \} &= \left[ \begin{array}{c}
A_{11} \\
A_{12} \\
A_{13} \\
A_{14} \\
A_{15}
\end{array} \right] \\
\{ FR_2 \} &= \left[ \begin{array}{c}
A_{21} \\
A_{22} \\
A_{23} \\
A_{24} \\
A_{25}
\end{array} \right] \\
&= \left[ \begin{array}{c}
DP_1 \\
DP_2 \\
DP_3 \\
DP_4 \\
DP_5
\end{array} \right]
\end{align*}
\]

where \( A_{ij} = (\partial F_i / \partial D_j) \), named as ‘stiffness’, represents the dependence of objective functions on design parameters. To get the feeling of these stiffness coefficients, dependence of objective function on individual design variable is shown in Figs. 4 and 5. Load = 20,000 N, speed = 50 rps and journal dia. = 0.1 m are used to get all results illustrated in Figs. 4 and 5. \( C = 40 \), \( A = 0.5 \), groove loc. = 25\(^\circ\), supply pressure = 100 kPa and \( \mu = 0.006 \) Pa s are used, wherever design parameter are not stated.

Fig. 4 shows \( A_{15} > A_{12} > A_{11} > A_{13} > A_{14} \). Further, \( A_{14} \rightarrow 0 \), for groove location lesser than 30\(^\circ\), \( A_{13} \rightarrow 0 \). Similarly Fig. 5, indicates \( A_{21} > A_{22} > A_{25} > A_{23} > A_{24} \) Further, \( A_{24} \rightarrow 0 \), for groove location lesser than 30\(^\circ\). Using these
observations, one can update Eq. (16) in following manner:

\[
\begin{align*}
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix}
&=egin{bmatrix}
x & X & 0 & 0 & X \\
X & x & 0 & 0 & X
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2 \\
DP_3 \\
DP_4 \\
DP_5
\end{bmatrix} \\
&= (17)
\end{align*}
\]

Here \(X\) indicates medium to large effect, \(x\) specifies low to medium effect and \(0\) shows insignificant effect. In other words, if groove is located in divergent domain, its effect on power loss and flow rate is insignificant. However, to stabilize the bearing, often the groove is partially located in the convergent region.

Eq. (17) shows more number of DPs (five) than FRs (two). In the terminology of axiomatic approach, such a design is called a ‘redundant design’. In axiomatic design view every design needs to be an ideal design. An ideal design has number of DPs equal to number of FRs, with the following relation:

\[
\begin{align*}
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix}
&=egin{bmatrix}
A_{11} & 0 \\
0 & A_{22}
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} \\
&= (18)
\end{align*}
\]

Therefore, the next task is to convert redundant design into ideal design. There are two ways: (1) increase the number of
functional requirements, or (2) reducing the number of design variables. If one selects second option of reduce design variables to control power loss and flow rate, then he/she considers the effect of individual parameter on the objective function. Discussion in the previous paragraph indicates insignificant effect of groove location, if groove is located in the divergent portion of circular bearing. Similarly, supply pressure has insignificant effect on the flow rate and negligible effect on the power loss. Further, supply pressure that is higher than atmospheric pressure always increases the oil flow rate. Therefore, groove location and supply pressure can be removed from the variable list.

Design parameter bearing length exerts low to medium effect on the flow rate and strong effect on power loss. Further, length value lesser than 0.6 times of diameter does not create any contradiction. Any increment in the value of bearing length from 0.3 to 0.6 times of the diameter will increase power loss and flow rate. Therefore, final value of bearing length will be decided by applied load and allowable maximum pressure.

Based on this discussion one can choose the values of three design variables as: groove location = 25°, supply pressure = 100 kPa, and bearing length in the range of 0.4–0.5. Using these considerations, Eq. (17) is modified as:

\[
\begin{align*}
\{ \text{FR}_1 \} &= \{ X \} [ \text{X} \} [ \begin{bmatrix} \text{DP}_5 \end{bmatrix} \\
\{ \text{FR}_2 \} &= \{ X \} [ \text{X} \} [ \begin{bmatrix} \text{DP}_1 \end{bmatrix} \\
\end{align*}
\]

Eq. (19) presents a very simplified problem, having only three design variables (clearance, and viscosity) and two...
objective functions (power loss and flow rate). The suggested design methodology, consists of

- closed form solution to guess eccentricity ratio, attitude angle and pressure,
- finite difference mass conserving algorithm for accurate prediction of lubricant flow and power loss,
- Pareto optimal concept to avoid subjective decision on priority of objective functions,
- genetic algorithm to deal with multimodal nature of hydrodynamic-bearing and develop a Pareto optimal front,
- fitness sharing to maintain genetic diversity of the population used in genetic algorithm, and
- separate pool scheme to preserve elite non-inferior solution.

One can find results illustrated in Fig. 6 and Table 3. Lower limit of radial clearance is reduced from 35 to 30 μm. However, results of Table 3 are superior compared to those indicated in Table 2 but they are not the best.

If one compares Eq. (19) with Eq. (18), he/she finds the existence of off-diagonal terms in Eq. (19). In axiomatic design terminology, Eq. (19) represents a coupled designed. For an ideal design, the design needs to be uncoupled. To understand this, let us optimize power loss and flow rate individually assuming two variable functions. Minimization of power loss provides a value of 435.21676 W for clearance 30.7 μm and viscosity =0.0029. Optimization of flow rate provides a value of 1.3934015 x 10^{-5} m^3/s for clearance 30.0 μm and viscosity=0.0065. These results indicate viscosity as a root of coupling and this coupling restricts one from achieving the true optima (\(W=435\) W, and \(Q=14\times 10^{-6} \) m^3/s). To resolve this coupling one needs to have a thorough understanding of oil viscosity.

In practice 20–50 mPa s value of oil viscosity is used to sustain 20,000 N load of shaft rotating at 50 rps. Though such a high value of viscosity will increase power loss by 5–10 times compared to the power loss predicted in the present study, it is still used by experienced engineers. Probably high viscosity is used to achieve the ‘zero wear’ [13]. Wear is assumed to be negligible if a change in the surface contour is less than or of the same order as the surface finish [13].

The design analysis presented in the current paper assumes a fully hydrodynamic operation of journal bearing. However, surface unevenness impairs the hydrodynamic performance and bearing has to run under partial hydrodynamic lubrication conditions, at least for first few cycles of journal rotation. In order to avoid possible bearing failure during this period an early establishment of a fully hydrodynamic lubrication is required. High value of oil viscosity suffices this requirement but gives way to high power loss. However, high power loss is of little worry in real practice. To understand, let us take example of results of 750 W power loss for journal bearing which support 22,000 N load and has relative speed of 50 rps and inner diameter of 0.1 m. This causes coefficient of friction as low as 0.002386. Even if one increases the oil viscosity by 5 times, coefficient of friction will be lesser than 0.02, which is acceptable to the industry.

![Fig. 6. Pareto optimal front for two variable problem.](image)

<table>
<thead>
<tr>
<th>Power loss (W)</th>
<th>Oil flow (m^3/s)</th>
<th>Clearance (μm)</th>
<th>Oil viscosity (Pa s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>443.8162</td>
<td>1.61x10^{-5}</td>
<td>30.32258</td>
<td>3.01x10^{-3}</td>
</tr>
<tr>
<td>474.4606</td>
<td>1.58x10^{-5}</td>
<td>30.29326</td>
<td>3.49x10^{-3}</td>
</tr>
<tr>
<td>478.56</td>
<td>1.58x10^{-5}</td>
<td>30.43988</td>
<td>3.54x10^{-3}</td>
</tr>
<tr>
<td>500.4021</td>
<td>1.54x10^{-5}</td>
<td>30.1173</td>
<td>3.83x10^{-3}</td>
</tr>
<tr>
<td>514.6745</td>
<td>1.53x10^{-5}</td>
<td>30.1173</td>
<td>4.04x10^{-3}</td>
</tr>
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Previous paragraph highlights an additional function of oil viscosity. This makes three objective functions, minimization of wear, power-loss and flow-rate for a specified load and journal speed. This additional function enhances the previously discussed coupling due to oil viscosity. To break this coupling due to third function, literature suggests three approaches:

1. Using lower viscosity with friction reducing additives (‘friction modifier’) [14].
2. Circumferential microgrooves [15]
3. Undulated surface [16]

Bartz [14] analyzed the influence of friction modifier by assuming 1/3 of total friction losses occur in the mixed film or boundary regime, whereas remaining losses occur in the hydrodynamic lubrication. He illustrated the reduction in engine friction by using low viscosity oils with friction modifiers, and showed the reduction in engine fuel consumption by 2.7–5.8%. Kumada et al. [15] made microgrooves in the circumferential direction by fine boring on the bearing inner surface. They kept circular cross section of microgrooves with a pitch 0.2–0.25 mm and a depth of 4.0–4.5 μm. They showed that micro-grooved bearing establishes hydrodynamic lubrication more readily (takes 20% of time) compared to a normal bearing. The key concept here is the retention of lubricant in the microgrooves that help in reducing friction and wear at the start of journal bearing operation. Oktay and Suh [16] worked on the basics of wear mechanism and developed a surface that reduces the particle agglomeration. As the main role of lubricant in the boundary lubrication mechanism is to prevent wear particle agglomeration or built up edge, a suitable surface having only well shaped valleys, but with no peaks (‘termed as undulated surface’) can do the same work by discarding minute wear debris in the valleys.

Three solutions reviewed in the previous paragraph indicate that coupling can be avoided by choosing a new variable (friction reducing agents [14], microgrooves [15], or undulated surface [16]). This new well designed variable will be effective at the start of bearing operation, and will be inactive once full hydrodynamic operation is resumed. This inspires us to think about a lubricant having high viscosity at one time/space (for low flow rate) and low viscosity at other time/space (for low power loss). This concept of variable viscosity reveals the shortcomings of the present algorithm. The present design approach assumes a uniform effective thermal analysis, and does not consider point-to-point temperature variation. For a bearing under static load condition a temperature difference of 15–20°C is generally observed between the supply and minimum film thickness points [17]. Further, experimental results indicate a high temperature (nearly equivalent to temperature at minimum film thickness) in the divergent region. If one assumes oil viscosity μ₀ at the point of lubricant supply, then viscosity at minimum film thickness and divergent region will be approximately μ = μ₀ e⁻κτ where

$$\kappa = 0.029$$

In the present analysis, an effective thermal analysis is used. If actual temperature rise is in the range of 0–20°C, then effective thermal analysis predicts temperature rise of approximately 10°C. Therefore, if supply oil has viscosity μ₀, the present analysis will use viscosity equivalent to 0.75μ₀. Hence, slightly higher power loss and oil flow will be predicted. To demonstrate this point, let us consider viscosity variation as μ = 0.5μ₀(1 + cos((θ - ϕ)/2)) for 0 ≤ (θ - ϕ)/2 ≤ π. In the divergent region, viscosity is 0.5μ₀. The optimized results obtained for such a viscosity distribution is shown in Fig. 7, where series 1, 2 and 3 represent the optimized results for viscosity variation (μ₀ to 0.5μ₀) only in convergent region, constant viscosity (μ₀), and viscosity variation convergent region (μ₀ to 0.5μ₀) and divergent region (0.5μ₀). Series 4 represents the results obtained on changing temperature coefficient χ from 0.029 to 0.04.

Fig. 7 clearly demonstrates that by playing with oil properties one can achieve better optima. Based on this observation one can tailor multi-grade oil with a high value of viscosity at low temperature (at supply point) and low viscosity at high temperature.
viscosity at high temperature, such as shown in Fig. 8. This will uncouple the bearing objective functions, and provide a win–win (absolute optima) situation. However, determining exact viscosity requirements needs an elaborate simulation that can keep track of temperature distribution between the bearing pair.

6. Conclusions

- A finite difference mass conserving algorithm is used to provide relatively accurate power loss and flow rate.
- A separate pool scheme is suggested to preserve non-inferior solution. By comparing solutions among those obtained over 25 generations, this separate pool is updated at every 25 generations. This scheme provides a high convergence rate without impairing the evolutionary genetic algorithm.
- A rank-based genetic algorithm, with niche operator and ‘separate pool scheme’ is used to optimize two objective functions under three constraints (maximum pressure, temperature rise, and minimum film thickness). The objectives, power loss and flow rate, are functions of five variables: radial clearance, slenderness ratio, groove location, supply pressure and viscosity. Obtained results show some irregular behavior of groove location and supply pressure.
- Axiomatic design is used to understand the bearing optimization problem in depth. Redundancy and coupling in journal bearing design is diagnosed. Redundancy is reduced by using sensitivity analysis. Negligible effect of supply pressure and groove, located in divergent region, is observed. Further, monotonic behavior of medium length of bearing is demonstrated.
- Tailoring of multifunctional oil viscosity is suggested. However, this requires a full thermohydrodynamic mass conserving algorithm.

References