Magnetic Bearing Configurations: Theoretical and Experimental Studies

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A radial magnetic bearing, consisting of two permanent magnets, is an attractive choice because of its zero wear, negligible friction, and low cost, but it suffers from low load capacity, low radial stiffness, lack of damping, and high axial instability. To enhance the radial load and radial stiffness, and reduce the axial thrust, we have made a theoretical and experimental study of various radial configurations, including hydrodynamic lubrication to improve dynamic performance of the magnetic bearing. We developed an experimental setup to investigate the performance of bearing configurations under different operating conditions. The motion of a rotating shaft is mapped by two displacement sensors with a data acquisition system and personal computer. The first critical speed of each configuration is determined experimentally and verified through frequency analysis. We present a polar plot of displacement data.

Index Terms—Hydrodynamic bearing, low-friction bearing, natural frequency of magnetic bearing, permanent-magnet bearing, wearless bearing.

I. INTRODUCTION

Static load capacity and stiffness of permanent-magnet bearing configurations are given by [1]

\begin{align}
\mathbf{F} &= F_x \hat{x} + F_y \hat{y} + F_z \hat{z} \\
\mathbf{K} &= - \left( \frac{dF_x}{dx} \hat{x} + \frac{dF_y}{dy} \hat{y} + \frac{dF_z}{dz} \hat{z} \right) \\
\text{or, } \mathbf{K} &= - (K_x \hat{x} + K_y \hat{y} + K_z \hat{z}),
\end{align}

(1)

(2)

(3)

As per Earnshaw theorem [2], sum of bearing stiffness must be equal to zero [1], [3], i.e.,

\[ K_x + K_y + K_z = 0. \]

(4)

For circular arc, \( K_x = K_y = K_r \), which leads to

\[ K_z = -2K_r. \]

(5)

For stable configuration of any bearing, stiffness needs to be positive. But, (5) indicates that the positive value of \( K_r \) results in a negative value of \( K_z \). There is a very strong coupling between axial and radial stiffness of cylindrical radial magnetic bearing. In other words, radial permanent-magnet bearing, i.e., designed to support radial load, will be unstable in axial direction. To assure stability of radial permanent-magnet bearing along the axis of rotation, two thrust bearings [4] (+z and -z direction) are required. Ball [4] and/or electromagnetic [5]–[8] bearings are commonly used to deal with the axial force. Use of electromagnets increases the initial and running costs [9], [10] and makes the system bulky and complex. Ball bearings amplify the friction of the system and eat away all the benefits (zero wear, low manufacturing cost, no maintenance, etc.) provided by the magnetic bearing. To eliminate the need of extra axial supports and increase the capacity of radial load, the present paper suggests an economical configuration of radial magnet bearing.

Permanent-magnet bearings generally have low damping [10] and limited load carrying capacity [11]. To add damping, the technology of hydrodynamic journal bearing [12]–[14] can be incorporated. The high load carrying capacity at high rotational speed, soaring damping, and simple manufacturing of hydrodynamic journal bearings make it admirably popular. The inadequacy of the hydrodynamic journal bearing lies in its lack of film formation at lower speed that makes it vulnerable to wear. The absorption of advantages of magnetic and hydrodynamic bearings and extraction of their demerits has been tried by Swanson et al. [11] and Tan et al. [4]. However, in their experimental work, magnetic bearing and hydrodynamic bearing were arranged side-by-side. Such an arrangement requires more space, which is undesirable. To increase the damping characteristics, the present paper proposes a hybridization of hydrodynamic and magnetic forces. To explore the viability of combining magnetic and hydrodynamic effects in a single bearing arrangement, we have hypothesized a few concepts. To validate the theoretical concepts, working models of bearing arrangements are made and presented in this paper. An experimental setup is designed and fabricated to perform a study on:

- axial arrangements of cylindrical magnets;
- radial bearing configurations;
- magnetic + hydrodynamic bearing arrangement.

II. MATHEMATICAL MODELING

Two different methods, namely dipole method and surface charge density method, have been used to understand and design the magnetic bearings. For quick observation of variation in axial force and axial stiffness with axial offset, the closed form equations [15] have been used. For detailed design calculation, the surface charge density method [4] has been used.

A. Dipole Method

Radial magnetic bearing imposes “undesirable” axial force. For a good design of such bearing, designers require the lowest...
value of axial force generated by arrangements of magnets. To understand the effect of geometry and relative position of stator and rotor magnets (Fig. 1), mathematical modeling of magnetic force is essential. The closed form expressions of “undesirable” axial force for concentric radial magnetic bearing are given by [15]

\[ F_z = -L \int_{S_1}^{S_2} \frac{J_1 J_2}{2\pi \mu_0 R_{12}} ds_1 ds_2 \sin \phi \tag{6} \]

or

\[ F_z \approx -L \frac{J_1 J_2}{2\pi \mu_0 R_0} S_1 S_2 \sin \phi \tag{7} \]

where \( \phi = \tan^{-1}(z/R) \) and “z” is axial offset as shown in Fig. 2.

Using (7), the results of axial force for a typical magnetic bearing (\( R_{11} = 10 \) mm, \( R_{20} = 16 \) mm, and \( R_{31} = 19 \) mm, \( R_{so} = 25 \) mm and \( L = 15 \) mm) are plotted in Fig. 3. This figure shows zero value of axial force at no axial offset position. The value of axial force increases with increase in axial offset between stator and rotor. To validate this theoretical finding, we examined the magnetic flux variation along the length of stator and rotor magnets in radial magnetic bearing using a Gaussmeter. Measurements have been taken on outer \((r = R_{11})\) and inner surfaces \((r = R_{so})\) of rotor and stator, respectively. Results of such measurements are plotted in Figs. 4 and 5. These figures show that the magnetic flux density is the maximum at the ends and decreases towards the center of the magnets. The polarity of both the stator and rotor magnets changes at the center of the length. Therefore, if rotor and stator magnets are aligned along the faces, then axial force will be zero. Fig. 5 depicts the magnetic flux variations for the arrangement of stator and rotor magnets with axial offset, \( z \). It is clear that if there is an offset between stator and rotor magnets, the middle zone of stator magnet comes to the opposite polarity of rotor magnet and magnetic pair experiences an attractive force. This phenomenon reduces resultant radial repulsion and develops the axial force, which results axial instability. To avoid axial force, ideally, the rotor and stator magnets should be arranged with zero axial offset. However, the zero axial offset leads to very high
negative axial stiffness, which is given by the following expression [15]:

$$K_z = L \int \int \frac{J_1 J_2}{2 \pi \mu_0 r_1^2} dS_1 dS_2 \cos 2\phi$$  \hspace{1cm} (8)

or, $$K_z \approx L \frac{J_1 J_2}{2 \pi \mu_0} \left( -\frac{6}{R_1^4} \right) S_1 S_2 \cos 2\phi$$  \hspace{1cm} (9)

and $$K_r = -\frac{K_z}{2}$$  \hspace{1cm} (10)

where $\phi = \tan^{-1}(z/R)$ and “z” is axial offset as shown in Fig. 2.

Axial stiffness using (9) is plotted in Fig. 6. This figure shows the highest value of axial stiffness at zero offset. With axial offset, this stiffness decreases. This behavior of magnetic bearing poses contradiction between “axial force” and “axial stiffness.” The designer requires minimum of “axial force” and “axial stiffness,” but “minimum of axial force” is coupled with “maximum of axial stiffness.” To resolve this contradiction, one needs to explore various configurations of rotor and stator magnets.

B. Surface Charge Density Method

One of the possible solutions to reduce axial force is to decrease the arc extent of the stator magnet. To arrive at suitable extent of stator arc, one needs to evaluate the radial magnetic force. This force may be evaluated using modeling discussed in Section II-A. However, “dipole method” can predict reliable results if all the magnet dimensions are smaller than the distance between the magnets. As in the present study, magnet dimensions are larger than distance between them, surface charge density method (using Coulomb law for magnetic) has been used to determine the radial magnetic force.

To explore the effect of stator arc on magnetic force, geometry of stator magnet can be discretised in punctual magnet. To derive the generalized force expression radial magnetic bearing with a configuration having a cylindrical rotor magnet and two arcs of stator magnet, as shown in Fig. 7, is considered. Here, magnetic forces between stator and rotor have been calculated by magnetic charge density method [4]. According to this method, it is assumed that magnetic charges are distributed on the faces of magnetic poles: 1, 2, 3, 4 as shown in Fig. 8 and magnetic forces are generated by interaction between charges of two faces. From the literature [16], elemental magnetic force between the faces 2 and 3 is given by the Coulomb law for magnetic poles

$$d\mathbf{F}_{23} = \frac{\mu_0 q_{m1} q_{m2} \mathbf{r}_{23}}{4 \pi r_{23}^2}$$  \hspace{1cm} (11)

where $q_{m1}, q_{m2}$ are the magnetic charge or “point pole” [16] on surface elements ds1 and ds2, respectively. Now $q_m$ can be expressed in terms of surface charge density ($\sigma$) like electrostatic field in the following way:

$$q_m = \sigma ds.$$  \hspace{1cm} (12)

Using (11) and (12)

$$d\mathbf{F}_{23} = \frac{\mu_0 \sigma_1 \sigma_2 ds_1 ds_2 \mathbf{r}_{23}}{4 \pi r_{23}^2}$$

or, $$d\mathbf{F}_{23} = \frac{\mu_0 \sigma_1 \sigma_2 ds_1 dr_2 \mathbf{r}_{23}}{4 \pi r_{23}^3}.$$  \hspace{1cm} (13)

The relationship [16] between magnetization ($\mathbf{M}$), magnetic field ($\mathbf{H}$), and magnetic induction ($\mathbf{B}$) for linear isotropic homogeneous magnetic material having permeability $\mu$ is given by

$$\mathbf{B} = \mu_0 (\mathbf{M} + \mathbf{H})$$  \hspace{1cm} (14)

$$\mathbf{M} = \chi \mathbf{H} + \mathbf{M}_r$$  \hspace{1cm} (15)

For rare earth material, $\mu_r \approx 1$.

Therefore, from (15)

$$\mathbf{M} = \mathbf{M}_r.$$  \hspace{1cm} (16)

And from (14) and (16)

$$\mathbf{B}_r = \mu_0 \mathbf{M}_r.$$  \hspace{1cm} (17)

For uniformly magnetized material

$$\mathbf{M}_r = \sigma.$$  \hspace{1cm} (18)

From (17) and (18)

$$\sigma = \mathbf{B}_r / \mu_0.$$  \hspace{1cm} (19)

Now from (13) and (19)

$$d\mathbf{F}_{rn} = \frac{\mu_0^2 R_0 \mathbf{R}_n \mathbf{R}_m (\mathbf{R}_{rd} \mathbf{R}_{nd} \mathbf{R}_{dd}) \mathbf{R}_m}{4 \pi \mu_0 R_{r_0}^3}$$  \hspace{1cm} (20)

where $R_{rn} = e + \mathbf{R}_r - \mathbf{R}_a$.

The magnitudes of the vector $R_{rn}$ between different magnetic faces (shown in Fig. 8) can be expressed by

$$R_{13} = \left[ z^2 + (e + \mathbf{R}_r \cos \alpha - \mathbf{R}_a \cos \beta)^2 \right] + (\mathbf{R}_r \sin \alpha - \mathbf{R}_a \sin \beta)^2 \right)^{1/2} = R_{24}$$  \hspace{1cm} (21)

$$R_{14} = \left[ (1 - z)^2 + (e + \mathbf{R}_r \cos \alpha - \mathbf{R}_a \cos \beta)^2 \right] + (\mathbf{R}_r \sin \alpha - \mathbf{R}_a \sin \beta)^2 \right)^{1/2}$$  \hspace{1cm} (22)

$$R_{23} = \left[ (1 + z)^2 + (e + \mathbf{R}_r \cos \alpha - \mathbf{R}_a \cos \beta)^2 \right] + (\mathbf{R}_r \sin \alpha - \mathbf{R}_a \sin \beta)^2 \right)^{1/2}.$$  \hspace{1cm} (23)
The force along the line of center is given by adding all radial components as shown in Fig. 8

\[ dF_{rs} = dF_{13r} + dF_{24r} - dF_{14r} - dF_{23r}. \]  

(28)

Resultant elemental force between rotor and stator is obtained by adding all radial components as shown in Fig. 8

\[ dF_{rs} = dF_{13} + dF_{24} + dF_{14} + dF_{23}. \]  

(27)

The force along the line of center is given by

\[ dF_{rs} = dF_{rs,j} = \frac{B_r B_{ss} R_{rs} R_{rss} R_{d} d\Omega \alpha}{4\pi \mu_0 R_{r13}^3} R_{rs,j} \]  

(28)

where \( R_{rs,j} = (e + R_r \cos \alpha - R_s \cos \beta). \)

Using (28) and considering the direction of force components along the line of center, the total elemental radial force is obtained as follows:

\[ dF_{rs}^{13} = dF_{13r} + dF_{24r} - dF_{14r} - dF_{23r}. \]  

(29)

On integrating (29), one gets the magnitude of total radial force along the line of center

\[ F_{rs} = \frac{1}{4\pi \mu_0} \int_0^{2\pi} R_{rs}^{13} R_{rs}^{14} R_{rs}^{23} \left[ \frac{2B_r^2}{(R_{r13}^3)} - \frac{B_r^2}{(R_{r14}^3)} - \frac{B_r^2}{(R_{r23}^3)} \right] e_1 d\beta \]

\[ \frac{\theta_i}{\theta_f} \left( \frac{2B_r^2}{(R_{r13}^3)} - \frac{B_r^2}{(R_{r14}^3)} - \frac{B_r^2}{(R_{r23}^3)} \right) e_1 d\beta \]  

(30)

where \( e_1 = (e + R_r \cos \alpha - R_s \cos \beta). \)

Similarly, the axial force can be determined by adding the axial components of elemental forces as shown in Fig. 8

\[ dF_{ra} = dF_{13r} + dF_{24r} + dF_{14r} + dF_{23r}. \]  

(31)

On integration (31), the resultant axial force is obtained as

\[ F_{ra} = \frac{1}{4\pi \mu_0} \int_0^{2\pi} R_{ra}^{13} R_{ra}^{14} R_{ra}^{23} \left[ \frac{2B_r^2}{(R_{r13}^3)} + \frac{B_r^2}{(R_{r14}^3)} \right] e_2 d\alpha \]

\[ \frac{\theta_i}{\theta_f} \left( \frac{2B_r^2}{(R_{r13}^3)} + \frac{B_r^2}{(R_{r14}^3)} \right) e_2 d\alpha \]  

(32)
stator arc, made of permanent 180° magnets) provides the best results. To resolve the arc is made of magnetic material, while remaining arcs are made of aluminum material. In “configuration 3” (θ₁ = 3π/8, θ₂ = 5π/8) 180°+45° arcs are made of magnetic material, while remaining parts are made of aluminum. The results of magnetic force for all three configurations are plotted in Fig. 10. This figure indicates that the “configuration 2” provides maximum radial force among all three configurations. The results of axial force with axial perturbation are plotted in Fig. 11. This figure also indicates the advantages of “configuration 2” (causing lesser axial force) over “configuration 1” and “configuration 3.”

High radial force and lower axial force are desirable for radial magnetic bearings as per Figs. 10 and 11, half ring magnet should be a choice for radial magnetic bearings. But as per Mukhopadhyay et al. [6], [7], “configuration 3” (180° and 45° arc magnets) provides the best results. To resolve the issue of “which configuration is better,” one needs experimental verification.

III. EXPERIMENTAL SETUP

An experimental setup, as shown in Fig. 12, has been designed and developed. This setup consists of motor-drive, flexible coupling, shaft-bearing subassembly, and lubrication subassembly [20]. An ac motor (1.5 hp) with a frequency controller is used to operate the setup at speed ranging between 115 to 6000 rpm. The flexible aluminum coupling is used to sustain axial load as well as to allow angular (up to 7°) misalignment.

The shaft-bearing subassembly consists of stainless steel (SS201) shaft, mild steel rotor; stand for sensors, two aluminum housings, rotor magnets, and stator magnets. The mild steel rotor of 160 mm diameter and 20 mm thickness is mounted on the shaft and located midway between the bearing supports. Two magnetic (neodymium Iron Boron) rotors having Rₚ₉₀ = 16 mm and Rₚ₉₀ = 16 mm are push fitted on the stainless steel shaft. The total weight of the magnetic rotors, shaft, and mild steel rotor is 3.8 kg.

The bearing pedestals, sensor stand, and motor are mounted on I-section steel beams which are firmly attached on a steel base plate. The base plate is mounted on rubber pads to isolate the system from its surrounding external disturbance.

The lubrication subassembly contains rotary pump, dc motor and its controller (to operate at variable speed), flexible pipe, and oil filter. The required amount of oil is supplied to the bearing clearance by the rotary pump. Before oil supplied to the clearance, it is filtered in each circulation.

The instrumentation used in the experiments includes: nonmagnetic fiber optic displacement sensor (0.556 V/mm, Model RC 90-OPQ, Philetic, Inc), a tachometer, data acquisition card (PC 1710, 100 KS/s, 12 Bit, Dynalog (India), Ltd.), and computer (Pentium IV). The two displacement sensors are used to measure the shaft displacements in vertical and horizontal directions. The tachometer is used to check the rotational speeds.

**TABLE I**

<table>
<thead>
<tr>
<th>Sr No</th>
<th>Bearing Configurations</th>
<th>Motor Side Bearing</th>
<th>End Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L (mm)</td>
<td>Cₗ (mm)</td>
</tr>
<tr>
<td>1</td>
<td>Full Cylindrical NdFeB Ring</td>
<td>15</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>Half NdFeB + Half Aluminum</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>225° (180° and 45°) NdFeB + Two 67.5° Aluminum arcs</td>
<td>15</td>
<td>2.7</td>
</tr>
</tbody>
</table>

**IV. EXPERIMENTAL PROCEDURE**

The experiments are performed in two stages. The first stage experiments are performed, for all three configurations with dimensions shown in Table I, without use of any oil supply. The aim of these experiments is to find out the effect of all three configurations on the natural frequency of the system. In the second stage, the validity of hybrid bearing concept is checked by carrying out orbital analysis in time domain.

**A. Observations in Stage 1 Experiments**

The critical speed of any assembly can experimentally be determined by measuring amplitude of vibration of that assembly at different operating speed. In the present study, vibration amplitudes in the midplane of bearing are measured using two displacement sensors. Real displacement data, collected by

**Fig. 12.** Experimental setup.

**Fig. 13.** Frequency spectrum of configuration 1.

**Fig. 14.** Frequency spectrum of configuration 2.
the displacement sensors and recorded on computer for each speed, are utilized for the frequency analysis (Figs. 13–15). Figs. 13–15 show the frequency spectrum observed for “configuration 1,” “configuration 2,” and “configuration 3” of stator, respectively. Full ring arrangement shows the lowest range of amplitude (around 38, Fig. 13) due to radial forces from stator acting on the rotor magnet through out the 360° periphery. The Fig. 14 shows the highest amplitude value (around 300) for “configuration 2,” as there is no downward force from the upper portion of stator magnet on rotor magnet to restrict the vibration of rotor. The upper 45° arc in “configuration 3” stator exerts downward force and results lesser vibration amplitude (about 250) as shown in Fig. 15. From the figures, it is seen that critical speeds for “configuration 1,” “configuration 2,” and “configuration 3” are 700 rpm, 310 rpm, and 600 rpm, respectively. The values of critical speed suggest “configuration 1” is preferred over “configuration 2” and “configuration 3.”

It is interesting to note that theoretical analysis proves preference of “configuration 2” over “configuration 1” and “configuration 3,” while experimental results demonstrated superiority of “configuration 1” over “configuration 2” and “configuration 3.” But Mukhopadhyay et al. [6], [7], established “configuration 3” (180° and 45° arc magnets) as the best configuration for radial permanent-magnet bearings. Scientifically, all three configurations have their own merits and demerits. Justification has been provided in Fig. 16. Fig. 16(a) shows the position of shaft without the bearing and also indicates the negligible load sharing of flexible aluminum coupling. This figure expresses the need of bearings to support the shaft weight. In Fig. 16(b), aluminum sleeves are provided to restrict static deflection of shaft. This figure shows mechanical touch between shaft and bearing surfaces, which will wear out relative moving (bearing and shaft) surfaces. Fig. 16(c) illustrates the static position of shaft center, when the shaft is supported on magnetic bearing having “configuration 2.” This configuration levitates the shaft by magnetic repulsion forces acting against the dead (static) weight of the shaft assembly. Finally, Fig. 16(d) demonstrates the position of shaft center, when shaft is supported by “configuration 3” magnetic bearings. Fig. 16(c) and (d) depicts that “configuration 2” and “configuration 3” do not assure concentricity between geometric center of shaft assembly and equilibrium point obtained by balancing magnetic force against the dead weight of shaft assembly. Eccentricity ($e_d$) between “geometric point” and “equilibrium point” generates dynamic force ($\propto e_d(\text{rotational speed})^2$), magnitude of which depends on rotating speed and $e_d$. Therefore, a careful design of magnetic bearings needs to account “static” as well as “dynamic” forces to minimize vibration amplitude. In other words, a particular configuration of a radial bearing depends on static weight to be levitated and angular speed of rotating shaft. If static weight is very high compared to the dynamic force, then “configuration 2” may provide the best results. If static weight is negligible compared to dynamic load, then “configuration 1” will be preferred. If static and dynamic forces are comparable, then configuration similar (top stator arc may be 15°, 30°, 60°, or 90°) to “configuration 3” will provide better results. One important point to be noted is that it is very difficult to make eccentricity $e_d$ equal to zero. Therefore, shaft will wobble if it is rotated. If sufficient damping is not provided, shaft may experience radial instability. To control the shaft dynamic deflection, use of “hydrodynamic lubrication” has been hypothesized in the present study. As combined effects of “magnetic repulsion” and “hydrodynamics” have been utilized in the proposed bearing, therefore such a bearing may be called a “hybrid bearing.”
B. Experimental Observations on Hybrid Bearing

To check the feasibility of hybrid bearing concept, the same experimental setup is used. “Configuration 3” (shown in Fig. 17) of magnetic bearing has been used. The only changes are made in bearing clearance between rotor and stator and displacement sensors. Clearance is reduced to micron level. Two sensitive displacement sensors, 3300XL Proximity Sensor (Make: Bentley Nevada) having high accuracy of 7.87 mV/μm are used to get the journal orbit.

To understand the effect of hydrodynamic action on conventional and proposed hybrid bearings, first, experiments have been performed on conventional hydrodynamic journal bearing with rotor 39.94 ± 0.02 mm and aluminum ring ID 40.0 mm. To check the performance of the aluminum bearing at low speed, the bearing is operated at a speed of 115 rpm. Within three hours of operation, the noise level and displacement sensors output changed drastically. On inspection, considerable wear of bearing as shown in Fig. 18 has been observed.

Experiments with hybrid configuration (configuration 3) are conducted for 3 h at 115 rpm. Interestingly, no wear of bearing surface is observed. However, this bearing configuration (Fig. 17) could not be operated at relatively high speed due to difficulty in confining the magnetic pieces within low clearance hybrid bearing. To solve this problem, the whole bearing assembly is coated with Teflon as shown in Fig. 19. With Teflon coated hybrid bearing, the setup is run at 3000, 4000, and 5000 rpm. Results are plotted in Fig. 20, which indicates that with increase in shaft speed, the orbit of the shaft reduces in size. This suggests that with increase in shaft speed, hydrodynamics play an important role by providing damping to the system. This leads to better radial stability of bearing.

V. Conclusion

- Radial magnetic bearing experiences axial instability. Judicial arrangement of magnet may help to provide preload to compliant coupling and reduce undesirable backlash.
- The 180° stator magnet provides higher load capacity compared to full cylindrical stator magnet, if static load is very high compared to dynamic load. However, full cylindrical stator will be the obvious choice if dynamic load is very high compared to static load. If static and dynamic forces are comparable, then configuration similar (top stator arc may be 15°, 30°, 60°, or 90°) to “configuration 3” may provide better results.
- Hydrodynamic action along with magnetic repulsion force provides wearless high damping bearing configuration.

APPENDIX

List of Symbols

\begin{align*}
z & \text{ Axial offset.} \\
B_r & \text{ Remanent induction of permanent magnet, Wb/m}^2. \\
C_r & \text{ Radial clearance, m.} \\
\end{align*}
\( d\mathbf{F}_{ij} (i = 1 \text{ to } 2; j = 3 \text{ to } 4) \)  
Force vector between point \( i \) and \( j \) on magnetic faces, N.

\( d\mathbf{F}_{ijy} (i = 1 \text{ to } 2; j = 3 \text{ to } 4) \)  
Radial force component, N.

\( d\mathbf{F}_{iza} (i = 1 \text{ to } 2; j = 3 \text{ to } 4) \)  
Axial force component, N.

\( d\mathbf{F}_{ta} \)  
Elemental force vector between rotor and stator element, N.

\( d\mathbf{F}_{tr} \)  
Total elemental repulsive force between rotor and stator element, N.

\( d\mathbf{F}_{rs} \)  
Resultant elemental magnetic force between rotor and stator element, N.

\( e \)  
Stiffness vector between bearing and journal center, m.

\( F \)  
Total magnetic force, N.

\( F_{xx}, F_{yy}, F_{zz} \)  
Force component in \( x, y, \) and \( z \) directions, N.

\( F_{rs} \)  
Resultant radial force per unit length between rotor and stator, N.

\( J_1, J_2 \)  
Polarization of stator and rotor magnets respectively, m^2/m.

\( K_x, K_z \)  
Stiffness in radial and axial direction, N/m.

\( L \)  
Stator magnetic length, m.

\( K \)  
Resultant magnetic bearing stiffness, N/m.

\( K_{xy}, K_{yz}, K_{zx} \)  
Stiffness component along \( x, y, \) and \( z \) directions, N/m.

\( K_r \)  
Radial stiffness, N/m.

\( R \)  
Average distance between \( S_1 \) and \( S_2 \), m.

\( R_{r_1}, R_{r_0} \)  
Position vector of \( ds_2 \) from the center of bearing, m.

\( R_{r_1}, R_{r_0} \)  
Position vector of \( ds_1 \) from the center of journal, m.

\( R_{r_1}, R_{rs} \)  
Position vector between rotor and stator, m.

\( R_{r_1} \)  
Rotor’s inner and outer radius respectively, m.

\( R_{r_1}, R_{so} \)  
Stator’s inner and outer radius respectively, m.

\( R_{ij} (i = 1 \text{ to } 2; j = 3 \text{ to } 4) \)  
Distance between point \( i \) and \( j \) on magnetic faces, m.

\( R_{ij} (i = 1 \text{ to } 2; j = 3 \text{ to } 4) \)  
Vector between point \( i \) and \( j \) on magnetic faces, m.

\( S_1, S_2 \)  
Cross-sectional area of rotor and stator magnets, m^2.

\( \hat{x}, \hat{y}, \hat{z} \)  
Unit vector in \( x, y, \) and \( z \) directions, respectively.

\( \alpha, \beta \)  
Rotor and stator angular variable, radians.

\( \phi \)  
Angular position of rotor’s polarization w.r.t. that of stator on axial plane, radians.

\( \theta_1, \theta_2 \)  
Start and end of the upper stator arc, radians.

\( \mu_0 \)  
Permeability of free space, H/m.

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