A parametric study on supersonic flutter behavior of laminated composite skew flat panels

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Abstract

Here, the system parameters effects on supersonic panel flutter behavior of laminated composite skew plates are investigated using a shear deformable finite element approach. The first-order high Mach number approximation to linear potential flow theory is employed for evaluating the aerodynamic pressure. The solutions of complex eigenvalue problem, formulated based on Lagrange's equation of motion are obtained using the standard method for finding eigenvalues. The variation of critical aerodynamic pressure is evaluated considering different parameters such as skew angle, fiber orientation, and boundary conditions. The influence of aerodynamic and structural damping, and thermo-mechanical loads on the critical aerodynamic speed is also examined.

Keywords: Composite skew plate; Flutter; Finite element; Structural damping; Thermal stress; In-plane loads

1. Introduction

The dynamic instability of structures exposed to airflow on one side of the surface is an important problem to be investigated in the design of aerospace vehicles. Jordan [1] is the first person to identify such problem. Since then, a large number of analytical, numerical, and experimental investigations on supersonic panel flutter characteristics have been reported in the literature. These studies are reviewed and well documented by Dowell [2] and Bismark-Nasr [3,4]. It is observed from the available literature that most of the work have been devoted to the panel flutter behavior of rectangular plates, and also a limited attention has been focused to examine the influence of system parameters such as structural damping, and existing in-plane/thermal load on the flutter boundary. Further, in practice, the geometry with non-rectangular plan-forms like skew plates find wide application in the aerospace industry [5]. Vibration and aero-elastic analyses of such structures, especially, thin-walled structural elements with relatively low flexural rigidity, have recently gained importance among researchers.

Durvasula [6] studied the flutter behavior of simply supported isotropic skew plates using Lagrange's equation and employing beam characteristics functions. Kariappa et al. [7] and Sander et al. [8] employed finite element method to investigate the flutter characteristics of skew panels. Srinivasan and Babu [9,10] solved the flutter problem of isotropic and cross-ply laminated panels using an integral equation technique. Lia and Sun [11] investigated the supersonic flutter behavior of stiffened composite skew plates and shells using a degenerated shell element. Recently, Chowdary et al. [12] used a shear deformable finite element method to investigate the supersonic flutter of composite skew panels. The effect of skew angle on the critical dynamic pressure is investigated for different boundary condition, and fiber orientation. Pidaparti and Chang [13] investigated the flutter characteristics of skewed and cracked composite panels. However, all these investigations have mainly been concerned with the skewed laminates under external airflow, neglecting the effect of damping and thermo-mechanical loads.

It would be more general and physically realistic to consider the effects of in-plane forces by edge restraints, aerodynamic heating, and structural damping. The effect
of aerodynamic damping on the supersonic flutter behavior of rectangular composite plates is studied adopting finite element procedure [14]. Ganapathi and Touratier [15], and Xue and Mei [16] have studied recently the influence of aerodynamic heating on the flutter characteristics of rectangular composite plates. However, to the best of the authors’ knowledge, the work on dynamic instability of composite skew plates subjected to supersonic airflow, considering the system parameters such as structural and aerodynamic damping, thermal effect and in-plane forces, appears to be scarce in the open literature.

In the present paper, a four-noded shear flexible quadrilateral high precision plate-bending element developed recently [17] is extended to analyze the supersonic panel flutter behavior of laminated composite skew plates. The hysteretic damping model is introduced in the formulation. The aerodynamic force is evaluated considering the first-order high Mach number approximation to linear potential flow theory. Normal modes approach is adopted to reduce the number of degrees of freedom of the finite element system and the QR method is employed for the solution of complex eigenvalue problem. The formulation developed herein is validated with the available solutions. A detailed parametric study has been carried out to bring out the influences of the skew angle, lay-up, and boundary conditions on the flutter behavior of laminated skew plates. The effect of structural and aerodynamic damping, thermal stress and in-plane loads on the critical dynamic pressure is also examined.

2. Formulation

Fig. 1 shows the rectangular Cartesian co-ordinate system along with the associated covariant base vectors \( \{ e_1, e_2, e_3 \} \) and contravariant base vectors \( \{ g_1, g_2, g_3 \} \) for the skew plate having \( a \) and \( b \) as the length and width, and \( \psi \) as the skew angle. \( \{ g_1, g_2, g_3 \} \) and \( \{ l^1 g, l^2 g, l^3 g \} \) are related to the rectangular Cartesian unit base vectors \( \{ e_1, e_2, e_3 \} \) by

\[
\begin{align*}
g_1 &= e_1, & g_2 &= (\sin \psi) e_1 + (\cos \psi) e_2, & g_3 &= e_3 \\
n^1 g &= e_1 - (\tan \psi) e_2, & n^2 g &= (\sec \psi) e_2, & n^3 g &= e_3
\end{align*}
\]

(1)

The covariant components of the displacement vector \( u = (u_1, u_2, u_3) \) of a shear deformable plate can be expressed in terms of the contravariant components of the position vector \( r = \{ g_1 r_1, g_2 r_2, g_3 r_3 \} \) as [17,18].

\[
\begin{align*}
u_1(^1 r, ^2 r, ^3 r) &= u_1^0(^1 r, ^2 r) + 3^1 \phi \{ ^1 r, ^2 r \} \\
u_2(^1 r, ^2 r, ^3 r) &= u_2^0(^1 r, ^2 r) + 3^2 \phi \{ ^1 r, ^2 r \} \\
u_3(^1 r, ^2 r, ^3 r) &= u_3(^1 r, ^2 r)
\end{align*}
\]

(2)

Here, \( ^1 \phi \{ ^1 r, ^2 r \} \) and \( ^2 \phi \{ ^1 r, ^2 r \} \) are the total rotations; \( \gamma_1 \{ ^1 r, ^2 r \} \) and \( \gamma_2 \{ ^1 r, ^2 r \} \) are the rotations due to shear deformation of the normal to the plate middle surface around \( l^2 g \) axis and \( l^1 g \) axis respectively; \( () \) represents the partial differentiation of the variable preceding it with respect to \( r \).

Fig. 1. (a) Oblique co-ordinate system for the skew plate. (b) Skew plate under initial uni-axial compression and airflow.

where

\[
\begin{align*}
^1 \phi \{ ^1 r, ^2 r \} &= \{ u_1 + \gamma_1 \} \\
^2 \phi \{ ^1 r, ^2 r \} &= \{ u_2 + \gamma_2 \}
\end{align*}
\]

Now the covariant components of strain tensor \( e \ (e = e_{ij} g_i g_j) \) can be written in terms of displacement components \( u \ (u = u^i g_i = ^i u g_i) \) as [17,18].

\[
e_{ij} = \frac{1}{2} (u_{ij} + u_{ji}) - z_{ij} T
\]

(3)

The normal strain component \( e_{33} \) is zero, \( z_{ij} \) is the thermal expansion coefficients in plate co-ordinate system and are related to the coefficients \( (z_{11}, z_{12}, 0) \) in the material principal direction, and \( T \) is the temperature change.

The contravariant components of stress tensor \( \sigma^{ij} \ (\sigma = \sigma^{ij} g_i g_j) \) and the covariant components of the strain tensor \( e_{ij} \ (e = e_{ij} g_i g_j) \) are related to their counterparts \( ^i \sigma_{mn} \) and \( ^c e_{mn} \) in the rectangular Cartesian co-ordinate system by

\[
\begin{align*}
^c e_{mn} &= \sigma^{ij} (g_j \cdot e_m) (g_i \cdot e_n)
\end{align*}
\]
and
\[ c_{em} = e_{ij} (g \cdot e_{ij}) (g \cdot e_{ij}) \]  
(5)

For a composite laminate of thickness \( h \), comprising of \( N \) layers with stacking angles \( \theta_i, (i = 1, 2, \ldots, N) \) and layer thicknesses \( h_i, (i = 1, 2, \ldots, N) \), the necessary expression to compute the stiffness coefficients are available in the literature [19]. The stress-strain relationship in the rectangular Cartesian co-ordinate system can be written as
\[ c\{\sigma\} = [Q]^T c \{e\} \]  
(6)
where \( [Q]^T \) represents plane stress reduced stiffness matrix of the \( k \)th laminate of the plate, \( [R] \) is the transformation matrix, and \( [Q] \) is the reduced stiffness matrix of an orthotropic lamina.

The strain energy \( W_E \) and kinetic energy \( W_I \) of the plate may be expressed as
\[ W_E = \frac{1}{2} \int_V \sigma^T \epsilon \; dV \]  
(7)
\[ W_I = \frac{1}{2} \int_V [u_i^2 \rho + (u_{j3} + \gamma_j) \rho \varepsilon^2] \; dV \quad i = 1, 3; \quad j = 1, 2 \]  
(8)
where \( dV = d^3r \cos \psi \).

The work done by the non-conservative hysteretic damping \( W_D \) is expressed as
\[ W_D = -\frac{1}{\omega} \int_V \sigma^T T_e \; dV = -\frac{1}{\omega} \int_V \varepsilon^T G_e \; dV \]  
(9)
where \( \omega \) is the frequency, \( T_e \) and \( G_e \) are the damping coefficient matrix, and \( [H] \) is a diagonal matrix with non-zero components \( h_{11} = \eta_1, \quad h_{22} = \eta_2, \quad h_{44} = \eta_3, \quad h_{55} = \eta_4, \quad h_{66} = \eta_2 \); \( \eta_1 \) and \( \eta_2 \) denote the loss factors in longitudinal and transverse directions, respectively with respect to the fibers, and \( \eta_12, \eta_13 \) and \( \eta_23 \) are the loss factors due to shear.

The work done by the non-conservative applied aerodynamic force is
\[ W_A = \int_A \Delta p u_3 \; dA = \int_A \left[ \frac{\partial u_3}{\partial r_1} + g_T \frac{\partial u_1}{\partial r} \right] u_3 \; dA \]  
(10)

Here, first-order high Mach number approximation to the linear potential flow theory is used to express the aerodynamic pressure \( \Delta p \), where \( \lambda = \rho_s U_s^2 / \sqrt{M^2 - 1} \) and \( g_T = \lambda (M^2 - 2) / \{U_s(M^2 - 1)\} \) are the dynamic pressure and aerodynamic damping parameter, respectively (\( \rho_s \): air density; \( U_s \): airflow velocity; \( M \): free stream Mach number).

Substituting Eqs. (7)-(10) in Lagrange’s equations of motion, one obtains the governing equations as
\[ M \ddot{\delta} + (g_T D + C + i Q) \delta + [K + K_G + \lambda A] \dot{\delta} = 0 \]  
(11)
where \( M, K, K_G, C, D, \) and \( A \) are mass, stiffness, geometric stiffness, structural damping, aerodynamic damping and aerodynamic load matrices respectively.

Here, a four-noded quadrilateral plate element with ten degrees of freedom per node, namely \( u_1^0, u_2^0, u_3^0, u_4^0, u_{11}, u_{12}, u_{13}, u_{14}, u_{21}, u_{22}, \) \( \gamma_1 \) and \( \gamma_2 \) are used [17]. The linear polynomial shape functions are employed to describe the field variables corresponding to in-plane displacements \((u_i^0, u_j^0)\) and rotations due to shear of the middle surfaces \((\gamma_1, \gamma_2)\), whereas quantic polynomial function is considered for the lateral displacement \((u_3)\) and are expressed as follows:
\[ u_i^0 = c_k (\lambda r)^i (\lambda r)^j; \quad i, j = 0, 1, 2, 3; \quad m = 0, 1; \quad n = 4, 5; \quad k = 0, 1, 2, 3 \]  
(12)
where \( c_k \) are constants and are expressed in terms of nodal displacements in the finite element discretization. The full integration scheme with \( 6 \times 6 \) Gaussian integration rule is adopted for computing the element mass matrix \( M \), aerodynamic damping matrix \( D \), and aerodynamic load matrix \( A \), whereas \( 4 \times 4 \) Gaussian integration rule is used to calculate the element stiffness matrix \( K \), geometric stiffness matrix \( K_G \), and structural damping matrix \( C \). This element is free from locking syndrome. It has good convergence properties and has no spurious rigid modes [17].

3. Solution procedure

Introducing harmonic motion in the form \( \delta \) = \( \delta e^{i\omega t} \), Eq. (11) is rewritten as
\[ \{-\omega^2 M + i \omega g_T D + [K + i C + K_G + \lambda A]\} \delta = 0 \]  
(13)

The damping matrix may be considered as the scalar multiple of mass matrix \( (D = M/\rho_0) \) by neglecting the shear and rotary inertia terms of the mass matrix \( M \), and the eigenvalue equation (13) becomes
\[ \{-\kappa M + [K + i C + K_G + \lambda A]\} \delta = 0 \]  
(14)
where the eigenvalue \( \kappa = \{-\omega^2 - g_T \omega / \rho_0\} \) includes the contribution of aerodynamic damping. Eq. (14) is solved for eigenvalues for a given value of \( \lambda \) by QR method. For the case of undamped vibration, neglecting aerodynamic and structural damping, any two of the eigenvalues \( \{\kappa\} \) will approach each other, as \( \lambda \) increases from zero, and coalesce to \( \kappa_{cr} = \lambda_{cr} \). Here, \( \lambda_{cr} \) is considered to be that value of \( \lambda \) at which first coalescence occurs, and is the critical flutter speed parameter.

In the presence of aerodynamic damping alone, the eigenvalues \( \{\kappa\} \) in Eq. (14) becomes complex with the
increase in the value of λ. The corresponding frequency may be obtained from

\[ \kappa = -\omega^2 - g_r \omega / \rho h = \kappa_R - i \kappa_I \]  

(15)

The flutter boundary is reached (λ = λ_cr) when the frequency ω becomes purely imaginary number, i.e. \( \omega = i \sqrt{\kappa_R} \) at \( g_r = \kappa_I / \sqrt{\kappa_R} \). In practice, the value \( \lambda_{cr} \) is determined from a plot of \( \omega_R \) vs. \( \lambda \) corresponding to \( \omega_R = 0 \). It may be observed that after computing the eigenvalue for a given \( \lambda \), the evaluation of \( \omega \) is very much simplified, as only the algebraic equation (15) is to be solved for various values of \( g_r \). For study of structural damping, the eigenvalues \( \{\kappa\} \) of Eq. (14) are complex for any values of aerodynamic pressure λ and they approach each other as \( \lambda \) increases from zero, but do not coalesce. In this case, flutter occurs when an imaginary part of an eigenvalue \( \kappa_I \) changes from positive to negative [23].

4. Results and discussion

The study, here, has been focused on the supersonic flutter characteristics of composite skew plates. Based on the progressive mesh refinement, 8 x 8 mesh is found to be adequate to model the full skew plate. The material properties, unless specified otherwise, used in the present analysis are

\[ E_L/E_T = 40.0, \quad G_{LT}/E_T = 0.6, \quad G_{TT}/E_T = 0.5, \quad \nu_{LT} = 0.25, \quad \nu_L = 10^{-06}, \quad \nu_T = 10^{-05} \]

where \( E \), \( G \), \( \nu \) and \( z \) are Young’s modulus, shear modulus, Poisson’s ratio and thermal expansion coefficient respectively. Subscripts L and T represent the longitudinal and transverse directions respectively with respect to the fibers. All the layers are of equal thickness. The boundary conditions considered here are:

**Simply supported on all sides (SS):**

\[ u_1 = u_2 = u_3 = 0 \] along the boundary nodes

**Clamped on all edges (CC):**

\[ u_1 = u_2 = u_3 = 0, u_{3,1} = 0 \] at \( r = 0, a \)

\[ u_1 = u_2 = u_3 = 0, u_{3,2} = 0 \] at \( r = 0, b \)

**Cantilever:**

\[ u_1 = u_2 = u_3 = 0, u_{3,2} = 0 \] at \( r = 0 \)

Before proceeding for the detailed study, the formulation developed herein is validated against free vibration of laminated composite skew plates. The non-dimensional natural frequencies \( (\sigma = \omega a^2 / \pi^2 h \sqrt{\rho / E_T}; a \) and \( h \) are length and thickness of the plate) obtained for simply supported, and clamped 5-layered angle-ply \( [45^\circ/-45^\circ/45^\circ/-45^\circ/45^\circ] \) skew laminates are presented in Table 1 along with the analytical solutions of Wang [20], and they match very well. The non-dimensional critical dynamic pressure \( \lambda_{cr} = \lambda a^3 / D \) are evaluated considering simply supported and cantilever isotropic skew plates \( (a/b = 1; a/h = 100.0) \) and are shown in Table 2 along with the available results [10–12]. The efficacy of the present element for thermal stress analysis has also been tested [17] and, for the sake of brevity, these results are not presented here. Further, the non-dimensional critical dynamic pressure of an isotropic square plate subjected to thermal load, and in-plane compressive stress is examined and the solutions obtained here are tabulated in Table 3 along with those of available results [15,16,21,22]. They are found to be in excellent agreement with the existing solutions.

Next, a detailed study is carried out considering multi-layered cross-ply and angle-ply skew laminates for

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**Table 1**

<table>
<thead>
<tr>
<th>B.C.</th>
<th>Skew angle</th>
<th>Modes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS</td>
<td>0°</td>
<td>Present</td>
<td>2.4343</td>
<td>4.9854</td>
<td>6.1823</td>
<td>8.4827</td>
<td>10.2464</td>
<td>11.6432</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>Present</td>
<td>4.5425</td>
<td>8.3787</td>
<td>9.8764</td>
<td>12.8428</td>
<td>15.6673</td>
<td>17.4547</td>
</tr>
</tbody>
</table>
which results are scarce in the literature. The critical aerodynamic pressure is evaluated based on the coalescence of modes and is demonstrated in Fig. 2 considering a simply supported 5-layered cross-ply \([0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]\) composite skew plate \((a/b = 1.0; a/h = 100.0)\). Here, the non-dimensional frequencies \((\omega = \omega_a \sqrt{h/\rho/E_T})\) is plotted against non-dimensional aerodynamic pressure \(\lambda (= \lambda a^3/E_2h^3)\) for different values of skew angle. The minimum value of aerodynamic pressure at which the first coalescence between any two modes occurs is termed as critical aerodynamic pressure. It is observed from Fig. 2 that the critical pair of modes corresponding to the coalescence is \((1,3)\) for skew angle \(0^\circ\), whereas it is \((9,10)\) for skew angle \(15^\circ\), \((8,9)\) for skew angle \(30^\circ\), and \((11,12)\) for skew angle \(45^\circ\).

Table 2
Comparison of non-dimensional critical aerodynamics pressure \(\lambda_a (= \lambda a^3/D)\) of isotropic skew plates \((a/b = 1, a/h = 100.0)\)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>(0^\circ)</td>
<td>5.27</td>
<td>5.12</td>
<td>5.25</td>
<td>57.89</td>
<td>59.51</td>
<td>58.35</td>
</tr>
<tr>
<td>(30^\circ)</td>
<td>6.47</td>
<td>6.31</td>
<td>6.82</td>
<td>40.04</td>
<td>41.12</td>
<td>42.33</td>
</tr>
</tbody>
</table>

Table 3
Flutter boundary \(\lambda_a (= \lambda a^3/D)\) of simply supported isotropic square plate subjected thermal and in-plane mechanical loads \((a/h = 240.0; \nu = 0.3; \alpha = 12.5 \times 10^{-6})\)

<table>
<thead>
<tr>
<th>Plate under thermal stress ((T/T_a = 0.8))</th>
<th>Plate under bi-axial compressive stress ((P/P_c = 1.0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>Present study</td>
</tr>
<tr>
<td>376.271</td>
<td>343.15</td>
</tr>
<tr>
<td>Ganapathi and Touratier [15]</td>
<td>Present study</td>
</tr>
<tr>
<td>376.125</td>
<td>342.0</td>
</tr>
<tr>
<td>Xue and Mei [16]</td>
<td>Ganapathi [22]</td>
</tr>
<tr>
<td>371.093</td>
<td>343.45</td>
</tr>
</tbody>
</table>

Fig. 2. Coalescence of frequencies of a simply supported 5-layered \([0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]\) composite plate for different skew angles \((a/b = 1; a/h = 100.0; \lambda = \lambda a^3/E_2h^3; \omega = \omega_a \sqrt{h/\rho/E_T}; \Psi \) is skew angle).
angle 30°, and (11,12) for skew angle 45°, respectively. However, it may be noted from the existing literature that, for isotropic case, the coalescence modes are, in general, the two fundamental modes from the lower order. The change of mode shapes with aerodynamic pressure for the coalescing cases are also demonstrated in Fig. 3 considering the first six natural mode shapes of a simply supported 5-layered cross-ply \([0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]\) square laminate \((a/b = 1.0; \ a/h = 100.0)\). It is observed that, with the increase in aerodynamic pressure, a pair of mode (1,3) approach to each other and coalesce at a non-dimensional pressure \((\lambda_{cr})\) value of 909.53. Similarly, the other pairs of modes (2,4) and (5,6) coalesce at the flutter speeds of 964.78, and 1053.83, respectively. It is further seen that the location of maximum displacement shifts with increase in the air speed. Similar study assuming a 5-layered angle-and cross-ply skew plate is carried out for different boundary conditions and the results are given in Table 4. It may be concluded from this table that the flutter mode corresponding to the coalescence strongly depends on the skew angle and lamination scheme, and boundary condition of the plate. Furthermore, it is noticed that the skew angle may enhance the critical aerodynamic pressure when the coalescing modes remain unchanged with respect to skew angle. However, the critical aerodynamic pressure, in general, decreases when the flutter mode pair changes with the skew angles.

The effect of thermal field on the critical aerodynamic pressure is demonstrated in Figs. 4 assuming a 5-layered simply supported cross- and angle-ply plates. The uniform temperature load \((T)\) is considered and the buckling temperature is denoted by \(T_{cr}\). The critical buckling temperatures are \((109.14, 104.52, 102.72, 119.33)\) and \((68.05, 71.19, 84.04, 119.32)\) for angle-ply \([45^\circ/45^\circ/45^\circ/45^\circ]\) and cross-ply \([0^\circ/90^\circ/90^\circ/0^\circ]\) laminates respectively. From this figure, it is observed that the flutter speed decreases with the increase in temperature.

---

Fig. 3. Natural and flutter mode shapes of a simply supported 5-layered \([0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]\) composite square plate \((a/b = 1; \ a/h = 100.0; \ \lambda = \lambda_{a}^{3}/E_{2}h^{3})\).
and the coalescing mode numbers remains unchanged. The rate of decrease of aerodynamic pressure with temperature is more when the participating modes in deciding the flutter boundary are from the lower ones. However, the rate of decrease reduces with higher modes coalescence. This may be due to the fact that the first fundamental frequency approaches zero with the increase in the temperature to the critical value whereas the higher frequencies reduce to some definite value. The influence of in-plane compressive stress on the critical aerodynamic pressure is highlighted for a 5-layered simply supported angle-ply plate in Fig. 5. The uniaxial in-plane compressive stress \( P \) is applied perpendicular to flow direction as shown in Fig. 1b, and the critical buckling stress is defined as \( P_{cr} \). The critical aerodynamic pressure decreases with the increase of uniaxial compression as expected.

The influence of non-dimensional aerodynamic damping \( gT = gT \sqrt{\frac{a^3}{E_3 h^3}} \) on the critical aerodynamic pressure \( \lambda = \frac{\lambda a^3}{E_3 h^3} \) is shown in Fig. 6 for cross- and angle-ply simply supported skew plate laminates. The influence of lay-up (symmetric and unsymmetric) is also investigated. It is observed that, the aerodynamic damping always increases the critical flutter speed. For the case in which the first coalesce occurs between the lower fundamental modes, depending on skew angle and lamination parameter, the rate of increase of aerodynamic pressure with aerodynamic damping is rather less. However, while the higher modes coalesce predicting the critical flutter boundary, the dynamic pressure increases significantly with the aerodynamic damping. It is also further noticed that, with the increase in the aerodynamic damping value, the participation of coalesce modes order can be different. It is also viewed from this figure that the behaviors of symmetric and unsymmetric laminates are found to be qualitatively same, irrespective of ply-angles.

Finally, the contribution of structural damping on the critical aerodynamic pressure is investigated considering HMS/DX-210 composite, for which properties are as follows [24]:

\[
E_L = 172.7 \text{ GPa}, \quad E_T = 7.2 \text{ GPa}, \\
G_{LT} = 3.76 \text{ GPa}, \quad \nu_{LT} = 0.3 \\
\eta_1 = 7.162 \times 10^{-4}, \quad \eta_2 = 6.716 \times 10^{-3}, \\
\eta_{12} = \eta_{13} = \eta_{23} = 1.1222 \times 10^{-2}
\]

### Table 4

<table>
<thead>
<tr>
<th>Skew angle</th>
<th>Simply supported</th>
<th>Clamped</th>
<th>Cantilever</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-ply ([0^\circ]/90^\circ/0^\circ/90^\circ/0^\circ])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0^\circ)</td>
<td>909.5 (1,3)</td>
<td>1570.2 (1,2)</td>
<td>60.5 (1,2)</td>
</tr>
<tr>
<td>(15^\circ)</td>
<td>690.6 (9,10)</td>
<td>1339.5 (7,9)</td>
<td>25.7 (3,4)</td>
</tr>
<tr>
<td>(30^\circ)</td>
<td>351.3 (8,9)</td>
<td>1144.9 (5,6)</td>
<td>23.6 (8,9)</td>
</tr>
<tr>
<td>(45^\circ)</td>
<td>412.6 (11,12)</td>
<td>1014.8 (3,4)</td>
<td>19.8 (1,2)</td>
</tr>
<tr>
<td>Angle-ply ([45^\circ]/-45^\circ/45^\circ/-45^\circ/45^\circ])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0^\circ)</td>
<td>547.5 (1,2)</td>
<td>834.9 (1,2)</td>
<td>48.5 (1,2)</td>
</tr>
<tr>
<td>(15^\circ)</td>
<td>613.3 (1,2)</td>
<td>899.4 (1,2)</td>
<td>108.3 (1,2)</td>
</tr>
<tr>
<td>(30^\circ)</td>
<td>746.79 (1,2)</td>
<td>1148.1 (1,2)</td>
<td>81.9 (9,10)</td>
</tr>
<tr>
<td>(45^\circ)</td>
<td>250.3 (11,12)</td>
<td>341.9 (11,12)</td>
<td>142.6 (2,3)</td>
</tr>
</tbody>
</table>

*The flutter modes are given in bracket.*
Fig. 7 shows the effect of structural damping on the flutter boundary of simply supported cross- and angle-ply skew laminates. The magnification factor $n$ on the material loss factors $\eta_{ij}$ is plotted against the critical aerodynamic pressure $(\lambda)$. Here, the eigenvalues $\kappa$ are complex for any values of aerodynamic pressure, $\lambda$. Here, the flutter speed is evaluated based on the criterion when an imaginary part of an eigenvalue change from positive to negative [23]. It is interesting to note that, the flutter modes can change with the increase in the loss factor magnitudes. It is further observed that, small magnitude of structural damping can destabilize the laminates and it depends on skew angle and ply-angles wherein lower modes coalesce for the critical situation. However, the structural damping can marginally increase the critical aerodynamic pressure if coalescence occurs between higher modes. Furthermore, for the higher value of loss factor, the change in the flutter speed, in general, is less.

5. Conclusions

Supersonic panel flutter behavior of composite skew plate has been investigated using a four-noded shear flexible quadrilateral high precision plate-bending ele-
ment. Numerical studies are conducted to examine the effect of skew angle, fiber angle orientation, and boundary conditions, damping and thermo-mechanical loads on the flutter characteristics of composite skew plate. Some observations made are as follows:

(a) The critical aerodynamic pressure and the coalescence of flutter modes depend on the lay-up sequence, skew angle and boundary condition.
(b) The flutter boundary decreases with the increase of thermal stress and in-plane compressive as expected.
(c) Aerodynamic damping always increases the critical aerodynamic pressure. However, the rate of increase in the flutter speed depends on the order of modes pairing for the coalescence.
(d) Structural damping may stabilize or destabilize the flutter instability of composite skew plates depending on mode pairing for the coalescence.

References