Interference Cancellation by Using Quasi-Orthogonal STBC in Two-User MIMO System

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Abstract—In this paper, we propose that by co-ordinated transmission of single user STBCs we can completely eliminate co-channel interference in a multiple cell scenario. A $4 \times 4$ two-user STBC code design is proposed which provides decoupled decoding of the data of each user. The proposed multiuser STBC performs better than the existing multiuser STBCs and also provide lower value of peak-to-average power ratio (PAPR) as compared to a trivial TDMA based STBC.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems are used to provide high capacity and diversity gains as compared to single-input single-output (SISO) systems [1]. Orthogonal space-time block codes (OSTBCs) is a popular choice for MIMO systems as they provide symbol wise decoupled decoding along with full diversity [1], [2]. However, they are available for limited number of transmit antennas. Space-time block code (STBC) design for multiuser has been explored recently in [3]. In [3], an example of a two-user $2 \times 2$ MIMO STBC is given based on Alamouti structure [4]. An algebraic construction of multiuser STBCs is presented in [5] to achieve the diversity-multiplexing trade-off for users with a single transmit antenna and arbitrary number of receive antennas. In [6], STBCs for multiuser MIMO system based on OFDM is designed for arbitrary number of transmit and receive antennas. However, none of these works consider the decoding complexity and co-channel interference issues which are very important in the multiuser multiple cells scenario. A trivial solution for all these issues is to coordinate the transmission from the users in TDMA manner, i.e., only one user is allowed to transmit at a time. However, in this case, the resulting joint space-time codeword will consist many zeros and result into a STBC with high peak-to-average power ratio (PAPR) [7] of each user.

The PAPR is important for several reasons. First, a high-PAPR signal is susceptible to clipping and other nonlinear distortion by the power amplifier, leading to both detection errors and out-of-band interference. Second, power consumption of the power amplifier depends mainly on the peak power rather than average power, and hence, high PAPR results in high power consumption. Finally, the transmission of a high-PAPR signal requires a power amplifier with a large back off, making it inefficient, bulky and expensive [8]. This point is more elaborated in [7] for STBC designs with single symbol decoding complexity.

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In this paper, we propose a $4 \times 4$ quasi orthogonal STBC (QOSTBC) which can be used in a two-user MIMO system, where, both users are in different cells but transmitting in same frequency band. This code provides decoupled decoding of each user’s data. Therefore, the co-channel interference is completely eliminated. If Alamouti code is used as a building block, then it is also possible to achieve the symbol wise decoding of the data. An important observation here is that the QOSTBCs of [9], [10] cannot be used in a two-user MIMO system based on MAC as they do not exchange the information. The proposed code provides better performance as compared to the existing multiuser STBCs. Moreover, the proposed STBC also provides significant reduction in PAPR of each user as compared to a trivial TDMA based two-user STBC.

II. SYSTEM MODEL AND CODE CONSTRUCTION

Consider a two-user MIMO system where each user is equipped with $n_t$ transmit antennas and two base-stations (BSs) each equipped with $n_r$ receive antennas as shown in Fig.1. Both users are in different cells utilizing same frequency band. User 1 and User 2 intend to transmit data to BS 1 and BS 2, respectively. Signals of User 1 generate interference to the User 2 and vice versa. The received data $Y_k \in \mathbb{C}^{n_r \times 2n_t}$ at the receiver in the BS 2 in $k$-the block over multiple access channels (MAC) can be given as

$$Y_k = H_k X_k + Q_k,$n_t$$

where $H_k$ is an $n_r \times 2n_t$ matrix consisting of uncorrelated zero mean complex Gaussian circularly distributed channel gain coefficients with component wise $\rho^2$ variance and $Q_k \in \mathbb{C}^{n_r \times 2n_t}$ contains additive white complex-valued Gaussian noise (AWGN) with $\sigma^2$ variance. Since both users transmit their space-time block code in multiple access manner, therefore, they are not allowed to exchange any information. Let $X_k \in \mathbb{C}^{n_t \times 2n_t}$ and $X_k^2 \in \mathbb{C}^{n_t \times 2n_t}$ be STBCs of User 1 and User 2, respectively. Hence, $X_k \in \mathbb{C}^{2n_t \times 2n_t} = \ldots$
\[
\begin{bmatrix}
(X_k^1)^T, (X_k^2)^T
\end{bmatrix}^T
\] is a joint two-user STBC for MAC channels designed in Subsections II-B and IV-A.

The PAPR for the \( m \in \{1, 2, \ldots, n_u\} \)-th transmit antenna of \( i \in \{1, 2\} \)-th user's STBC \( X_k \) is given as [7]

\[
PAPR_m = 2n_t \max_{t \in \{1, 2, \ldots, 2n_t\}} \left\{ \left| X_k^i \right|_{m,t} \right\}^2,
\]

where \( E \) is expectation over all codewords and \( X_k^i \) is element of \( X_k\) belonging to \( m\)-th row and \( t\)-th column.

A. Problem Statement

We aim at designing a non-trivial STBC for two-user MIMO system which can provide decoupled decoding of the data of each user over uplink channels such that the interuser interference is completely eliminated.

B. Code Structure

For simplicity we will explain the multiuser STBC design for \( n_u = 2 \) throughout the paper. However, the results presented in this paper can be extended to higher number of transmit antennas. Let \( s_k^1 = \left[ s_k^1, s_k^2 \right]^T \), where \( \left[ \cdot \right]^T \) denotes the transpose of a matrix, \( \{ s_k^1, s_k^2 \} \subseteq A, A \) is an arbitrary constellation be the symbols of user 1, and \( s_k^2 = \left[ s_k^3, s_k^4 \right]^T \), where \( \{ s_k^3, s_k^4 \} \subseteq A \) be the symbols of user 2, to be transmitted in \( k\)-th block. Before transmission \( s_k^1 \) and \( s_k^2 \) are encoded into the STBCs \( A_k \in \mathbb{C}^{2 \times 2} \) and \( B_k \in \mathbb{C}^{2 \times 2} \), respectively. We propose to transmit the following \( 4 \times 4 \) matrix over uplink channels:

\[
X_k = \begin{bmatrix}
A_k & A_k \\
B_k & -B_k
\end{bmatrix}.
\]

We name the STBC of (3) as AAB-B STBC. It can be seen from (3) that the proposed AAB-B requires coordinated transmission from both users. The coordination can be achieve by the X2 interface among base stations by transmitting very small amount of signaling overhead.

The STBCs \( A_k \) and \( B_k \) must be chosen such that the decoupled decoding is possible. One unanimous candidate for \( A_k \) and \( B_k \) is Alamouti OSTBC [4]. If \( A_k \) and \( B_k \) are chosen as Alamouti code, then \( X_k \) can be written as follows:

\[
X_k = \begin{bmatrix}
\hat{s}_k^1 - (\hat{s}_k^2)^* & \hat{s}_k^1 - (\hat{s}_k^2)^* \\
\hat{s}_k^3 - (\hat{s}_k^4)^* & \hat{s}_k^3 - (\hat{s}_k^4)^*
\end{bmatrix}.
\]

After a few manipulations it can be shown that

\[
X_kX_k^T = \begin{bmatrix}
2 \left\| s_k^1 \right\|^2 & 0_2 \\
0_2 & 2 \left\| s_k^2 \right\|^2
\end{bmatrix}.
\]

where \( I_2 \) is a \( 2 \times 2 \) identity matrix, \( 0_2 \) is an all zero matrix of \( 2 \times 2 \), and \( \left[ \cdot \right]^H \) stands for Hermitian of the input matrix. It can be seen from (5) that \( X_kX_k^T \) is diagonal matrix.

**Proposition 1**: The proposed AAB-B STBC provides lower value of PAPR as compared to a trivial TDMA based STBC.

**Proof**: By using the definition of PAPR (2) it can be verified that a trivial TDMA based two user STBC will provide significantly higher value of PAPR for each user as compared to the proposed STBC given in (4). 

As all transmit antennas have same value of PAPR, we drop the subscript \( m \) and \( i \) in (2). In order to support Proposition 1 we have tabulated few values of PAPR of the proposed AAB-B code and a trivial TDMA based STBC in Table I for different constellations. It can be seen from Table I that the proposed AAB-B provides significant reduction in PAPR as compared to the trivial two-user STBC.

<table>
<thead>
<tr>
<th>Code</th>
<th>PAPR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-QAM</td>
<td>3.0103</td>
</tr>
<tr>
<td>16-QAM</td>
<td>5.5630</td>
</tr>
<tr>
<td>64-QAM</td>
<td>6.6901</td>
</tr>
<tr>
<td>Trivial</td>
<td>0</td>
</tr>
<tr>
<td>AAB-B</td>
<td>2.5527</td>
</tr>
<tr>
<td>3.6798</td>
<td>3.6798</td>
</tr>
</tbody>
</table>

It is further shown in next section that it is possible to perform symbol wise decoding of the proposed AAB-B code similar to the trivial TDMA based STBC. Therefore, the proposed AAB-B provides significant reduction in PAPR without effecting the decoding complexity.

III. Decoding and Performance Analysis of the Proposed QOSTBC

By assuming that the receiver knows the channel \( H_k \) perfectly, we can write the maximum likelihood (ML) decoding of \( s_k \) as follows:

\[
\hat{s}_k = \arg \min_{\hat{s}_k \in A^4} \left\| Y_k - H_k \hat{X}_k \right\|
\]

where \( \left\| \cdot \right\| \) denotes the Frobenius norm and

\[
\hat{X}_k = \begin{bmatrix}
\hat{s}_k^1 - (\hat{s}_k^2)^* & \hat{s}_k^1 - (\hat{s}_k^2)^* \\
\hat{s}_k^3 - (\hat{s}_k^4)^* & \hat{s}_k^3 - (\hat{s}_k^4)^*
\end{bmatrix}.
\]

After few manipulations, it can be shown that

\[
\hat{s}_k = \arg \min_{\hat{s}_k \in A^4} \sum_{m=1}^{n_t} \left( \left\| \hat{s}_k^1 \right\|^2 \left| h_{m,1,1} \right|^2 + \left| h_{m,1,2} \right|^2 \right) + \left\| \hat{s}_k^2 \right\|^2 \left( \left| h_{m,2,1} \right|^2 + \left| h_{m,2,2} \right|^2 \right) - \Re \left\{ (h_{m,1,1}\hat{s}_k^1 + h_{m,1,2}\hat{s}_k^2) \left( y_{m,1} + y_{m,3} \right) + (h_{m,1,1}\hat{s}_k^1 + h_{m,1,2}\hat{s}_k^2) \left( y_{m,1} - y_{m,3} \right) + (h_{m,2,1}\hat{s}_k^2 + h_{m,2,2}\hat{s}_k^2) \left( y_{m,2} + y_{m,4} \right) + (h_{m,2,1}\hat{s}_k^2 + h_{m,2,2}\hat{s}_k^2) \left( y_{m,2} - y_{m,4} \right) \right\},
\]

where \( h_{m,i}, k = 1,2, i = 1,2,3,4, m = 1,2, \ldots, n_r \), is the channel gain between the \( m\)-th receive antenna and \( i\)-th transmit antenna of user \( k \), \( y_{m,l} \), \( l = 1,2,3,4 \) is the data received at \( m\)-th receive antenna in \( t\)-th time interval, and \( \Re \{ \cdot \} \) and \( \left( \cdot \right)^* \) stand for real part and complex conjugation, respectively, of a complex number. It can be seen from (8) that the decision variable does not contain any multiplication of \( s_k^1, s_k^2, s_k^3, s_k^4 \) and \( s_k \). Therefore, the decoding of \( s_k^1, s_k^2, s_k^3, s_k^4 \), and \( s_k \) can be performed independently as follows:

\[
\hat{s}_k^1 = \arg \min_{s_k^1 \in A} \sum_{m=1}^{n_t} \left( \left\| s_k^1 \right\|^2 \left| h_{m,1,1} \right|^2 + \left| h_{m,1,2} \right|^2 \right) - \Re \left\{ h_{m,1,1}\hat{s}_k^1 (y_{m,1} + y_{m,3}) + h_{m,1,2}\hat{s}_k^1 (y_{m,1} - y_{m,3}) \right\},
\]

\[
\hat{s}_k^2 = \arg \min_{s_k^2 \in A} \sum_{m=1}^{n_t} \left( \left\| s_k^2 \right\|^2 \left| h_{m,1,1} \right|^2 + \left| h_{m,1,2} \right|^2 \right) - \Re \left\{ h_{m,2,1}\hat{s}_k^2 (y_{m,1} + y_{m,3}) - h_{m,2,1}\hat{s}_k^2 (y_{m,2} + y_{m,4}) \right\},
\]
\[ s_3^k = \arg \min_{s_3^k \in \mathbb{C}^{d \times 1}} \sum_{m=1}^{n_r} \left( |\hat{s}_3^k|^2 \left( |h_{m,1,1}|^2 + |h_{m,2,1}|^2 \right) - \text{Re}\left\{ h_{m,1,1} \hat{s}_3^k \left( y_{m,1} - y_{m,3}^* \right) + h_{m,2,2} \left( \hat{s}_3^k \right)^* \left( y_{m,2} - y_{m,4}^* \right) \right\} \right) \] (11)

\[ s_4^k = \arg \min_{s_4^k \in \mathbb{C}^{d \times 1}} \sum_{m=1}^{n_r} \left( |\hat{s}_4^k|^2 \left( |h_{m,1,1}|^2 + |h_{m,2,1}|^2 \right) - \text{Re}\left\{ h_{m,2,2} \hat{s}_4^k \left( y_{m,1} - y_{m,3}^* \right) - h_{m,1,1} \left( \hat{s}_4^k \right)^* \left( y_{m,2} - y_{m,4}^* \right) \right\} \right) \] (12)

It can be seen from (9), (10), (11), and (12) that the proposed multiuser QOSTBC of (4) allows symbol wise decoding of the data of both users and results into a minimum complexity decoder at the base station. Further, it can be noticed that the previously proposed QOSTBCs [9], [10] can only provide pairwise decoding complexity and are not applicable in two-user MIMO system utilizing MAC strategy. The proposed AAB-B not only completely eliminates the interuser interference but also completely eliminates the inter symbol interference as well which is not possible in the existing schemes [3], [5], [6], [11].

### A. Performance Analysis of AAB-B Code

Let \( X_0^k \) is the transmitted codeword and \( X_k \) is the decoded codeword. Using the procedure given in [12, Section 4.2.2], (5), and the properties of the Kronecker product [13], an upper bound of pairwise error probability of the proposed AAB-B code can be obtained as follows:

\[ \mathbb{E}_{X_k} \left[ \Pr \left( X_0^k \rightarrow X_k \right) \right] \leq \left| I_4 + \frac{\rho^2}{4\sigma^2} X_k X_k^H \right|^{-n_r}, \] (13)

where \( X_k = X_0^k - X_k \).

**Lemma 1:** The proposed AAB-B code can achieve the single user diversity 2n_r.

**Proof:** By using the inequalities \(|A| \leq |I + A|\) and equivalently \(|A|^{-m} \geq |I + A|^{-m}, m \geq 0\) of a positive definite matrix \(A\) [12, Eq. (A.4.26)], we can further upper bound the PEP given in (13) as follows:

\[ \mathbb{E}_{X_k} \left[ \Pr \left( X_0^k \rightarrow X_k \right) \right] \leq \left( \frac{\rho^2}{4\sigma^2} \right)^{-n_r} \prod_{i=1}^{r} \lambda_i^{-n_r}, \] (14)

where \(r\) is the rank of \(X_k X_k^H\) and \(\lambda_i, i = 1, 2, 3, 4,\) is the finite eigenvalue of \(X_k X_k^H\).

Unlike the single-user point-to-point system where joint encoding across all transmit antennas is possible, individual users cannot coordinate their transmission in multiple access case. Therefore, out of all possible codeword difference matrices it is possible that in many cases one user decodes the data successfully while other is in outage. Hence, the minimum distance codeword difference matrix will be determined when one symbol out of \(s_1^k, s_2^k, s_3^k, s_4^k\) is decoded wrongly. It can be seen from (5) that in this case rank of \(X_k X_k^H\) is \(r = 2\). With this observation and from (14) it is apparent that the proposed AAB-B code provides the diversity order of \(rn_r = 2n_r\).

### IV. Optimized AAB-B Code

Full rate (1 symbol per channel use (SPCU)) OSTBC is available for 2 transmit antennas only. In order to achieve optimized coding gain over more than two transmit antennas with 1 SPCU data rate, we propose an optimized ABB-A code. We will describe optimized AAB-B code for \(n_t = 2\) only for simplicity, however, the AAB-B code for higher number of transmit antennas can be obtained by following the suggested code structure.

**A. Code Structure of Optimized AAB-B Code**

Let \( \Theta \in \mathbb{C}^{2 \times 2} \) and \( s_k = [s_1^k, s_2^k, s_3^k, s_4^k]^T \) be a vector of symbols of two users to be transmitted in block number \(k\). Then, the proposed \(4 \times 4\) optimized AAB-B code is constructed as follows:

\[ X_k = \begin{bmatrix} \text{diag} \{ \Theta s_1^k \} \Phi & \text{diag} \{ \Theta s_3^k \} \Phi \\ \text{diag} \{ \Theta s_2^k \} \Phi & -\text{diag} \{ \Theta s_4^k \} \Phi \end{bmatrix}, \] (15)

where \(\text{diag} \{a\}\) operator returns a diagonal matrix with the elements of vector \(a\) on its main diagonal, \(\Phi\) is an arbitrary \(2 \times 2\) unitary matrix, \(\Theta\) is \(2 \times 2\) optimized unitary rotation matrix which can be chosen as Vandermonde matrix [14]. It can be verified by using [8, Eq. (3)] that Proposition 1 is also valid for the optimized AAB-B code.

**B. Decoding of the Optimized AAB-B Code**

It can be shown after a few manipulations that \(X_k X_k^H\) is a diagonal matrix. By defining \(\Theta s_1^k \triangleq [\hat{\theta}_{1,1}, \hat{\theta}_{1,2}]^T\) and \(\Theta s_3^k \triangleq [\hat{\theta}_{2,1}, \hat{\theta}_{2,2}]^T\), using (6), and after few manipulations, it can be shown that the decoding of \(s_1^k\) and \(s_3^k\) can be performed as follows:

\[ s_1^k = \arg \min_{\hat{s}_1^k \in \mathbb{C}^2} \sum_{m=1}^{n_r} \left( |\hat{\theta}_{1,1}|^2 |h_{m,1,1}|^2 + |\hat{\theta}_{1,2}|^2 |h_{m,2,1}|^2 \right) - \text{Re}\left\{ h_{m,1,1} \hat{\theta}_{1,1} \left( y_{m,1} + y_{m,3} \right) + h_{m,2,1} \hat{\theta}_{1,2} \left( y_{m,2} + y_{m,4} \right) \right\}, \] (16)

\[ s_3^k = \arg \min_{\hat{s}_3^k \in \mathbb{C}^2} \sum_{m=1}^{n_r} \left( |\hat{\theta}_{2,1}|^2 |h_{m,3,1}|^2 + |\hat{\theta}_{2,2}|^2 |h_{m,4,1}|^2 \right) - \text{Re}\left\{ h_{m,3,1} \hat{\theta}_{2,1} \left( y_{m,3} \right) + h_{m,4,1} \hat{\theta}_{2,2} \left( y_{m,3} - y_{m,4} \right) \right\}. \] (17)

It can be seen from (16) and (17) that the proposed optimized AAB-B code allows pairwise decoding of the symbols. Therefore, the interuser interference is also completely eliminated by using the proposed optimized AAB-B code.

### V. Simulation Results

**A. Performance of Alamouti based and Optimized AAB-B codes with Single Receive Antenna**

Simulations are performed over uplink MAC channels with two users each having \(n_t = 2, 3\) transmit antennas and a base-station with one receive antennas, BPSK and QPSK constellations, Rayleigh fading, and using \(10^3\) channel realizations. Performance of the Alamouti based AAB-B code with QPSK constellation is shown in Fig. 2. We have also plotted the performance of the following multiuser STBC [6] with BPSK...
scheme over all SNRs. The proposed multiuser STBC outperforms the existing ABLAST scheme to decode the data of each user in decoupled manner. It can be seen from Fig. 2 that the proposed AAB-B code utilizes QPSK constellation. The ABLAST scheme for ABLAST scheme the multiuser code given in (18) is transmitted with two transmit antennas and the BS also has two receive antennas. It is taken into consideration where each user consists of two antennas.

For ABLAST scheme the multiuser code given in (18) is transmitted with two transmit antennas and the BS also has two receive antennas. It is taken into consideration where each user consists of two antennas.

\[ X_k = \begin{bmatrix} s_k^1 & -(s_k^2)^* \\ s_k^2 & (s_k^1)^* \\ s_k^3 & (s_k^4)^* \\ s_k^4 & -(s_k^3)^* \end{bmatrix}. \]  

(18)

For decoupled (suboptimal) decoding of multiuser STBC of (18) base-station considers the signals related to data of other user as noise. It can be seen from Fig. 2 that the decoupled decoding of the proposed Alamouti based AAB-B code performs better than the decoupled decoding of multiuser STBC of (18) from moderate to high SNR. Moreover, we have also shown the maximum likelihood (ML) decoding of the multiuser STBC of (18) which performs approximately same as the decoupled decoding of the proposed Alamouti based AAB-B code. However, the ML decoding of the existing multiuser STBC is much more complex than the decoupled decoding of the proposed multiuser STBC. The performance of the optimized AAB-B code with \( n_t = 3 \) and QPSK constellation is also shown in Fig. 2. It can be seen from Fig.2 that the proposed optimized AAB-B code provides third order diversity.

We have also plotted the performance of the trivial TDMA based STBC with QPSK constellation and \( n_t = 2 \) in Fig. 2. It can be seen from Fig. 2 that the proposed AAB-B code performs similar to the trivial scheme. However, the AAB-B code provides significant reduction in PAPR as compared to the trivial scheme without loosing coding gain.

B. Performance of Alamouti based AAB-B codes with Two Receive Antennas

In Fig. 3 we have shown the performance comparison of the proposed Alamouti based AAB-B code and existing Alamouti BLAST (ABLAST) scheme [11]. A two-user MIMO system is taken into consideration where each user consists of two transmit antennas and the BS also has two receive antennas. For ABLAST scheme the multiuser code given in (18) is transmitted with BPSK constellation. Whereas, the proposed AAB-B code utilizes QPSK constellation. The ABLAST scheme of [11, Section IV] utilizes a MMSE based BLAST detector to decode the data of each user in decoupled manner. It can be seen from Fig. 3 that the proposed AAB-B code provides better diversity than the ABLAST scheme. In addition, the proposed multiuser STBC outperforms the existing ABLAST scheme over all SNRs.

VI. CONCLUSIONS

We have proposed a quasi-orthogonal space-time block code (QOSTBC) which can be transmitted over uplink channels to reduce the co-channel interference of one user to other user in multiple cells based mobile communication system. Due to the quasi-orthogonal structure of the proposed code, the decoding of the data of each user can be performed separately. Hence, the cochannel interference is completely eliminated in the proposed scheme. The proposed code is able to provide \( 2n_t \) diversity gain to each user. In addition, the proposed code performs better than one of the existing multiuser space-time block codes.

REFERENCES


