Precoded DDOSTBC with Non-Unitary Constellations over Correlated Rayleigh Channels with Carrier Offsets

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Abstract—In this paper, we consider the effect of carrier offsets in arbitrarily correlated flat-fading multiple-input multiple-output (MIMO) Rayleigh channels. We propose a double-differential coding for square orthogonal space-time block code (OSTBC) with non-unitary constellations over flat-fading MIMO channels with carrier offsets. In order to improve the performance of the proposed double-differential orthogonal space-time block code (DDOSTBC) over arbitrarily correlated Rayleigh channels we introduce a full memoryless precoder matrix in the transmitter. For the proposed precoded DDOSTBC, a pair-wise error performance (PEP) analysis is conducted. Then we propose a precoder design based on the PEP upper bound. The proposed DDOSTBC differs from the previously proposed DDOSTBC by: 1) The previously proposed DDOSTBC is applicable to only a specific class of OSTBC which follows diagonal group property, whereas, the proposed DDOSTBC is applicable to any square OSTBC. 2) The previously proposed DDOSTBC can work on M-PSK constellation only, whereas, the proposed DDOSTBC is applicable to M-QAM, M-PAM, or M-PSK constellations. The proposed DDOSTBC provides higher performance gain as compared to the same rate previously proposed DDOSTBC. In addition, the proposed precoded DDOSTBC is able to perform better than the unprecoded DDOSTBC over arbitrarily correlated Rayleigh channels.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems have a substantial potential in terms of increasing the data rate, capacity, and diversity of wireless communication systems. Space-time block codes (STBC) are one of the main coding techniques for MIMO systems, which enable them to exploit full diversity without any channel knowledge at the transmitter. The decoding complexity can be further reduced by using orthogonal STBC (OSTBC). The receiver may use training data for the estimation of the channel coefficients at the cost of reduced effective data rate. However, training-based methods do not work well when the channel remains constant for relatively small number of channel usages. Differential coding is proposed in [1]–[4] to avoid the need of channel estimation at the receiver end.

Another challenge is the presence of the carrier offsets, which result because of the movement of the receiver or transmitter and/or scatterers, and mismatch between the transmit and receive oscillators. The differential systems in [1]–[4] fail to perform well in the presence of carrier offsets, because the carrier offsets make the channel behave as a time-varying channel. Double-differential (DD) coding [5], [6] is a key technique which could be used to avoid the need of carrier offset and channel estimation. DD coding enables the receiver to make decisions based on three consecutively received data blocks/matrices without any carrier offset or channel knowledge. DD coding for MIMO system was proposed in [7], however, it can only be applied to a specific class of STBC, which belong to the diagonal unitary group.

In this paper, our main contributions are as follows: 1) We propose a precoded double-differential coding, which is applicable to square OSTBC with M-QAM, M-PAM, and M-PSK constellations. 2) A low-complexity linear decoder of precoded DDOSTBC is obtained. 3) We derive an upper bound of the PEP of the proposed DDOSTBC for arbitrarily correlated Rayleigh channel. 4) Based on this PEP bound, we propose a precoder designed to improve the performance of DDOSTBC over correlated MIMO channels.

The rest of the paper is organized as follows: In Section II, DD encoding for OSTBC with non-unitary constellation is explained while the channel model is discussed in Section III. Decoding of precoded DDOSTBC is presented in Section IV. Section V performs the theoretical performance analysis of the precoded DDOSTBC scheme. The precoder is designed in Section VI. Simulation results and comparisons are discussed in Section VII. Some conclusions are drawn in Section VIII.

II. DOUBLE-DIFFERENTIAL ENCODING FOR OSTBC WITH NON-UNITARY CONSTELLATIONS

Let \( S_k, k \geq 2 \), be an \( n \times n \) OSTBC matrix obtained from \( s_k = [s_1, s_2, \ldots, s_n]^T \), \( n_s \leq n \), \( s_i \in M\text{-QAM}, M\text{-PAM}, \) or \( M\text{-PSK} \) constellation. The first order differential matrix \( C_k \) of size \( n \times n \) can be obtained as

\[
C_k = \frac{C_{k-1}}{s_{k-1}} S_k, \quad k \geq 2,
\]

with \( C_1 \in \mathbb{C}^{n \times n} \) as the initialization matrix. Next, the second order differential matrix \( D_k \in \mathbb{C}^{n \times n} \) can be obtained from \( C_k \) as

\[
D_k = \frac{D_{k-1}}{s_{k-1}} C_k, \quad k \geq 1,
\]

where \( D_0 \in \mathbb{C}^{n \times n} \) is the initialization matrix. For \( p \geq 2 \), the second order differential matrix \( D_p \) is obtained from the
current OSTBC matrix $S_p$ from:

$$D_p = \frac{D_0}{\|s_0\|} \frac{C_1}{\|s_1\|} \cdots \frac{C_{p-2}}{\|s_{p-2}\|} \frac{C_{p-1}}{\|s_{p-1}\|} S_p.$$  

(3)

III. CHANNEL MODEL

Consider a MIMO system with $n_t$ transmit and $n_r$ receive antennas. Let $\omega_m \in [-\pi, \pi]$ be the random carrier offset between the transmit antennas and the $m$-th receive antenna, which is independent of time $k$, and assumed to be constant over the transmission period of at least three DD encoded matrices $D_k$. The received data $y_{m,k} \in \mathbb{C}^{1 \times n}$ at the $m$-th receiver antenna corresponding to the $D_k$, $k \geq 0$, is [8, Eq. (9.7.14)]

$$y_{m,k} = \exp (j \omega_m n k) h_m F D_k \Omega_m + q_{m,k},$$  

(4)

where $\Omega_m \in \mathbb{C}^{n \times n}$ is the diagonal matrix given by $\Omega_m = \text{diag} \{h_m, \ldots, h_m\}$, $h_m = [h_{m,1}, \ldots, h_{m,m}]$ is an $1 \times n_t$ row vector consisting of channel coefficients between the transmit antennas and $m$-th receive antenna, $q_{m,k} \in \mathbb{C}^{1 \times n}$ is an additive white complex-valued Gaussian noise (AWGN), whose elements are i.i.d. Gaussian random variables with zero mean and variance $\sigma^2$, and $F \in \mathbb{C}^{n \times n}$ is a precoder matrix which is used to improve the performance of DDOSTBC over correlated Rayleigh channels. The channel $H = [h_1^T, h_2^T, \ldots, h_n^T]^T$ is assumed to be constant over the transmission period of at least three DD encoded matrices.

A. Model of Correlated Rayleigh Channels

We assume flat block-fading correlated Rayleigh channel model [9]. Let the channel $H$ have zero mean, complex Gaussian circularly symmetric distribution with positive semi-definite autocorrelation given by $R = \mathbb{E} \{\text{vec} (H) \text{vec}^H (H)\}$ of size $n_t n_r \times n_t n_r$. A channel realization of the correlated Rayleigh channels can be found by $\text{vec} (H) = R^{1/2} \text{vec} (H_w)$, where $R^{1/2}$ is the unique positive semi-definite matrix square root [10] of $R$ and $H_w$ is $n_r \times n_t$ matrix consisting of complex circular Gaussian distributed elements with zero mean and variance $\gamma^2$.

Kronecker Model: A special case of the model given above is the Kronecker model, which can be represented as [9, Eq. (3.26)] $R = R_0 \otimes R_r$, where $R_r$ is the $n_r \times n_r$ receive correlation matrix and $R_t$ is the $n_t \times n_t$ transmit correlation matrix.

IV. DECODING OF DDOSTBC

From (4), it can be seen that if $\omega_m, h_m, F$, and $D_k$ are known for all $k$ and $m$, $y_{m,k}$ is distributed as

$$f (y_{m,k} | \omega_m, h_m, F, D_k) = \frac{1}{\pi^{n} \text{det} (\Psi)} \times \exp \left( - \left[ y_{m,k} - \exp (j \omega_m n k) h_m F D_k \Omega_m \right]^* \Psi^{-1} \left[ y_{m,k} - \exp (j \omega_m n k) h_m F D_k \Omega_m \right] \right),$$  

(5)

where $\Psi$ is the covariance matrix of $q_{m,k}$. Since $q_{m,k}$ is AWGN, $\Psi = \sigma^2 I_n$. Let $y_m = [y_{m,k-2} y_{m,k-1} y_{m,k}]$ be the row vector consisting of the data samples received in three consecutive time intervals at the $m$-th receive antenna. When $\omega_m, h_m, F, D_{k-2}, C_{k-1},$ and $S_k$ are known,

$$f (y_m | \omega_m, h_m, F, D_{k-2}, C_{k-1}, S_k) = \frac{1}{\pi^{3n} (\sigma^2)^3 n} \times \exp \left( - \frac{1}{\sigma^2} \sum_{l=k-2}^k \| y_{m,l} - \exp (j \omega_m n l) h_m F D_l \Omega_m \|^2 \right).$$  

(6)

In order to find a maximum-likelihood (ML) estimate of the unknown data $S_k$, we need to maximize (6). If we assume that $\sigma^2$ is known, then the following metric is first minimized with respect to (w.r.t.) all unknown quantities $\omega_m, h_m, F, D_{k-2}$, and $C_{k-1}$, and, subsequently, over $S_k$ for $k \geq 2$:

$$I_k = \sum_{l=k-2}^k \| y_{m,l} - \exp (j \omega_m n l) h_m F D_l \Omega_m \|^2,$$  

(7)

which results into maximization of (6). Minimization of (7) w.r.t. $h_m, \omega_m, F, D_{k-2},$ and $C_{k-1}$, and $S_k$ is very complicated, therefore, we focus on a suboptimal decoder which makes the decision independent of the channel and carrier offset knowledge.

A. Suboptimal Decoder of DDOSTBC

To simplify the decision process, we consider a degenerated decision metric from (7) as follows for $k \geq 2$:

$$D_{m,k} = \sum_{l=k-2}^{k-1} \| y_{m,l} - \exp (j \omega_m n l) h_m F D_l \Omega_m \|^2,$$  

(8)

i.e., out of the three data vectors received at time $k - 2$, $k - 1$, and $k$, we consider only first two data vectors received at time $k - 2$ and $k - 1$. By means of the results given in [11] for matrix derivatives, (8) is minimized w.r.t. $g_{m,k} = \exp (j \omega_m n (k - 2)) h_m F D_{k-2} \Omega_m$ as

$$\hat{g}_{m,k} = \mathbb{E}_{\omega_m} y_{m,k-1} C_{k-1}^H + y_{m,k-2} | s_{k-2} |^2,$$  

(9)

where $\mathbb{E}_{\omega_m} = \exp (j \omega_m n)$, $C_{k-1} = \Omega_m^H C_{k-1} \Omega_m$, and $D_{k-2} = D_{k-2} / \| s_{k-2} \|$. By substituting (9) into (8) and using the unitary property of normalized OSTBC [8], for $k \geq 2$ (8) reduces into

$$D_{m,k} = \| \mathbb{E}_{\omega_m} \| s_{k-2} \|^2 \| y_{m,k-1} C_{k-1}^H - s_{k-1} \|^2 \|^2 y_{m,k-2} \|^2,$$  

(10)

From (10), it can be seen that if $C_{k-1}$ and $D_{k-2}$ are unknown, it is impossible to find the estimate of $\mathbb{E}_{\omega_m}$.

**Lemma 1:** If $D_{k-2} = C_{k-1} = I_n$, the estimator of $\mathbb{E}_{\omega_m}$ is given by

$$\hat{\mathbb{E}}_{\omega_m, k-1} = \exp \left( j \text{arg} \{ y_{m,k-1} y_{m,k-2}^H \} \right).$$  

(11)
Lemma 1 can be proved by differentiating (10) w.r.t. $\mathcal{E}_{\omega_m}$.

However, we cannot take the liberty of assuming $D_{k-2}$ and $C_{k-1}$ as the identity matrix as we are using OSTBC matrices with non-unitary constellation. Nevertheless, we may assume that the initialization matrix $D_0$ and $C_1$ are equal to the identity matrix. Then, the estimate of $\mathcal{E}_{\omega_m}$ can be found from (11) at $k = 2$, such that $\mathcal{E}_{\omega_m}$ can be used in the place of $\mathcal{E}_{\omega_m}$ in the further analysis. From (1), (2), and (4), the data received at the $k$-th time instant and at the $m$-th receive antenna can be written in the terms of $\mathcal{E}_{\omega_m}$, $g_{m,k}$, and $C_{k-1} = C_{k-1}/\| s_{k-1} \|$ as

$$y_{m,k} = \mathcal{E}_{\omega_m} g_{m,k} C_{k-1}^2 S_k \Omega_m + q_{m,k}.$$  

Hence, we can minimize the following decision metric to find the estimate of $S_k$,

$$\tilde{S}_k = \arg \min_{s_k \in \Xi} \left\| y_{m,k} - \mathcal{E}_{\omega_m,1} g_{m,k} C_{k-1}^2 \Omega_m \tilde{s}_k \right\|^2,$$  

(13)

where $\Xi$ is the set of all OSTBC matrices consisting of symbols from the $M$-PAM, $M$-QAM, or $M$-PSK constellation, $\Omega_m$ is obtained from $\mathcal{E}_{\omega_m,1}$, $C_{k-1}$ is the estimate of $C_{k-1}$ obtained using (1), and $\tilde{S}_k = \hat{\Omega}_m^* S_k \hat{\Omega}_m$. Let $h_m^\perp \triangleq \mathcal{E}_{\omega_m,1} g_{m,k} \hat{\Omega}_m^* C_{k-1} \hat{\Omega}_m$, then (13) can be written as

$$\tilde{S}_k = \arg \min_{s_k \in \Xi} \left\| y_{m,k} - h_m^\perp \tilde{s}_k \right\|^2.$$  

(14)

Equation (14) can be seen as the decoding of $S_k$ when the receiver knows $h_m$, the received vector $y_{m,k}$, and carrier offset matrix $\Omega_m$. By using the orthogonal properties of $S_k$, it can be shown that the decoding of $S_k$ can be done linearly. Apparently, the decoding of $S_k$ from (13) requires the knowledge of $C_{k-1}$. It can be assumed that the receiver has full knowledge about $C_1$ and in the subsequent time intervals it obtains $C_{k-1}$ from (1). The decoding of (13) can be generalized to the $n_r$ receive antennas case for $k \geq 2$ as

$$\tilde{S}_k = \arg \min_{s_k \in \Xi} \frac{1}{n_r} \sum_{m=1}^{n_r} \left\| y_{m,k} - \mathcal{E}_{\omega_m,1} g_{m,k} \hat{\Omega}_m^* C_{k-1}^2 S_k \hat{\Omega}_m \right\|^2.$$  

(15)

V. Performance Analysis of Precoded DDOSTBC

In this section, we analyze the pairwise error probability (PEP) of DDOSTBC over flat fading MIMO channels with carrier offset. From (15), the probability of detecting $S_k$ in place of $S_k^0$, where $S_k \neq S_k^0$, can be written as

$$\Pr \left\{ S_k^0 \rightarrow S_k \right\} = \Pr \left\{ \sum_{m=1}^{n_r} \left\| y_{m,k}^0 - \mathcal{E}_{\omega_m,1}^\perp g_{m,k} \hat{\Omega}_m^* C_{k-1}^2 S_k \hat{\Omega}_m \right\|^2 < \sum_{m=1}^{n_r} \left\| y_{m,k}^0 - \mathcal{E}_{\omega_m,1}^\perp g_{m,k} \hat{\Omega}_m^* C_{k-1}^2 S_k^0 \hat{\Omega}_m \right\|^2 \right\},$$  

(16)

where $y_{m,k}^0 = \exp (j\omega_m n_k) h_m \mathcal{F} D_{k-2} C_{k-1}^2 S_k^0 \Omega_m + q_{m,k}$ and $C_{k-1}$ is the estimate of $C_{k-1}$. After many manipulations, using [8, Theorem 4.2], the Chernoff bound [12, Eq. (2.172)], and Fisher’s matrix inequality [13, Eq. (11.8.1)], it can be shown that

$$\Pr \{ S_k^0 \rightarrow S_k \} \leq \exp \left\{ - \frac{\sum_{m=1}^{n_r} \left( \left\| \mathcal{E}_{\omega_m,1}^\perp \mathbf{v}_{m,k} S_k \right\|^2 \right)^{\frac{3}{2}}}{6\sigma^2 \sum_{m=1}^{n_r} \left\| \mathbf{v}_{m,k} S_k \right\|^2} \right\},$$  

(17)

where $\bar{S}_k = S_k^0 - S_k$, $W_{m,k} = \mathcal{E}_{\omega_m} \hat{\Omega}_m^* C_{k-1}^2$, $V_{m,k} = \bar{S}_k \hat{\Theta}_{m,k}$, $\hat{\Theta}_{m,k} = g_{m,k} \hat{\Omega}_m \mathbf{v}_{m,k}$, and $\lambda_{\min}(\mathbf{X}_{m,k})$ is the minimum eigenvalue of $\mathbf{X}_{m,k}$.

Lemma 2: The pairwise error probability of precoded DDOSTBC over arbitrarily correlated Rayleigh channel is bounded by

$$EH \left[ \Pr \{ S_k^0 \rightarrow S_k \} \right] \leq \left| I_{n_r} + \frac{1 + \lambda_{\min}(\mathbf{X}_{m,k})^2}{12\sigma^2} \right|,$$

$$\times \left[ (S_k^0 - S_k)^T \mathcal{F}^T \otimes I_{n_r} \right] \mathcal{F}^T \otimes I_{n_r}^{-1}.$$  

(18)

where $\mathbf{X}_{m,k} = \mathbf{X}_{m,k} |_{\omega_m = \pi/2, \omega_m = \pi/4}$.

Lemma 2 can be proved by assuming that all links are perturbed by the same worst carrier offset ($\omega_m = \pi/2$) and the estimator provides the worst estimate of the carrier offset ($\omega_m = \pi/4$) and then averaging (17) over $H$.

VI. DESIGN OF PRECODER FOR DDOSTBC

Since the channel statistics varies far more slowly than the channel coefficients, we assume that the receiver can perfectly estimate the channel correlation matrix and noise variance, and feed these back to the transmitter perfectly. By using the properties of OSTBC [8], it can be shown that the average power transmitted per DDOSTBC block is $E \left\{ D_k D_k^H \right\} = an_s\sigma^2 I_n$, where $\sigma^2$ is the average power of each symbol in the data vector $s_k$ to be encoded into DDOSTBC and $n_s$ is the total number of $M$-PAM, $M$-QAM, or $M$-PSK symbols transmitted over a DDOSTBC block. For DDOSTBC, we fix $a = 1$ and $\sigma^2 = 1/n_s$ to avoid power fluctuations. The average power constraint on the transmitted block $\mathcal{F} D_k$ can be expressed as $Tr \left\{ \mathcal{F} D_k \mathcal{F}^H \right\} = P$, where $P$ is the average power of the transmitted block $\mathcal{F} D_k$. We can express this optimization problem as [14]–[17]

$$\min_{\mathcal{F} \in \mathbb{C}^{n \times n}} \{ Tr \left\{ \mathcal{F} \mathcal{F}^H \right\} = P \} \text{ UBPEP}.$$  

(19)
Since the objective function should be minimized, to find for the general case of arbitrary correlation. Hence, we minimize problem by introducing a Lagrange multiplier.

From (18), we can rewrite the objective function as follows:

\[ \ln(UBPEP) = (S_0^k - S_k)^T F^T \Omega I_{n_r} \]

where \( F \) is the precoder matrix and \( I_{n_r} \) is the identity matrix of size \( n_r \times n_r \). The maximum value of \( UBPEP \) can be found by using the results in [11]. We can summarize the results of the derivative as follows:

\[ D_{F^*} \cdot \frac{\partial UBPEP}{\partial F} = \begin{pmatrix} R^T \Omega \end{pmatrix}^T \Omega \]

The channel is assumed to be uncorrelated, i.e., \( R = I_{n_r} \), and \( F = I_{n_t} \). The proposed DDOSTBC is applied over Alamouti STBC [18] with \( n_t = 2 \) and \( n_r = 1 \). Symbols are drawn from 16-QAM and 16-PSK constellations. The carrier offset is assumed to be randomly distributed over \([ -\pi, \pi ]\). The DD code of [7] utilizes diagonal STBC matrices, which can transmit only one M-PSK symbol per STBC block, hence, the DDSTBC of [7] provides low data rates. Therefore, for a fair comparison at the same data rate, the simulations are performed for (256:7,13) DDSTBC of [7] which transmits the following diagonal matrix in place of 256-PSK symbols

\[ C = \begin{pmatrix} z_{256}^7 & 0 \\ 0 & z_{256}^{13} \end{pmatrix} \]

where \( z_{256} = \exp(2\pi j/256) \). It can be seen from Fig. 1 that the proposed DDSTBC works well with unitary and non-unitary constellations. The proposed DDSTBC with 16-PSK constellation performs 4 dB better than the similar rate.
existing DDOSTBC [7] at SER=10^{-2}. In addition, it can be seen from Fig. 1 that the proposed DDOSTBC with non-unitary constellation outperforms the DDOSTBC with unitary constellation. At SER=10^{-2}, the DDOSTBC with non-unitary constellation performs approximately 4 dB better than the DDOSTBC with unitary constellation.

B. Performance of Precoded DDOSTBC over Correlated Rayleigh Channels

In Fig. 2, we have plotted simulation results for the Alamouti code with \( n_t = 2 \) and \( n_r = 2 \), and 16-QAM constellation. The channel is assumed to be a correlated Rayleigh channel with \( |\mathbf{R}_{i,j}| = (\rho)^{|i-j|}, \ 1 \leq |i,j| \leq n_t n_r \), with \( \rho \in \{0.9, 0.99999\} \). Each receive antenna is assumed to have different carrier offsets randomly distributed over \([-\pi, \pi] \). It can be seen from Fig. 2, that the proposed precoded DDOSTBC achieves performance gain over the correlated Rayleigh channel as compared to the unprecoded DDOSTBC.

VIII. CONCLUSIONS

We have proposed precoded double-differential coding for orthogonal space-time block codes with non-unitary constellations over arbitrarily correlated Rayleigh channels. The proposed double-differential codes are able to decode the space-time data without knowing the carrier offsets or channel coefficients and achieve higher performance gain as compared to the previously proposed double-differential space-time block codes. We have also proposed a PEP based precoder design criterion for the proposed double-differential codes over arbitrarily correlated Rayleigh channels.

REFERENCES


