Relay Strategies for High Rate Space-Time Code in Cooperative MIMO Networks

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Abstract— In this paper, we combine the idea of high rate Space-Time codes with the cooperative MIMO wireless networks to reduce the overall delay incurred in relaying signals to more than one receiver. The relay structure is optimized in order to achieve maximum SNR at the receiver nodes. This combination provides a significant reduction in the delay required for the relaying and transmission of the signals to the multiple receivers with a minute loss in performance. We have also shown that this loss in the performance could be recovered by selecting more number of relays. We propose three different relaying strategies for high rate Space-Time codes, which are very useful in providing faster handoffs for wireless networks.

I. INTRODUCTION

A. Motivations

It has been proved that cooperative communications, in particular, cooperative relay and cooperative MIMO, provides benefits of spatial diversity without physical antenna arrays [1]. A good relay strategy improves the performance of wireless networks by increasing coverage and by reducing end-to-end path losses. It has been shown in the literature that MIMO techniques and cooperative relaying may be combined to provide spatial and temporal transmit diversity and path loss compensation [1], [2]. There have been several useful works on relay strategies for cooperative MIMO systems [3], [4], [5]. A multiple relay network with an appropriate cooperative code construction that guarantees full spatial diversity gain is discussed in [4]. In [5], an optimum relay strategy for Alamouti S-T code is given for any number of relay nodes and single source-destination pair. In [6], High Data Rate Alamouti Codes [HDRAC] are proposed, which provide data rates of more than one, with linear decoding.

B. Contributions

In this work, we incorporate high rate space-time codes [6] in cooperative MIMO networks. We propose relaying strategies for high data rate Space-Time codes in two-user cooperative communication system. We also propose a full strategies for two-user cooperative communication system, which provide a significant reduction in time slots required for the relaying of the data of the two users.

Notations: We have used the following notations throughout the paper:  is used for scalar, is used for row/column vector, is used for matrix, is used for identity matrix, is used for complex transpose of matrix or vector, is used for complex conjugate of matrix or vector, and is used for transpose of matrix or vector

II. MODEL OF COOPERATIVE MIMO SYSTEM

We consider a simple cooperative MIMO system with one source, two destinations (users) and two relay nodes at equal distance from the source and destinations (users), shown in Fig. 1. Base station (BS) is equipped with two transmit antennas and relay nodes (R1 and R2) consist of one transmit and one receive antenna. Both users have one receive antenna. It is also assumed that each antenna can either transmit or receive signals at a time. The BS and relay transmit the data through TDMA. In addition, we have also assumed that users are almost at the edge of coverage area of BS and direct signals from BS to them are very weak in strength because of the heavy path loss. Nevertheless, the channels between BS and Relays and between Relay and users do not suffer from significant path loss. Next, let us first briefly review the high rate S-T codes [6].

III. HIGH RATE ALAMOUTI CODES FROM FIELD EXTENSION [6]

Proposition 1: Let be a field of characteristic zero and be an indeterminate. Also, let be the rational function field over in the indeterminate , that is, is the set of quotients of polynomials in with entries from . Then for any integer , the polynomial is irreducible in the ring .

Proof: See [8].

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**Proposition 2:** Let $K = F(\alpha)$ be an extension of field $F$ of degree $n$, and suppose the minimal polynomial of $\alpha$ over $F$ is $x^n + \alpha_{n-1}x^{n-1} + \ldots + \alpha_1x + \alpha_0$. Then the set of all matrices of the form $f_0I_n + f_1M + f_2M^2 + \ldots + f_{n-1}M^{n-1}$, where $M$ is the companion matrix of $\alpha$ and $f_0, f_1, \ldots, f_{n-1}$ are elements of $F$, has a property that the difference of any two matrices in it has full rank.

**Proof:** See [8] for proof.

These propositions are used to construct full rank high rate space-time code matrices as follows [8]: Let $Q$ be the set of rational numbers and $Q(\omega_n)$ denotes extension of $Q$ using a minimal polynomial $x^n - \omega_n$, where $\omega_n$ is an element of the set of $m$th roots of unity. If we extend $Q(\omega_m)$ using a transcendental number $\omega$, we get a rational function field $Q(\omega_m, \omega)$. Examples of transcendental numbers include $e$, $\pi$, and $e^{\pi}$ for any algebraic number $u$ [8]. The elements of $Q(\omega_m, \omega)$ are of the form $a(z)/b(z)$, where $a(z)$ and $b(z) \neq 0$ are polynomials over $Q(\omega_m)$. Then, from Proposition 1, $x^n - \omega_n$ is irreducible over $Q(\omega_m, \omega)$ [8]. Next, by using Proposition 2, with $F = Q(\omega_m)$ and $\alpha = \omega$, we can construct full rank Toeplitz STBC matrices as

$$X = \begin{bmatrix} f_0 & zf_1 & \ldots & zf_{n-1} \\ f_1 & f_0 & \ldots & z^2f_1 \\ \vdots & \vdots & \ddots & \vdots \\ f_{n-1} & f_{n-2} & \ldots & f_0 \end{bmatrix}, \quad (1)$$

where $(f_0, f_1, \ldots, f_{n-1}) \in F$. The $(i, j)$th element of $X$ is transmitted using the $i$th transmitter antenna at the $j$th time instant. Since $n$ symbols $f_0, f_1, \ldots, f_{n-1}$ are transmitted in $n$ time instants this is a rate-1 STBC. A rate-$R$ STBC can be obtained by replacing each symbol $f_i$ in (1) with a $R$th degree polynomial of the form

$$f_i(z) = \sum_{k=0}^{R-1} f_{i,k} z^k \quad \text{where } f_{i,k} \in Q(\omega_m). \quad (2)$$

Using (1) and (2), full rank rate-$R$ STBC can be obtained as [8]

$$X = \begin{bmatrix} f_0(z) & zf_{0,1}(z) & \ldots & zf_{0,n-1}(z) \\ f_1(z) & f_0(z) & \ldots & zf_{1,n-2}(z) \\ \vdots & \vdots & \ddots & \vdots \\ f_{n-1}(z) & f_{n-2}(z) & \ldots & f_0(z) \end{bmatrix}, \quad (3)$$

where $f_i(z) \in Q(\omega_m)[z], i = 0, 1, \ldots, n-1$.

The code matrix for the famous Alamouti code is given by

$$X = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}. \quad (4)$$

These matrices form a set, which has the property that the difference of any two such matrices is of full rank and hence, it satisfies the Proposition 2 [6]. Replacing $s_i$ with $f_{i-1,0} + f_{i-1,1}z$, where $f_{i,j} \in Q(\omega_m)$, we get the following code matrix [6]:

$$X = \begin{bmatrix} f_{0,0} + f_{0,1}z & -(f_{0,0} + f_{0,1}z)^* \\ f_{1,0} + f_{1,1}z & (f_{0,0} + f_{0,1}z)^* \end{bmatrix}. \quad (5)$$

**HDRAC** [6] proposed in (5) has a code rate of 2 because 4-symbols $f_{i,j}, i, j \in \{0, 1\}$ are transmitted in two time instants. In addition, because it is derived from the standard Alamouti code, it retains the simple decoding complexity of OSTBCs. These codes provide full diversity and better coding gain as compared to the other high rate codes [6].

**IV. FIRST RELAYING STRATEGY FOR HDRAC**

We consider a wireless system with two relay terminals in between source and destinations as shown in Fig. 1. Let us consider a scenario, where the users are at such a distance from BS that quality of the received signal drops a lot. Therefore, the signals are relayed from other nodes to them in order to improve the quality of reception. Let $h_1$ be channel gain vector between first antenna of BS and R1 and R2, and $h_2$ be channel gain vector between second antenna of BS and R1 and R2. The channel gain matrix between BS and relays can be expressed as

$$H = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}. \quad (6)$$

Moreover, let $h_1, h_2$ denotes channel gain vector between R1, R2 and User 1 and $h_3$ denotes channel gain vector between R1, R2, and User 2. All channel gains are assumed Rayleigh
distributed. If \( f_{0,0} \) and \( f_{1,0} \) are symbols of User 1 and \( f_{0,1} \) and \( f_{1,1} \) are symbols of User 2, data received at the relay terminals in first two time-intervals (I and II), will be

\[
R = X^H H + V_R,
\]

where \( R \) is a 2x2 matrix of received data and \( V_R \) is AWGN noise. Relays transform the received data matrix \( R \) by a 2x2 linear beamforming matrix \( B \) as

\[
Z = RB,
\]

where \( B \) can be determined in order to maximize SNR. This beamformed received data is further relayed to the users sequentially as shown in Fig. 2. Received data at the first user in next two-time intervals (III and IV) will be

\[
y = Z_{0,1} + w_1,
\]

where \( w_1 \) is circular AWGN noise for first user. We can write (9) in matrix form as

\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} =
\begin{bmatrix}
(f_{0,0} + f_{0,1}) z \\
(f_{0,1} + f_{1,1}) z
\end{bmatrix} + \begin{bmatrix}
h_{0,1} b_0 \\
h_{1,1} b_0
\end{bmatrix} + \begin{bmatrix}
w_1 \\
w_2
\end{bmatrix},
\]

which leads to

\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} =
\begin{bmatrix}
h_{0,1} b_0 \\
h_{1,1} b_0
\end{bmatrix} + \begin{bmatrix}
(f_{0,0} + f_{0,1}) z \\
(f_{0,1} + f_{1,1}) z
\end{bmatrix} + \begin{bmatrix}
v_1 b_1 \\
v_2 b_1
\end{bmatrix} + \begin{bmatrix}
w_1 \\
w_2
\end{bmatrix},
\]

or alternatively we may write (11) as

\[
\bar{y} = \bar{H} f + \bar{v} + \bar{w}_1
\]

or

\[
\bar{y} = \bar{H} f + \bar{q}
\]

where \( \bar{q} \) is the total noise. It can be shown that \( \bar{q} \) is still circular AWGN and

\[
\bar{H}^H \bar{H} = \left( |h_{0,1} b_0|^2 + |h_{1,1} b_0|^2 \right) I.
\]

Therefore, least squared (LS) estimates of \( f \) can be obtained as

\[
\hat{f} = \bar{H}^H \bar{y} \left( |h_{0,1} b_0|^2 + |h_{1,1} b_0|^2 \right)^{-1}
\]

Let \( B \) be a diagonal matrix, i.e., we are not assuming any dependence between the two relay terminals. Let \( g = B h_{0,1} \), and under the assumption of independence between all noise components SNR of LS estimates for User 1 could be found as [7]

\[
\text{SNR} = \frac{|h_1 g_1|^2 + |h_1 g_2|^2}{\sigma^2} E\left[\|f\|^2\right].
\]

Alternatively

\[
\text{SNR} = \frac{\|H g\|^2}{\sigma^2} E\left[\|f\|^2\right],
\]

where \( \sigma^2 \) is total noise power. Let us now maximize the SNR with subject to a constraint that

\[
\|g\|^2 \leq \gamma^2.
\]

Hence, the optimization problem takes on the following form:

\[
\max_g \|H g\|^2,
\]

s.t. \( \|g\|^2 \leq \gamma^2 \).

From Cauchy-Schwartz inequality [7]

\[
\frac{\|H g\|^2}{\|g\|^2} = \gamma^2 \quad \text{where} \quad \frac{\|H g\|^2}{\|g\|^2} \leq \lambda_{\text{max}} \left( H^H H \right),
\]

with equality if \( g \) is proportional to the eigenvector of \( H^H H \) that corresponds to the largest eigenvalue. Hence, optimum \( g \) is given by

\[
g_{\text{opt}} = \gamma \phi.
\]

Similarly, we can also find \( B \) matrix for relaying of the data to second user in next two-time intervals (V and VI), which would result into the same relation as (22) except \( H_2 \) replaced with \( H_1 \).

A. Comparison of First Relaying Scheme with Alamouti Relaying [5]

If we use conventional Alamouti relaying [5] for transmission of the data to two different users then the resulting timing diagram is shown in Fig. 3.

![Fig. 3 Timing diagram of Alamouti code relaying [5] for two users; time-intervals are shown as I, II, ..., VIII and transmission from single antenna on each node (BS and relays) is shown on y-axis by a rectangular block. Transmissions from BS are shown by green and from relay by yellow.](image)
we are able to save 33.33% time slots as compared to the conventional Alamouti relaying [5].

B. Performance Evaluation of First Relaying Strategy

Performance of the proposed scheme is investigated with one source node (base station) and two destination nodes. We assume that all relay terminals are essentially at the same distance from the source and users. Moreover, we use zero-mean unit variance complex Gaussian channel models for channel gains, which remain stationary for two symbol periods. The symbols of both the users are taken from QPSK constellation. γ is kept as 1/5. Fig. 4 shows the SER vs. SNR plots for Alamouti relaying for single user and new relaying scheme for two users. Apparently, the new scheme is only performing slightly more than 2dB poorer than the conventional Alamouti relaying scheme.

This loss in performance could be recovered if we use more than two relays between BS and users as shown in Fig. 5. However, the new high rate-relaying scheme provides the extremely useful reduction in the delay, which is very much crucial in handoff situations.

![Fig. 4 Comparison of first relaying scheme with the Alamouti relaying scheme.](image)

![Fig. 5 Performance of first relaying scheme with different number of relays.](image)

V. SECOND RELAYING STRATEGY FOR HDRAC

Timing diagram of second strategy is shown in Fig. 6. In this scheme, BS transmits the signals of both the users in first two time intervals (I and II) using HDRAC to relay terminals and the relay terminals relay data of the two users in next two time intervals (III and IV).

Model of system would remain same in this case, as in Section II. The SNR analysis for both the users would also remain same as that in first relay strategy in Section IV. However, the problem of maximization changes as here as we are relaying signals of both users simultaneously, therefore, maximization would be done for the sum of the SNRs of the two users as follows:

\[
\max_{\mathbf{g}} \left[ \frac{1}{2} \mathbf{H} \mathbf{g} \right] \quad \text{s.t.} \quad \| \mathbf{g} \|_2 \leq \gamma_1^2, \quad \| \mathbf{g} \|_2 \leq \gamma_2^2, \\
\| \mathbf{g} \|_2 \leq \gamma_1^2, \quad \text{and} \quad \| \mathbf{g} \|_2 \leq \gamma_2^2.
\]

where \( \mathbf{g}_1 = \mathbf{B} \mathbf{h}_1 \) and \( \mathbf{g}_2 = \mathbf{B} \mathbf{h}_2 \). Since \( \mathbf{B} \) remains same in both the cases and \( \mathbf{h}_1 \) and \( \mathbf{h}_2 \) are different, we could only optimize \( \mathbf{B} \) for any one of the two users at a time. It results into the loss of diversity. If we optimize \( \mathbf{B} \) for two users alternately, performance of such system is shown in Fig. 7. Apparently, we are getting diversity of the order of \( nr \), where \( n_r \) is number of receive antennas at the users.

A. Two User Full Diversity Strategy for Relaying Of HDRAC

Here, we reconsider the maximization problem of (24). We can rewrite (24) as follows:

\[
\| \mathbf{H} \mathbf{g}_1 \|_2^2 \leq \lambda_{\max} \left( \mathbf{H}^H \mathbf{H} \right), \\
\| \mathbf{g}_1 \|_2^2 \leq \lambda_{\max} \left( \mathbf{H}^H \mathbf{H} \right),
\]

(25)

Here, we have separated the problem of maximization of (24) into two different maximization problems given in (25) and (26). The optimum value of \( \mathbf{g}_1 \) and \( \mathbf{g}_2 \) can be found by following same procedure as followed in Section IV, as

\[
\mathbf{g}_{1opt} = \gamma_1 \mathbf{\phi} \quad \text{and} \quad \mathbf{g}_{2opt} = \gamma_2 \mathbf{\phi}.
\]

(27)

(28)
After substituting the value of $g_1 = Bh_1$ and $g_2 = Bh_2$ in (27) and (28), we get

$$Bh_1 = \gamma_1 \phi_1,$$
(29)

$$Bh_2 = \gamma_2 \phi_2,$$
(30)

where $B = \begin{bmatrix} h_1 & b_1 \\ b_2 & h_2 \end{bmatrix}$, $h_{11} = [h_{111} \ h_{112}]^T$ and $h_{22} = [h_{211} \ h_{222}]^T$.

Substituting, these value in (29) and (30) and solving the two equations we get the following values of $b_1$, $b_2$, $b_3$, and $b_4$:

$$h_1 = \frac{\gamma_1 \phi_1 - b_2 h_{112}}{h_{111}},$$
(31)

$$h_2 = \frac{\gamma_2 h_{211} - h_{212} - b_2 h_{211}}{h_{111}},$$
(32)

$$h_3 = \frac{\gamma_1 \phi_2 - b_2 h_{112}}{h_{111}},$$
(33)

$$h_4 = \frac{\gamma_2 h_{211} - h_{212} - b_2 h_{211}}{h_{111}}.$$
(34)

However, this solution is a non-optimal solution ($B$ is non-diagonal) but it preserves the full diversity (slope of SER versus SNR plot compared to the other schemes). The performance of this strategy is shown in Fig. 8.

VI. CONCLUSIONS

We have incorporated high rate Space-Time codes in cooperative MIMO networks to enable them to handle situations when multiple receivers are asking for handoff or require relaying of data almost simultaneously. As the base station can only transmit signals to one user at a time, the other users need to wait for first user to receive its signals. This delay involved in relaying increases call drop rate when multiple users cross the cell boundary almost simultaneously. We have proposed three relaying strategies for high data rate codes, which can significantly reduce the delay involved in the relaying of the data of different users.

REFERENCES


