Amplify-and-Forward Cooperative Communications Using Double-Differential Modulation over Nakagami-$m$ Channels

Manav R. Bhatnagar, Are Hjørungnes, and Lingyang Song
UniK – University Graduate Center,
University of Oslo, Instituttveien 25, P. O. Box 70, NO-2027 Kjeller, Norway
Email: {manav,arehj,lingyang}@unik.no

Abstract—In this paper, we propose double-differential (DD) modulation for an amplify-and-forward protocol based on cooperative communication over Nakagami-$m$ fading channels. The proposed scheme is able to achieve performance gain in the presence of random carrier offsets and without channel knowledge at relay or destination. The proposed scheme reflects its utility for the Nakagami-$m$ type flat fading channels with carrier offsets, where the conventional single differential scheme breaks down. In addition, the proposed scheme outperforms the conventional direct transmission double-differential system.

I. INTRODUCTION

Cooperative communications has several promising features to become a main technology in future wireless communications systems. It has been shown in the literature [1], [2] that the cooperative communication can avoid the difficulties of implementing actual antenna array and convert the single-input single-output (SISO) system into a virtual multiple-input multiple-output (MIMO) system. In this way, cooperation between the users allows them to exploit the diversity gain and other advantages of MIMO system at a SISO wireless network.

Various cooperation protocols for wireless systems were proposed in [3]. In the most basic decode-and-forward (DAF) protocol, a user (source) is needed to select another user who agrees to relay its data to the destination. The source sends information to the relay and destination. The relay decodes the data sent by the source and retransmits the decoded data to the destination. Hence, the destination has two received replicas of the same data and the quality of reception is expected to improve. However, if the channel between the source and relay is corrupted with a lot of noise, the relay cannot decode the data perfectly and relays erroneous data to the destination. This causes error floor in the performance, and, hence, the DAF protocol cannot achieve full diversity in its pure form and needs some intelligence added to the relay or destination for improving the performance. This may cause increase in the cost and power consumption at the relays and destination.

Another form of basic relaying protocol is the amplify-and-forward (AAF) protocol. In this scheme, the relay simply amplifies the received data without decoding it and retransmits the amplified data toward the destination. The destination now has two different copies of the same data and applies signal processing techniques to estimate the actual data. The AAF protocol does not need any intelligence at the relaying node or destination and generally performs better than the DAF protocol in the signal-to-noise ratio (SNR) range of the most practical interest—from low to medium SNR.

It is normally assumed that the destination and relay have perfect knowledge of the channel. However, in practice, it is difficult for relays and users to estimate the channel gains accurately. This problem becomes worse for fast fading channels.

In order to avoid channel estimation, differential modulation can be implemented in cooperative systems [4], [5], [6], [7]. The differential system requires the channel to be constant over at least two symbol durations. However, the presence of carrier offset due to the mismatch between the transmit and receive oscillators or relative motion of the receiver and transmitter, makes the block fading channel behave as time-varying channel, which does not remain constant over two consecutive time-periods. Hence, the performance of the differential scheme degrades substantially. This is an important practical problem for cooperative wireless communications which has not yet been given much consideration.

In this paper, our main contributions are: 1) We apply double-differential (DD) modulation in AAF cooperative wireless communications to achieve the performance gain in flat fading Nakagami-$m$ channels with carrier offsets. 2) We derive analytical BER of the DD modulation with AAF protocol (DDAAF). 3) Based on the BER analysis we also determine the power allocation for the DD cooperative systems.

The rest of this paper is organized as follows: In Section II, the system model, channel model, and DD modulation for a SISO link are discussed. Section III implements DD modulation in the AAF cooperative communications. The BER performance analysis of DD modulation with AAF protocol is performed in Section IV. In Section V, the analytical and simulation results are discussed and details of the power allocation to minimize the BER of DDAAF cooperative system are provided. Section VI contains some conclusions. The article also contains two appendices, which provide detailed derivations.

II. SYSTEM MODEL

We consider a basic cooperative communication system, which consists of one source, one relay, and one destination terminal as shown in Fig. 1. Each of them can either transmit
or receive a signal at a time. The transmission of the data from the source to the destination terminal is furnished in two phases. In the first phase, the source broadcasts data to the destination and the relay. The relay amplifies the received data and retransmits it to the destination, in the second phase. To avoid the interference, source and relay use orthogonal channels for transmission [3]. For ease of presentation we assume that in both phases, the source and relay transmit data through time-division multiplexing (TDMA). In the TDMA scheme, the source has to remain silent in the second phase in order to maintain the orthogonality between the transmissions. However, in the frequency-division multiplexing (FDMA) or the code-division multiplexing (CDMA) schemes, the source and the relay can transmit at the same time.

A. Channel Model

All links are assumed to be Nakagami-\(m\) distributed with the following probability density function (p.d.f.) [8, Eq. (2.21)]

\[
 f_{\gamma,p,q}(\gamma_{p,q}) = \frac{m_{p,q}^{m_{p,q} - 1}\gamma_{p,q}^{m_{p,q} - 1}}{\bar{\gamma}_{p,q}^m \Gamma(m_{p,q})} \exp\left(-\frac{m_{p,q}\gamma_{p,q}}{\bar{\gamma}_{p,q}}\right), \quad \gamma_{p,q} \geq 0,
\]

where \((p, q) \in \{(s, d), (s, r), (r, d)\}\), \(m_{p,q} \geq 0.5\) is the Nakagami-\(m\) fading parameter, \(\gamma_{p,q}\) is the instantaneous signal to noise ratio (SNR), and \(\bar{\gamma}_{p,q}\) is the average SNR of the link. The channel of each link is assumed to be a block fading channel, which remains constant for at least three consecutive time intervals and all the channel coefficients are assumed to be independent of each other. It is also assumed that all three links are perturbed by the different carrier offsets, which are randomly distributed and independent of each other. The random offsets are assumed to be uniformly distributed over \([-\pi, \pi]\), however, generally there is no restriction over the probability distribution of the offsets and they could have any probability distribution. We have assumed that these offsets remain fixed for at least three consecutive time-intervals. The presence of carrier offsets makes all three block fading channels behave as time-varying channels, which do not remain constant over two consecutive time-intervals.

B. Double-Differential Modulation

Let \(z[n]\) denote the symbols belonging to the unit-norm \(M\)-PSK constellation \(\Xi\) to be transmitted at the time \(n\). In a DD modulation based system, the transmitted signal \(v[n]\) is obtained from \(z[n]\) as shown in Fig. 2 (a):

\[
x[n] = \sqrt{\rho} e^{j\omega n} v[n] + e[n], \quad n = 0, 1, \ldots,
\]

where \(x[n]\) is the received signal, \(\rho\) is the transmitted signal power, \(h\) is the channel gain, \(e[n]\) is complex-valued additive white Gaussian noise (AWGN), and \(\omega \in [-\pi, \pi]\) is an unknown frequency offset. The receiver makes a decision variable, \(d[n] = X[n]X^*[n-1]\), where \(X[n] = x[n] x^*[n-1]\) as shown in Fig. 2 (b). The maximum likelihood (ML) optimal decoding of \(z[n]\) is performed in the following way [9, Eq. (15)]:

\[
\hat{z}[n] = \text{arg max}_{z \in \Xi} \text{Re} \{X[n]X^*[n-1]z^*\}. \quad (4)
\]

III. DOUBLE-DIFFERENTIAL MODULATION FOR AAF

COOPERATIVE COMMUNICATION SYSTEM

If we use DD modulation in the cooperative communication system, the data received during the first phase at the destination is

\[
x_{s,d}[n] = \sqrt{P_1} h_{s,d} e^{j\omega_{s,d} n} v[n] + e_{s,d}[n], \quad n = 0, 1, \ldots,
\]

and at the relay is

\[
x_{s,r}[n] = \sqrt{P_1} h_{s,r} e^{j\omega_{s,r} n} v[n] + e_{s,r}[n], \quad n = 0, 1, \ldots,
\]

where \(P_1\) is the power transmitted by the source, \(h_{s,d}\) and \(h_{s,r}\) are the channel gains, and \(\omega_{s,d}\) and \(\omega_{s,r}\) are the carrier offsets between source and destination, and source and relay, respectively, and \(e_{s,d}[n]\) and \(e_{s,r}[n]\) are AWGN noise on the two links. During the second phase the source remains silent and the relay amplifies the received data of (6) and retransmits such that the received signal by the destination in the second phase is:

\[
x_{r,d}[l] = \sqrt{P_2} h_{r,d} e^{j\omega_{r,d}m} x_{s,r}[l] + e_{r,d}[l], \quad l = 0, 1, \ldots,
\]

where \(P_2\) is the power transmitted by the relay.
where $l$ is the time index which is used in the place of $n$ to show the difference in time of first and second phases, \( h_{r,d} \) is the channel gain, \( \omega_{r,d} \) is the carrier offset between relay and destination, \( e_{r,d}[l] \) is the AWGN noise, and \( \hat{P}_2 \) is the amplification factor which ensures constant average transmission power during the second phase. It can be seen from (6), that the average power of \( x_{s,r}[l] \) is \( P_1 \sigma^2_{s,r} + \sigma^2 \), where \( \sigma^2_{s,r} \) is the variance of \( h_{s,r} \), and \( \sigma^2 \) is the variance of the AWGN noise \( e_{s,r}[l] \), hence, \( \hat{P}_2 \) is defined as

\[
\hat{P}_2 = \frac{P_2}{P_1 \sigma^2_{s,r} + \sigma^2},
\]

where \( P_2 \) is the average power transmitted by relay. It is also assumed that \( P_1 + P_2 = P \), where \( P \) is the total transmitted power. Next, we propose the following maximal ratio combining (MRC) [10] based scheme for a DDAAF receiver

\[
d[k] = \alpha_1(x_{s,d}[n] x^*_{s,d}[n-1])(x_{s,d}[n-1] x^*_{s,d}[n-2])^* + \alpha_2(x_{r,d}[l] x^*_{r,d}[l-1])(x_{r,d}[l-1] x^*_{r,d}[l-2])^*,
\]

where \( k = n = l \), i.e., the data received by the destination during the same time interval with respect to the beginning of the each phase is combined, and \( \alpha_1 \) and \( \alpha_2 \) are given as

\[
\alpha_1 = \frac{1}{\sqrt{2P_1 |h_{s,d}|^2 + \sigma^2}},
\]

\[
\alpha_2 = \frac{P_1 \sigma^2_{s,r} + \sigma^2}{\kappa},
\]

where

\[
\kappa = 2P_1 P_2 |h_{r,d}|^4 |h_{s,r}|^2 \sigma^2 + 2P_1 P_2 (P_1 \sigma^2_{s,r} + \sigma^2) |h_{r,d}|^2 |h_{s,r}|^2 \sigma^2 + P_2 |h_{r,d}|^4 \sigma^4 + 2P_2 (P_1 \sigma^2_{s,r} + \sigma^2)^2 \sigma^4.
\]

A detailed derivation of the MRC scheme is given in Appendix I. However, as we intend to use DD modulation, the destination and relay are not expected to have knowledge of the channel gains, therefore, we can emulate the MRC by replacing the channel coefficients \( |h_{s,r}|^2 \), \( |h_{r,d}|^2 \), and \( |h_{s,d}|^2 \) by their variances \( \sigma^2_{s,r}, \sigma^2_{r,d}, \) and \( \sigma^2_{s,d} \), respectively, in (8), (10), and (11). Then, the data is decoded as

\[
\hat{z}[n] = \arg \max_{z \in \mathbb{Z}} \text{Re} \{d[k] z^*\},
\]

where \( n = k \).

IV. BER PERFORMANCE ANALYSIS

Admittedly, the emulated maximum ratio combining (EMRC) obtained by replacing channel gains by their variances in (9) will perform poorer to the ideal MRC scheme given by (9), (10), and (11) [10]. For the simplicity of the analysis, we assume that the instantaneous signal to noise ratio (SNR) of the EMRC scheme is

\[
\gamma = \gamma_{s,d} + \gamma_{s,r,d},
\]

where \( \gamma_{s,d} \) and \( \gamma_{s,r,d} \) are the instantaneous SNRs of the direct link between source and destination, and cooperative link between source and destination through relay, respectively. This assumption is justified by the simulation results in Section V as the EMRC scheme performs very close to the ideal MRC scheme.

A. Analogy between Double-Differential and Single Differential Modulation

In a single differential modulation based system, \( p[n] \) is obtained from \( z[n] \) as shown in the first line of (2) with \( p[0] = 1 \). The data received corresponding to the transmitted signal \( p[n] \) over a channel \( h \) unperturbed by the carrier offset is

\[
x[n] = \sqrt{\gamma} p[n] + e[n], \quad n = 0, 1, \ldots
\]

The ML decoding of \( z[n] \) is performed as follows [11]:

\[
\hat{z}[n] = \arg \max_{z \in \mathbb{Z}} \text{Re} \{x[n] z^* \}.
\]

It can be observed that the optimal decoding of double differentially modulated signal is analogous to the single differential modulation and this fact could be verified by comparing (4) and (16). Therefore, we can approximate the performance of DDMPSK by the BER expressions of DMPSK with the SNR of \( X[n] \). This connection is shown in more detail in [11], [12], [13]. We can find the SNR of \( X[n] \) as

\[
\frac{E_s}{E_N} = \frac{\gamma'}{2 + (\gamma')^{-1}},
\]

where \( E_s \) is the signal power, \( E_N \) is the total noise power, and \( \gamma' \) is SNR of \( X[n] \) in (15). We may further take the following high SNR approximation to maintain the mathematical feasibility of the analysis

\[
\frac{\gamma'}{2 + (\gamma')^{-1}} \approx \frac{\gamma'}{2} - \frac{1}{4}.
\]

As a cross-check, we have compared the exact and approximate SNRs in Fig. 3 and it is satisfying to see that the approximate SNR follows closely the exact one for \( \gamma' > 5 \) dB, which is the region of \( \gamma' \) values of most practical interest.
B. Average BER of DDAF system

From (6), (7), and (8) it can be shown that the SNR of the link between relay and destination under double-differential modulation is

\[
\gamma_{s,r,d} = \frac{P_1 P_2 [h_{s,r}]^4 [h_{r,d}]^4}{\kappa},
\]

(19)

where \( \kappa \) is defined in (12). After some manipulations, it can be shown that

\[
\gamma'_{s,r,d} = \frac{\gamma'_{s,r,d}}{2 + (\gamma'_{s,r,d})^{-1}} \approx \frac{\gamma'_{s,r,d}^2}{2 - \frac{1}{4} \gamma'_{s,r,d}},
\]

(20)

where

\[
\gamma'_{s,r,d} = \frac{P_1 P_2 [h_{s,r}]^2 [h_{r,d}]^2}{(P_1 \sigma_{s,r}^2 + P_2 [h_{r,d}]^2 + \sigma^2)^2}.
\]

(21)

It can be seen from [14, Eq. (6)], that \( \gamma_{s,r,d} \) is the instantaneous SNR of dual-hop fixed gain relay transmission scheme. Let

\[
\gamma_{s,d} = \frac{P_1 [h_{s,d}]^2}{\sigma^2}, \quad \gamma_{s,r} = \frac{P_1 [h_{s,r}]^2}{\sigma^2}, \quad \gamma_{r,d} = \frac{P_2 [h_{r,d}]^2}{\sigma^2}, \quad \gamma_{s,d} = \frac{P_1 \sigma_{s,d}^2}{\sigma^2}, \quad \gamma_{s,r} = \frac{P_1 \sigma_{s,r}^2}{\sigma^2}, \quad \text{and} \quad \gamma_{r,d} = \frac{P_2 \sigma_{r,d}^2}{\sigma^2}.
\]

(22)

It can be seen from (1), that for Nakagami-m independent fading channels, \([h_{s,d}]^2 / [h_{s,r}]^2\) and \([h_{r,d}]^2\) are independent gamma random variables with parameters \(m_{s,d}\) and \(1/\sigma_{s,d}^2\), \(m_{s,r}\) and \(1/\sigma_{s,r}^2\), and \(m_{r,d}\) and \(1/\sigma_{r,d}^2\), respectively. The p.d.f. of \(\gamma_{s,r,d}\) for integer values of \(m_{s,r,d}\) can be written as [7, Eq. (16)]

\[
P_{\gamma_{s,r,d}}(n) = \frac{2^{m_{s,r,d}+1}}{\Gamma(m_{s,r,d}) \Gamma(m_{r,d}) \Gamma(P_1 \sigma_{s,r}^2)} \frac{m_{s,r} m_{r,d}^2}{P_1^2},
\]

(23)

where \(K_{\gamma}(\cdot)\) denotes \(\gamma\)-th modified Bessel function of second kind [15, Eq. (9.6.2)] and \(C_k^p\) is binomial coefficient [15, Eq. (3.1.2)].

It can be seen from (17) and (18), that the SNR of the direct link from source to destination under DD modulation can be expressed as

\[
\gamma_{s,d} \approx \frac{\gamma'_{s,d}}{2} - \frac{1}{4},
\]

(23)

where \(\gamma'_{s,d}\) is the SNR of the link under single differential modulation.

From the analogy between double and single differential modulation in Subsection IV-A, it is clear that the BER expressions of DD modulation can be obtained by replacing the SNR of single differential system by the SNR of DD system. For differential BPSK using two independent (but not identically) distributed channels, the BER conditioned on \(\gamma = \gamma_{s,d} + \gamma_{s,r,d}\) is given by [16, Eq. (12.1.13)]:

\[
P_b(\gamma) = \frac{1}{8} (4 + \gamma) \exp(-\gamma).
\]

(24)

Substituting the values of \(\gamma_s, \gamma_{s,d}, \text{and} \gamma_{s,r,d}\) from (14), (20), and (23), respectively, in (24) we can have the BER for DDAF system as

\[
P_b(h_{s,d}, h_{s,r}, h_{r,d}) = \frac{1}{8} \left(\frac{7}{2} + \gamma_{s,d} + \gamma_{s,r,d}\right)
\times \exp\left(-\frac{1}{2} - \gamma_{s,d} - \gamma_{s,r,d}\right).
\]

(25)

**Theorem 1:** BER of DDAF system with BPSK modulation averaged over all channel can be written as

\[
P_b = \frac{\exp\left(\frac{3}{16}\right)}{16} \left\{P_{b1} P_{b2} + P_{b2} P_{b3} + P_{b1} P_{b4}\right\},
\]

(26)

where

\[
P_{b1} = \frac{1}{\Gamma(m_{s,r}) \Gamma(m_{r,d}) \Gamma(P_1 \sigma_{s,r}^2)} \frac{m_{s,r}^2 m_{r,d}^2}{P_1^2 \sigma_{s,r}^2} \times \exp\left(-\frac{m_{s,r} m_{r,d} (P_1 \sigma_{s,r}^2 + \sigma^2)^2}{2 P_1^2 \sigma_{s,r}^2}\right)
\times \sum_{k=0}^{m_{s,r}} C_k^{m_{s,r}} \left(\frac{m_{s,r} \sigma^2 + P_1^2 \sigma_{s,r}^2}{m_{r,d} P_1 \sigma_{s,r}^2}\right) \frac{m_{s,r} - k}{2},
\]

(27)

\[
P_{b2} = \left(\frac{2 m_{s,d} m_{r,d}}{2 m_{s,d} \sigma^2 + P_1^2 \sigma_{s,d}^2}\right),
\]

(28)

\[
P_{b3} = \frac{1}{\Gamma(m_{s,r}) \Gamma(m_{r,d}) \Gamma(P_1 \sigma_{s,r}^2)} \frac{m_{s,r}^2 m_{r,d}^2}{P_1^2 \sigma_{s,r}^2} \times \exp\left(-\frac{m_{s,r} m_{r,d} (P_1 \sigma_{s,r}^2 + \sigma^2)^2}{2 P_1^2 \sigma_{s,r}^2}\right)
\times \sum_{k=0}^{m_{s,r}} C_k^{m_{s,r}} \left(\frac{m_{s,r} \sigma^2 + P_1^2 \sigma_{s,r}^2}{m_{r,d} P_1 \sigma_{s,r}^2}\right) \frac{m_{s,r} - k}{2},
\]

(29)

\[
P_{b4} = \frac{1}{\Gamma(m_{s,r}) \Gamma(m_{r,d}) \Gamma(P_1 \sigma_{s,r}^2)} \frac{m_{s,r}^2 m_{r,d}^2}{P_1^2 \sigma_{s,r}^2} \times \exp\left(-\frac{m_{s,r} m_{r,d} (P_1 \sigma_{s,r}^2 + \sigma^2)^2}{2 P_1^2 \sigma_{s,r}^2}\right)
\times \sum_{k=0}^{m_{s,r}} C_k^{m_{s,r}} \left(\frac{m_{s,r} \sigma^2 + P_1^2 \sigma_{s,r}^2}{m_{r,d} P_1 \sigma_{s,r}^2}\right) \frac{m_{s,r} - k}{2},
\]

(30)
A performance gain of more than $5 \, \text{dB}$ is observed at SER of diversity as compared to the direct transmission scheme and (Fig. 4, it can be seen that the proposed scheme has higher performance gain over the links as will be shown in the Section V-C. From Fig. 4, we can always achieve performance gain by power distribution over the links as will be shown in the Section V-C. It can also be observed that there is a collapse in the performance of the conventional differential scheme [4], because of the presence of random carrier offsets.

### B. Analytical and Experimental Performance

Fig. 5 shows the analytical and experimental performance of the proposed DDAAF based cooperative scheme with random carrier offsets. We have plotted the approximate analytical BER (25) for BPSK constellation, $P_1/P = P_2/P = 0.5$, $\sigma_{s,d}^2 = \sigma_{s,r}^2 = \sigma_{r,d}^2 = 1$, and $m_{s,d} = m_{s,r} = m_{r,d} \in \{1, 2, 3\}$. The simulation results of the proposed DDAAF scheme are also shown under the same conditions. From Fig. 5, it is seen that the experimental data closely follows the analytical results from moderate to high SNRs. However, there is a mismatch at the SNRs $< 6 \, \text{dB}$. Hence, this justifies the assumption taken in (14) and (18).

### C. Power Allocation for DDAAF System

It can be seen from (26) that the BER of DDAAF system nonlinearly depends upon $P_1$ and $P_2$. Therefore, with power constraint $P_1 + P_2 = P$ we can obtain the values of $P_1$ and $P_2$ which minimize the BER. We have calculated power distribution for $\text{SNR} = 20 \, \text{dB}$ by numerically minimizing (26) with power constraint $P_1 + P_2 = P$, $\sigma_{s,d}^2 = \sigma_{s,r}^2 = 1$, and $\sigma_{r,d}^2 = 10$. Fig. 6 shows the performance of DDAAF scheme with equal and numerically calculated power allocation over Nakagami-$m$ channels with $m = 2, 3$. It can be seen from Fig. 6, that DDAAF scheme with the calculated power distribution outperforms the DDAAF scheme with uniform power distribution $P_1 = P_2 = 0.5P$.

### VI. Conclusions

We have implemented double-differential modulation in cooperative communication system with the amplify-and-
forward protocol to avoid the problem of carrier offsets in Nakagami-\(m\) fading channels. Our scheme performs well in the practical scenario, where the conventional differential modulation schemes fail. With our scheme, the users are still able to decode their data without knowing the channel gains or carrier offsets. We have performed the BER analysis to predict the behavior of the cooperative system. In addition, we have also proposed a numerical power allocation based on this analysis to further improve the performance of the system.

**APPENDIX I**

**DERIVATION OF \(\alpha_1\) AND \(\alpha_2\)**

Similar to the single differential systems [4], [5], [16], it can be seen from (4) that for implementing MRC combining for the DDAF system we need to find the total noise of \(X_{s,d}[n] = x_{s,d}[n] + \sigma^2_{s,d}[n - 1]\) and \(X_{s,r,d}[n] = x_{s,r,d}[n] + \sigma^2_{s,r,d}[n - 1]\), and normalize the decision variables of each diversity branch by its noise before combining. The normalization factors \(\alpha_1\) and \(\alpha_2\) can be found as follows: The average noise power over the direct link under double-differential modulation can be calculated from (5) and written as

\[
E_1 = 2P_1 |h_{s,d}|^2 \sigma^2 + \sigma^4.
\]  

(31)

From (7), the noise power over the cooperative link under double-differential modulation can be calculated after some manipulations as

\[
E_2 = \frac{1}{(P_1\sigma^2_{s,r} + \sigma^2)} \left\{ 2P_1P_2^2 |h_{r,d}|^4 h_{s,r}^2 \sigma^2 + 2P_1P_2 \right. \\
\times \left. (P_1\sigma^2_{s,r} + \sigma^2) |h_{r,d}|^2 h_{s,r}^2 \sigma^2 + P_2^2 |h_{r,d}|^4 \sigma^4 + 2P_2 \right. \\
\times \left. (P_1\sigma^2_{s,r} + \sigma^2) |h_{r,d}|^2 \sigma^4 + (P_1\sigma^2_{s,r} + \sigma^2)^2 \sigma^4 \right\}.
\]  

(32)

The normalization factors can be found as \(\alpha_1 = 1/E_1\) and \(\alpha_2 = 1/E_2\).

**APPENDIX II**

**PROOF OF THEOREM 1**

(25) can be averaged over all the channels as follows:

\[
P_b = \frac{e^{(1/2)}}{16} \left\{ 7 \int_0^\infty e^{-\frac{(\alpha - \beta)}{2}} p_{\gamma,s,r,d}(\alpha) d\alpha \int_0^\infty e^{-\frac{(\beta - \alpha)}{2}} p_{\gamma,s,r,d}(\beta) d\beta \\
+ \int_0^\infty \alpha e^{-\frac{(\alpha - \beta)}{2}} p_{\gamma,s,r,d}(\alpha) d\alpha \int_0^\infty e^{-\frac{(\beta - \alpha)}{2}} p_{\gamma,s,r,d}(\beta) d\beta \\
+ \int_0^\infty e^{-\frac{(\alpha - \beta)}{2}} p_{\gamma,s,r,d}(\alpha) d\alpha \times \int_0^\infty \beta e^{-\frac{(\beta - \alpha)}{2}} p_{\gamma,s,r,d}(\beta) d\beta \right\}.
\]  

(33)

Each integral in (33) can be represented by \(P_{bi}, i \in \{1, 2, 3, 4\}\) as shown by (26). These integral can be solved with the help of change of variable and [17, Eqs. (6.631.3) and (3.478.1)].

**REFERENCES**


