Optimal maintenance schedule decisions for machine tools considering the user's cost structure

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Optimal maintenance schedule decisions for machine tools considering the user’s cost structure

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This paper proposes a model for obtaining optimal preventive repair and replacement intervals of a machine tool subassembly considering the user’s cost structure and the effect of major overhauls. Corrective actions are considered as minimal, whereas preventive repair and major overhauls are considered as imperfect in this paper. The objective is to obtain optimal repair/replacement decisions for the machine tool subassembly such that the expected life-cycle cost contribution over the whole life of the system is minimised. The proposed strategy improves over other existing strategies since it simultaneously considers the effects of the user’s cost structure and major overhauls, while optimising the preventive repair and replacement intervals. A numerical illustration is provided to demonstrate the general application of the proposed approach.

Keywords: maintenance scheduling; life cycle costing; machine tools; major overhauls

1. Introduction

Failure of a machine tool may cost a lot of money to users in terms of down time and lost quality. Once a machine tool has been designed, its inherent reliability is fixed; it is the maintenance that then determines the profitability of the machine tools in the long term by reducing the unplanned down time and maintaining the quality of the products produced. It is because of this that maintenance optimisation of production equipment has received due attention from researchers. For a machine tool, maintenance costs in general include down-time cost, reduced production rate cost, cost of poor quality and labour and material cost (Lad and Kulkarni 2010). Quantifying labour and material cost is straightforward since these are registered by standard accounting procedures (Pascual et al. 2008). On the other hand, down-time cost, reduced production rate cost, and the cost of poor quality may be hard to estimate because they depend on several external factors such as production rate, types of jobs manufactured on the machine, the criticality of the job, available redundant machines, etc. Thus, these costs may vary from user to user and from one application to another. For example, if a machine is used to machine a higher-precision and costlier job, then the cost of lost quality due to failure of any of the machine components will be higher than that of a machine used for machining a low-precision and cheaper job. Similarly, if a user has alternative machines available to bear the load of a failed machine, then the down-time cost of that machine may not be as significant as in the case where there is no alternative machine available to bear the load of the failed machine. Thus the maintenance schedule will greatly depend on the cost profile of the user.

Machine tools, apart from regular preventive repair, also receive major overhauls once or twice a year. For example, the spindle subassembly of the work head of a CNC grinding machine, which consists of bearings, cartridge, housing, flange, etc., receives regular preventive repairs such as oiling, greasing, cleaning, re-setting, etc., followed by major maintenance during overhauls, which, in addition, involves replacement of some of the components, such as bearings in this example, of the subassembly. Major overhauls also take longer than regular preventive actions. It is clear that the degree of restoration during overhaul is greater than during regular preventive repair. A major overhaul is thus expected to have a different effect on the degradation rate of the system than regular preventive action and must be considered separately. Regular preventive repair intervals are generally decided based on the failure and repair characteristics and the cost of corrective and preventive action. However, a major overhaul is relatively a strategic decision. The overhauling interval is decided by considering the Overall Equipment Manufacturer (OEM) recommendations and the maintenance requirements of the other machine tools on the shop floor. The implication is that the designer can consider a fixed overhaul schedule of his own or that

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provided by the user and optimise the regular preventive repair/replacement decisions. After a few preventive repairs or overhauls it may be economical to replace the subassembly with a new one. In this paper, it is assumed that the subassembly is replaced after some overhauls. Overhauling also affects the degradation profile of the subassembly, thereby affecting optimal preventive repair and replacement decisions. Thus, a model is required that simultaneously optimises the preventive repair and replacement intervals, considering imperfect overhauling of the subassembly.

The aim of this paper is to provide a model for obtaining optimal preventive repair and replacement intervals for a machine tool subassembly considering the user’s cost profile and the effect of a major overhaul. Thus, four kinds of maintenance actions are considered during the life of a system: corrective repair, regular preventive repair, yearly overhaul and preventive replacement. Corrective repair is considered as minimal, while preventive repairs and overhauls are considered as imperfect maintenance. The restoration achieved in overhauling is more than that achieved in preventive repair. Kijima’s (1989) first virtual age model is used to describe the system improvement due to imperfect maintenance. Replacement of the subassembly brings it back to as-good-as-new condition. Thus, replacement is perfect repair. A ‘conditional number of failures’ model is proposed in this paper that, along with Kijima’s virtual age model, can be used to formulate the optimisation problem that minimises the expected life-cycle cost contribution of the subassembly over the whole life of the system.

In the next section, a literature review is presented and observations made from the literature are summarised. Based on the observations, the problem statement and assumptions are provided and the problem is formulated in Section 3. Section 4 provides a ‘conditional number of failures model’. The maintenance cost model is presented in Section 5. In section 6 a numerical example is provided to demonstrate the applicability of the proposed model. Finally, comments on the results and possible directions for further research are provided in the concluding remarks.

2. Literature review

Based on the preceding discussion it seems logical to classify the literature review into two categories, viz. the maintenance cost model and system improvement due to imperfect maintenance.

2.1 Maintenance cost model

Pascual et al. (2008) classified the cost of production equipment failure and overhaul into two categories: the intervention cost and the down-time cost. Intervention cost includes labour and material, while down-time costs include the cost of lost production as well as other consequential costs such as reduced product quality, lost raw material, etc. Komonen (1998, 2002) proposed a classification of costs related to maintenance considering two groups: (1) direct (intervention) costs, due to maintenance operations (administrative costs, labour, material, subcontracting); and (2) lost production costs due to equipment failure and poor quality production due to equipment malfunction.

Vorster and De la Garza (1990) present a model that has the capability of quantifying the consequential costs of down time and lack of availability in four categories. The first, associated resource impact costs, deals with the costs that arise when failure in one machine affects the productivity and cost-effectiveness of other machines working in close association with it. The second category, lack-of-readiness costs, addresses the cost that may be incurred when a capital asset is rendered idle by the down time resulting from a prior failure. The third cost category, service level impact costs, deals with the situation that arises when one machine in a pool of resources fails to the extent that other machines in the pool must work in an uneconomical manner to maintain a given service level. The fourth cost category, alternative method impact costs, deals with the consequential costs that arise when failure causes a change in the method of operations.

Sondalini (2006) classified the failure cost into three categories: fixed cost, variable cost and lost profit. The fixed cost includes overheads such as the manager’s salary, the permanent staff and employees’ wages, insurance, equipment leases, etc. Variable costs are the cost of fuel, power, hired labour, raw materials to make the product, maintenance, etc. Lost profit includes lost sales for the down-time period.

There are many other models available in the literature that suggest that the preventive maintenance cost also depends on the user’s operating parameters. For example, Khalil et al. (2005) proposed a failure cost model for machine tools in terms of lost production cost, production damage cost, bottleneck penalty cost and booked
labour cost. Similarly, Akturk and Gurel (2007) modelled the preventive maintenance cost as a function of the production rate and other user-specific parameters such as work material, hardness of the cutting tool, etc.

Lad and Kulkarni (2010) classified machine failure costs into the following five categories: (1) down-time cost; (2) reduced quality cost; (3) reduced speed (reduced production rate) cost; (4) fixed cost of repair/replacement; and (5) maintenance labour cost.

Pandey et al. (2010a) presented a model that considers the cost of machine failure in terms of down time and reduced quality.

2.2 System improvement due to imperfect maintenance

The use of mathematical modelling for evaluating, improving, and optimising the performance of repairable equipment is well documented in the literature. A survey of such models can be found in Pierskalla and Voelker (1976), Wang (2002) and Cui (2008). The vast majority of these models assume either perfect repair (renewal) or minimal repair. Perfect repair implies that the equipment is ‘as good as new’ after repair. Minimal repair implies that the equipment is ‘as bad as old’ after repair, i.e. the equipment has the same age as it did at the time of the failure. Recently, more attention has been given to the concept of imperfect maintenance (Cassady et al. 2005). Imperfect repair makes a system ‘better than old’ but not ‘as good as new’. Research focusing on imperfect repair has been summarised in a survey by Pham and Wang (1996) and Wang and Pham (2006). Imperfect maintenance includes a wide variety of models. In general, these models can be classified into two groups: virtual age models and failure rate models.

Malik (1979) introduced the concept of virtual age, which essentially says that the system is younger than that before the action by some interval $T_f$. A similar formulation is offered by Kijima (1989). Kijima presented two system improvement models that are used to describe preventive maintenance and the preventive overhaul process in this paper. These models can be summarised as follows. Consider a unit of equipment that, at any point in time, is in one of two states, functioning or failed (under repair), and assume that the unit is initially (at time $t = 0$) functioning. Let $X_n$ denote the duration of the period between the $(n-1)$th repair completion and the $n$th failure, and let $V_n$ denote the virtual age of the unit at the end of the $n$th repair completion. Then, Kijima’s first model of virtual age is

$$V_n = V_{n-1} + (1 - a) \cdot X_n,$$

where $a$ is some constant (called the improvement factor) such that $0 \leq a \leq 1$, and $V_0 = 0$. Thus the model assumes that the $n$th repair cannot remove the damage incurred before the $(n-1)$th repair, while Kijima’s second model assumes that each repair removes a portion of the total accumulated equipment age. It can be written as

$$V_n = (1 - a) \cdot (V_{n-1} + X_n).$$

Kijima (1989) used the first model to obtain the optimal replacement interval under imperfect corrective repair. The model aims at minimising the long-run expected cost per unit time. Several other studies have added to the body of knowledge on virtual age. Uematsu and Nishida (1987) use a non-homogenous Poisson process to determine interval reliability, and develop optimal replacement models based on various costs. They use a more general repair model, including the Kijima models as special cases, where each interval of equipment function is subject to the influence of all previous failure history. Subsequently, Dagpunar (1997, 1998) extended Kijima’s second model by showing repair rates with respect to both chronological and virtual age. In recent work, Cassady et al. (2005) defined a model of repairable equipment behaviour based on the concept of virtual age with specific focus on availability function behaviour. A simulation model was used to estimate the availability performance of equipment described by this model and, based on the results of the simulation, a generic approximate availability function is proposed. Linear regression is used to estimate the parameters of this function. They further provided a meta-model of the approximate availability function parameters in terms of the reliability and maintainability parameters of the equipment.

On the other hand, Nakagawa’s failure rate model assumes that an imperfect repair returns the system to ‘as old’ with probability $a$ and ‘as good as new’ with probability $1 - a$ (Nakagawa 1979). It says that the failure rate function after an imperfect repair is different from the function before repair. However, this model has some limitations: if the original failure rate without overhauls of the system is a power function, the failure rate of the system with overhauls is always bounded. This characteristic restricts the applicability of the model. Zang and
Jardine (1998) proposed a new system improvement model by considering a direct reduction in the system’s failure rate due to the maintenance action of an overhaul. The improvement model assumes that each imperfect preventive action makes the system’s failure rate between ‘bad as old’ and ‘good as previous overhaul period’ with a fixed degree. As a result, the model allows the system’s failure rate function to change from overhaul period to overhaul period. Also, the author later proved that if the original system failure rate is unbounded (follows a power law), the system failure rate after overhauls is also unbounded. This property conforms to a situation encountered in many maintenance cases where, although maintenances are performed, a system needs to be replaced when it is too old. Zang and Jardine (1998) used system improvement models to establish two optimisation models to determine the optimal preventive maintenance interval and life cycle of the system: one minimises the expected unit-time cost and the other minimises the total discounted cost. Recently, Pascual et al. (2008) used the Zang and Jardine (1998) model and formulated a nonlinear mixed-integer problem that minimises the expected overall cost rate with respect to repair, overhauls and replacement times. The model considers a production system that is protected against demand fluctuations and failure occurrence with elements such as stockpiles, line and equipment redundancy, and the use of alternative production methods, thereby making the cost functions discontinuous.

Samet et al. (2009, 2010) consider the preventive maintenance optimisation problem of a single-unit system subject to random failures. Both corrective and preventive maintenance are considered as imperfect repair. The imperfection of repair actions is modelled by a decreasing quasi-renewal process based on a deterministic repair efficiency factor ‘a’. Further, a generalised formulation of the quasi-renewal process considering non-negligible repair duration is proposed and a numerical algorithm is developed that allows the computation of the quasi-renewal function for any quasi-renewal process with systems whose time to first failure follows any given distribution.

Lui and Huang (2010) used the concept of imperfect repair to obtain an optimal replacement policy for a multi-state system (MSS). A quasi-renewal process is used to describe the stochastic behaviour of each individual multi-state element after repair. This is then used to quantify the quality of imperfect maintenance. The authors also derived an explicit expression for the long-run expected profit per unit time, which is then maximised to obtain the optimal failure number to replace the entire system.

In addition, the problem of repairable system maintenance is also approached in the literature using shock models. For example, Tang and Lam (2006) proposed a δ-shock maintenance model for a deteriorating system, in which it is assumed that shocks arrive according to a renewal process and the inter-arrival times of shocks have a Weibull or gamma distribution.

Chen and Li (2008) proposed a shock model for a deteriorating system that is suffering random shocks from its environment. Assume that, in the system’s operating stage, whenever a shock arrives, it will do some damage to the system. A replacement policy, by which the system is replaced at the time of the Nth failure, is adopted. An explicit expression for the long-run average cost per unit time is derived, and an optimal policy N* for minimising the long-run average cost per unit time is determined analytically.

Nodem et al. (2010) simultaneously optimised the preventive repair and replacement decisions of machine tools considering an imperfect repair process. A semi-Markov decision process is used to solve such a problem. However, the focus of their paper was mainly on reduction of the repair time using preventive maintenance.

### 2.3 Observations from the literature

Most maintenance optimisation models reported in the literature are, in general, illustrated by considering a fixed value for the cost of corrective and preventive maintenance. However, as mentioned earlier, these costs may vary from user to user and from application to application, thereby affecting the maintenance schedule optimisation decisions. Further, the time spent on corrective and preventive repairs is generally considered negligible in most of the literature. The assumption seems valid while calculating the number of failures since the mean time to failure is generally very large compared with the repair or replacement time. However, the cost of down time during repair may be significant and a short repair time may affect the maintenance policy decisions. Thus the repair time must be considered explicitly while calculating the down-time cost. In particular, the down time due to maintenance logistic delay is generally very high, thereby significantly affecting the maintenance schedule optimisation decisions.

Machine tools, in addition to regular preventive repairs, also receive major overhauls once or twice a year that greatly affects the degradation pattern of the system. The effect is quite different from the regular preventive
maintenance in terms of the improvement made to the system. System improvement due to major overhaul may significantly affect the decisions of optimal preventive repair and replacement intervals. System improvement due to major overhauls must be considered separately in maintenance optimisation decisions. However, not many approaches are available in the literature in this direction.

Machine tool users are increasingly focusing on life-cycle costs of machines (Lad and Kulkarni 2008). Thus, a maintenance optimisation model should also provide a maintenance schedule that minimises the life-cycle cost contribution of the machine tool component/subassembly over the whole life of the system.

It is therefore necessary to base optimal maintenance policy decisions concerning the machine tool subassembly on the life-cycle cost contribution, which explicitly considers the user’s cost profile and the effect of major overhauls.

3. The problem

Let the time to failure of a machine tool subassembly follow a two-parameter Weibull distribution having shape and scale parameters $\beta$ and $\eta$, respectively. Corrective repairs are applied when the machine tool fails due to failure of any of the subassemblies. Failure of the machine tool has economic consequences in terms of down-time cost, slower production cost, lost quality cost, and repair/replacement cost. As mentioned earlier, these costs depend on the user’s cost structure. In order to reduce the cost of failures, periodic preventive repairs and yearly overhaul of the subassembly are performed. After some time it may be economically convenient to replace the subassembly by a new one. The overhauling schedule is generally decided based on the OEM recommendations, the operating pattern of the machine, maintenance department capacity, maintenance requirements of other machines on the shop floor, etc. Overhauling is thus relatively a strategic decision. Thus the problem for the machine tool user or the designer is to obtain the optimal preventive repair ($t^*_{\text{Repair}}$) and replacement duration ($t^*_{\text{Replace}}$) for the machine tool subassembly, considering the user’s cost structure and the effect of the overhaul such that it minimises the expected life-cycle cost contribution of the subassembly for a given life of the system (machine tool).

In this paper, it is assumed that corrective actions result in minimal repair. This assumption seems reasonable for subassemblies consisting of many components, each having their own failure rate, since the repair/replacement of the failed component will not influence the subassembly failure rate significantly. Preventive repair and yearly overhaul is considered imperfect, since it generally involves repair/replacement of more than one component. As mentioned earlier, an overhaul has a different improvement effect on the degradation rate of the subassembly than regular preventive repair and both need to be considered separately. Kijima’s first model of imperfect maintenance is used in this paper to measure the improvement due to imperfect repair/overhaul (Kijima 1989).

Consider the following conditions.

- The degree of restoration of regular preventive repair is $a$, which remains constant throughout the analysis.
- The degree of restoration of a major overhaul is $b$, which remains constant throughout the analysis.
- Overhaul (OH) is performed at the end of each year.
- The component/subassembly can generally be replaced at one of the overhaul intervals during the life of the machine.
- No regular preventive repair is performed at the time of the overhaul.
- The effective life of the machine on the shop floor is known, let it be $L$ years.
- Let the discounting factor (interest rate) be $r$ and it remains constant throughout the life of the machine.
- Let the machine be planned to operate $T_{\text{opr}}$ hours per year.
- Let the acquisition cost of the subassembly for which the optimal maintenance schedule is required be $C_{\text{aq}}$.

3.1 Problem formulation

From the above description, the maintenance decision-making problem for the machine tool subassembly can be formulated as

$$\text{minimise } PV_C,$$  \hspace{1cm} (3)
where $PV_C$ is the life-cycle cost contribution of the subassembly of the system, measured in terms of the present value of the cost. It can be calculated as

$$PV_C = C_{aq} + \sum_{i=1}^{L} \left\{ \left( \frac{1}{(1+r)^i} \right) \times \left[ EC_{CM} \times N_{CM} + EC_{PRepair} \times N_{PRepair} + EC_{OH} \times N_{OH} + EC_{PReplace} \times N_{PReplace} \right] \right\},$$

(4)

where $EC_{CM}$, $EC_{PRepair}$, $EC_{OH}$, and $EC_{PReplace}$ are the expected cost per corrective maintenance, preventive repair, overhaul, and preventive replacement, respectively. $N_{CM}$, $N_{PRepair}$, $N_{OH}$, and $N_{PReplace}$ are the expected numbers of corrective maintenances, preventive repairs, overhauls and preventive replacements in the $i$th year.

The number of preventive repairs and the number of replacements in any year are related as follows:

$$N_{PRepair} = \frac{T_{opr}}{N_{PRepair} + 1}. \quad (5)$$

At the end of each year there will be one overhaul and at one of the overhauls the subassembly will be replaced. Thus, the number of overhauls in any year can be either 1 or 0. Similarly, the number of replacements in any year will be either 1 or 0. For example, if a replacement is made in the third year, then there will be one overhaul in the first and second year and the number of replacements in the first two years will be zero. The number of overhauls in the third year will be zero and the number of preventive replacements in the third year will be 1. If the life of the machine is 12 years, then the same three-year cycle will be repeated for the remaining nine years. Similarly, the number of failures (corrective maintenances) cycle will also be repeated after replacement of the subassembly. The number of failures in each year until the end of the replacement year needs to be calculated considering the age at the start of that year and the restoration achieved due to each preventive repair in that year. The age at the start of each year depends on the age at the end of the previous year and the degree of restoration achieved by the overhaul at the start of that year. In general, the number of failures in any year is affected by the number of preventive repairs and the degree of restoration achieved by preventive repairs and overhauls. Every time a preventive repair or overhaul is performed, the age changes. Therefore, the number of failures (corrective maintenances) in any year can be calculated as

$$N_{CM} = \sum_{k=0}^{N_{PRepair}} \left\{ [N_{CM}]_{PRepair} | V_{k} \right\}, \quad (6)$$

where $[N_{CM}]_{PRepair} | V_{k}$ represents the conditional number of failures or the number of failures for time period $t_{PRepair}$ in the $i$th year given that the age at the start of that period is $V_{k}$. Thus, $V_{k}$ is the age after the $k$th preventive repair in the $i$th year and $k = 0$ indicates the starting point of any year, where

$$V_0 = \begin{cases} 0, & \text{if } i = 1, \\ V_{0,i-1} + [V_{N_{PRepair},i-1} + t_{PRepair} - V_{0,i-1}] \cdot (1 - b), & \text{if } i > 1, \end{cases} \quad (7)$$

and $V_{k}$, for $k > 0$, can be expressed as

$$V_{k} = V_{(k-1)} + t_{PRepair} \cdot (1 - a). \quad (8)$$

It is clear from Equations (7) and (8) that preventive maintenance acts on the age gained after the last preventive action, while the overhaul acts on the total age gained in one year, thereby following Kijima’s models I and II, respectively (Kijima et al. 1988, Kijima 1989). However, an overhaul also follows Kijima’s model I with respect to the previous overhaul.

The following section presents the model to calculate $[N_{CM}]_{PRepair} | V_{k}$.

4. Conditional number of failures model
The failure process $N(t)$ used in this paper models the number of failures as a function of the starting age and time, as shown in Equation (6), thereby making the extremely tedious and laborious calculation of the optimisation problem easier.
The probability that a system will survive an additional time \( t \) given that it has already survived until its current age \( V \) can be written as

\[
R(t|V) = 1 - \frac{F(t + V) - F(V)}{1 - F(V)}.
\] (9)

For a Weibull time to failure distribution it can be expressed as Equation (10) and in the same way the conditional probability density function can be represented as Equation (11):

\[
R(t|V) = \frac{1}{C_0} F(t + V) - \left[ \frac{(t + V)^{\beta} \cdot e^{\left(-\frac{(t + V)^{\beta}}{C_1}\right)}}{C_1^{\beta + 1}} \right].
\] (10)

\[f(t|V) = \left[ \frac{dR(t|V)}{dt} \right] = \left[ \frac{(t + V)^{\beta} \cdot e^{\left(-\frac{(t + V)^{\beta}}{C_1}\right)}}{C_1^{\beta + 1}} \right].
\] (11)

For minimal corrective repair, the number of failures can be calculated as (Kumar et al. 2006)

\[
N_{CM}(t) = \int_0^t \lambda(t) dt.
\] (12)

Thus, the conditional number of failures at any time \( t \) when the starting age is \( V \) can be written as

\[
N(t|V) = \int_0^t \lambda(t|V) dt = \int_0^t \left[ \frac{(t + V)^{\beta} \cdot \beta}{t + V} \right] dt,
\] (13)

or

\[
N(t|V) = -(V/\eta)^{\beta} + \left( \frac{t + V}{\eta} \right)^{\beta}.
\] (14)

The above simplified expression for the number of failures, along with Kijima’s model of system improvement due to imperfect maintenance, can be used to calculate the number of failures for a subassembly in any year when subject to imperfect preventive repair and imperfect overhaul.

5. Maintenance cost model

Lad and Kulkarni (2010) classified machine failures based on the failure consequences. They classified failure consequences into three categories. Failure Consequence 1 (FC1): Machine immediately down. Failure Consequence 2 (FC2): Machine running, but with increased cycle time. Failure Consequence 3 (FC3): Machine running, but with increased rejection. Similarly, Panagiotidou and Tagaras (2008) also considered out-of-control as one of the failures of production equipment and emphasised considering the same while deciding the maintenance policy.

In the present paper, models for cost per corrective maintenance, preventive repair, overhaul, and replacement are proposed based on the three failure categories proposed by Lad and Kulkarni (2010). These models are then used to calculate the life-cycle cost contribution of the machine tool subassembly as shown in Equation (4). The model is described below.

Consider the following conditions.

- The machine is required to operate at the designed production rate \( DPR \) (jobs/hour).
- Machine failure leads to any of the above three failure consequences, viz. FC1, FC2 and FC3, with probability \( P_{FC1} \), \( P_{FC2} \), and \( P_{FC3} \), respectively.
- The cost of lost production per job is \( C_{lp} \) and the cost of rejection per job is \( C_{rej} \).
- Maintenance labour cost per hour is \( C_{lc} \).
- The average fixed cost per corrective maintenance is \( C_{fixCM} \).
- The average fixed costs per overhaul, preventive repair and replacement are \( C_{fixOH} \), \( C_{fixPRepair} \), and \( C_{PReplace} \), respectively.
- The average active time to perform corrective maintenance is \( MACMT \).
- The average active time to perform an overhaul is \( MAOHT \).
The average active times to perform preventive repair and replacement are $MAP_{\text{Repair}} T$ and $MAP_{\text{Replace}} T$, respectively.

The average logistic delay in performing corrective maintenance is $MCDMT$.

The average logistic delays in overhaul, preventive repair and replacement are negligible.

Whenever the failure of the subassembly leads to Failure Consequence 2 (FC2), it reduces the production rate by RPR and the same is detected after $t_{FC2}$ hours.

Users use a control chart to monitor the process quality.

The process is centered with upper and lower control limits at $\pm 3\sigma$.

Whenever a subassembly fails and leads to Failure Consequence 3 (FC3), it shifts the process mean by $\delta \times \sigma$.

The sample size and the time between samples for the control chart are $S$ and $t_S$, respectively.

$N_s$ is the number of units produced during the time required to detect a shift in the process mean.

$R_s$ is the proportion of non-confirming units due to the process shift.

Thus the cost per corrective maintenance when failure leads to FC1, FC2 and FC3, respectively, can be written as follows:

\begin{align*}
C_{FC1} & = (MACMT + MCDMT) \times (DPR \times C_{lp} + C_{lc}) + C_{\text{fixCM}}, \\
C_{FC2} & = DPR \times RPR \times t_{FC2} \times C_{lp} + C_{FC1}, \\
C_{FC3} & = DPR \times IRR \times t_{FC3} \times C_{rej} + C_{FC1}.
\end{align*}

$IRR$ and $t_{FC3}$ can be calculated as follows. If we assume that the process is being monitored by a $\bar{X}$ control chart with control limits at $\pm 3\sigma$, the $\beta'$ (type II) error can be expressed as (Montgomery 2004)

\begin{equation}
\beta' = \varphi \left[ 3 - \delta \times \sqrt{S} \right] - \varphi \left[ -3 - \delta \times \sqrt{S} \right].
\end{equation}

The expected number of samples taken before the shift is detected, i.e. the average run length, is

\begin{equation}
ARL_{\beta} = \frac{1}{1 - \beta'},
\end{equation}

thus

\begin{equation}
t_{FC3} = \left( \frac{1}{1 - \beta'} \right) \times t_s.
\end{equation}

Similarly, the increase in rejection due to FC3 can be written as

\begin{equation}
IRR = 1 - \left[ \varphi [3 - \delta] - \varphi [3 + \delta] \right].
\end{equation}

The total expected cost per corrective maintenance $EC_{\text{cm}}$ to the user will be

\begin{equation}
EC_{\text{cm}} = P_{FC1} \times C_{FC1} + P_{FC2} \times C_{FC2} + P_{FC3} \times C_{FC3},
\end{equation}

the expected cost per preventive repair to the user will be

\begin{equation}
EC_{\text{PRepair}} = MAP_{\text{Repair}} T \times (DPR \times C_{lp} + C_{lc}) + C_{\text{fixPRrepair}},
\end{equation}

the expected cost per preventive replacement to the user will be

\begin{equation}
EC_{\text{PReplacement}} = MAP_{\text{Replace}} T \times (DPR \times C_{lp} + C_{lc}) + C_{\text{fixPRreplacement}},
\end{equation}

and the expected cost per preventive repair to the user will be

\begin{equation}
EC_{\text{OH}} = MAP_{\text{OH}} T \times (DPR \times C_{lp} + C_{lc}) + C_{\text{fixOH}}.
\end{equation}
6. Numerical example

Consider the example of a spindle subassembly used in a CNC grinding machine work head. The time-to-failure of the spindle follows a two-parameter Weibull distribution with values for the shape parameter $\beta$ and scale parameter $\eta$ of 2 and 7000 hours, respectively.

It is required to obtain optimal preventive repair and replacement intervals $t_{P\text{Repair}}^*$ and $t_{P\text{Replace}}^*$ for the spindle. Every time the spindle fails, the probabilities that it leads to FC1, FC2, and FC3 are 0.3, 0.2, and 0.5, respectively. The machine is operated for 4800 hours per year and the Designed Production Rate (DPR) is 45 jobs/hour. The mean active corrective and preventive repair time is 1 hour. It is assumed that the corrective repair is minimal. Preventive repair is considered to be imperfect with a restoration factor of 0.3. Apart from preventive repair, the subassembly also receives maintenance during major overhauls. This is performed yearly. The degree of restoration achieved for a spindle in overhaul is 0.7. The mean active time required per overhaul is 2 hours. The subassembly is replaced after a certain number of overhauls. The delay for preventive repair and overhaul, being planned activities, is considered to be negligible.

Users use a $\bar{X}$ control chart to monitor the process quality. The process is centered with upper and lower control limits at $\pm 3\sigma$. Whenever the spindle fails and leads to failure consequence 3 (FC3), it shifts the process mean by $0.5 \times \sigma$. The sample size and the time between samples are 4 and 8, respectively.

Let spindle failure that leads to FC2 reduce the production rate by 30%. The reduction in production rate is detected by the user after 6 hours. Table 1 shows the cost structure of the user. The discounting factor is considered to be 0.1 and is assumed to remain constant throughout the life of the machine. The effective life of the machine is taken as 12 years.

It is assumed that the user wants the optimal preventive repair interval at the end of the week and one week consists of 100 hours. This means that the optimal repair intervals must be in multiples of 100 hours.

Using the above data, the optimisation model given by equation (3) was solved using the simulation-based RISK Optimizer (http://www.palisade.com/RISKoptimizer/), which uses genetic algorithms (GAs) to solve the optimisation problem. The number of failures in each year is calculated using Equation (14). The optimal values of the decision variables obtained are as follows:

$$t_{P\text{Repair}}^* = 1600 \text{ hours}, \quad t_{P\text{Replace}}^* = 2 \text{ years}.$$

The minimum value obtained for the objective function ($PV_C$) is 1,523,626. This means that when the component/subassembly receives yearly overhauls, it must be preventively repaired every 1600 hours and must be replaced at the end of the second year.

In order to increase confidence in the solution, the optimisation algorithm was repeated using different starting solutions and population sizes. It was observed that the solutions obtained with different starting solutions and population sizes did not differ from each other.

6.1 Model validation

The model parameters $C_{lp}$ and $C_{rej}$ are assumed to be constant throughout the life of the machine. However, these values may change many times. In order to evaluate the robustness of the solution, the optimisation procedure is repeated with a small variation in these two model parameters. The cost of lost production and rejection per job is varied in the range of $\pm 15\%$. The results are shown in Tables 2 and 3. It can be seen from these two tables that the optimum repair/replacement schedule does not change with a small variation in the cost of rejection and cost of lost production. Thus, the model is robust for small variations of these parameters. However, it also shows that $PV_C$ is relatively more sensitive to the change in cost of rejection per job ($C_{rej}$) than to the cost of lost production per job ($C_{lp}$).

Table 1. Cost data (in Indian Rupees).

<table>
<thead>
<tr>
<th>$C_{aq}$</th>
<th>$C_{lp}$</th>
<th>$C_{lc}$</th>
<th>$C_{rej}$</th>
<th>$C_{fixCM}$</th>
<th>$C_{fixP\text{Repair}}$</th>
<th>$C_{fixOH}$</th>
<th>$C_{fixP\text{Replace}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>80,000</td>
<td>80</td>
<td>500</td>
<td>5000</td>
<td>2000</td>
<td>3000</td>
<td>30,000</td>
<td>80,000</td>
</tr>
</tbody>
</table>
The cost of rejection per job (\(C_{rej}\)) depends on the cost of raw materials, the cost of pre-operations performed on the job, etc. These costs, for a given type of job, do not change significantly. Therefore, for a given type of job, the cost of rejection per job can be assumed to remain constant or vary within a very small range. However, if the same machine is used to machine different types of jobs (such as crank shaft, gears, etc.), having entirely different requirements in terms of raw material, pre-machining, etc., these costs may vary significantly and in turn may affect the optimal preventive repair/replacement schedules. This can be a common situation in a job shop kind of environment. In such situations, the user needs to consider the average cost of rejection per job, while obtaining the optimal maintenance schedule. The average cost of rejection per job can be calculated based on the fraction of time the machine is generally used for a particular type of job. Moreover, the control chart policy may also differ for different types of products. Therefore, the same must also be considered while obtaining the preventive repair/replacement schedule for the machine.

For example, a machine is used for machining two types of jobs. A job of type 1 is processed for 30% of the time in a year, while a job of type 2 consumes the remaining 70% of the time of the machine. Let the cost of rejection per job for type 1 and type 2 jobs be \(C_{rej1}\) and \(C_{rej2}\), respectively. For both types of jobs, the user uses different control chart policies. If the other parameters remain the same, the cost per corrective maintenance when failure leads to FC3, i.e. Equation (17), can now be written as

\[
C_{FC3} = 0.3(DPR \times IRR_1 \times t_{FC31} \times C_{rej1}) + 0.7(DPR \times IRR_2 \times t_{FC32} \times C_{rej2}) + C_{FC1},
\]

where \(IRR_1\), \(IRR_2\) and \(t_{FC31}\), \(t_{FC32}\) should be calculated from Equations (20) and (21) using the respective control chart parameters for type 1 and type 2 jobs.

Similarly, the cost of lost production per job (\(C_{lp}\)), which mainly depends on the profit margin per job, market demand or demand from the next machine in the production line, etc., may also vary for different types of jobs. The variation in \(C_{lp}\) due to different types of jobs can be modelled using an approach similar to Equation (26).

### 6.2 Effects of a change in the user’s cost structure and shop floor policy

As mentioned earlier, the model is robust against small changes in the cost of lost production and rejection. The condition of a small variation in these parameters may hold good for a given user. However, these costs, along with the user’s shop floor policy parameters, such as control chart parameters, may take entirely different values for different users. For example, if a user is using the machine in a production line, then the cost of lost production will...
be higher than for a stand-alone machine or a machine used in a job shop kind of environment. Similarly, different users may use different control chart policies. Table 4 and 5 show the effects of the user’s cost structure and control chart parameters (illustrated through the variation in $t_{S}$) on the optimisation results and life-cycle cost of the component/subassembly. It can be seen from Tables 4 and 5 that optimal maintenance decisions vary with the user’s cost structure and shop floor policy parameters. Therefore, it is recommended that the cost structure of each user must be considered when designing a machine tool for optimal reliability and maintenance.

Although Table 5 illustrates the effect of only one of the control chart policy parameters, i.e. the time between two samples ($t_{S}$), the results provide a useful insight from the machine tool user’s point of view. A user can see the effect of different control chart policy parameters on the preventive maintenance schedule and $PVC$. For example, Table 5 clearly indicates that the control chart policy used by user 3 gives significant savings in cost. However, it requires more frequent sampling, which may result in additional costs. Thus the user needs to make a trade-off between the cost of quality control and the cost of preventive maintenance. This leads towards a new area of research called the joint optimisation of preventive maintenance and control chart policy parameters. This area of research is now receiving increasing attention from researchers. The interested reader may refer to Zhou and Zhu (2008) and Pandey et al. (2010b).

### 7. Conclusion and future work

A model to simultaneously obtain optimal preventive repair and replacement intervals for a machine tool subassembly during the whole life of the machine is presented in this paper. The model also considers the effect of the user’s cost structure and yearly overhauls on the maintenance schedule decisions. Thus the modelling approach provides more accurate and cost-effective decisions. The results clearly indicate that, for a given overhaul interval, optimal preventive repair and replacement decisions depend upon the user’s cost structure and shop floor policy parameters. However, for a small variation in the user’s cost structure the optimal preventive repair and replacement decisions remain unaffected.

The model can be extended in several ways and some of these are indicated below.

1. In the proposed approach, the overhauling interval was considered as fixed as one year; the same approach can be repeated with different overhauling intervals such as 6 months, 1.5 years, 2 years, etc. Thus an optimal overhaul interval can also be selected.

2. The same approach can be extended to the system level to obtain the optimal repair and replacement intervals of all the subassemblies such that the life-cycle cost of the machine tool is minimised.
The same approach can be used for the simultaneous optimisation of the reliability and maintenance schedule of the machine tool, where the designer can simultaneously choose, from the available alternatives for different subassemblies, an optimal subassembly and corresponding maintenance schedules.

Some of these problems are currently under investigation by the authors.

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