Joint consideration of production scheduling, maintenance and quality policies: a review and conceptual framework

Divya Pandey* and Makarand S. Kulkarni

Department of Mechanical Engineering, Indian Institute of Technology Delhi (IIT Delhi), Hauz Khas, New Delhi, 110016, India
Fax: +911126582053
E-mail: divya1823@yahoo.co.in
E-mail: mskulkarni@netearth.iitd.ernet.in
*Corresponding author

Prem Vrat
Management Development Institute, Mehrauli Road, Sukhrali, Gurgaon – 122 007, India
E-mail: premvrat@mdi.ac.in

Abstract: Performance of a manufacturing system depends on its ability to simultaneously meet several requirements such as: quick response to market demand, high product quality, low manufacturing costs and timely deliveries; which in turn strongly depend on performance at the shop floor level. In the past, the shop floor level operational policies in the context of scheduling, maintenance and quality have been examined in isolation. However, it is hypothesised that these three aspects of operations planning may also have some interaction effect and hence joint consideration of various policy options pertaining to quality, maintenance and scheduling on the performance of the manufacturing system is an important area for investigation. In this paper, a review of literature addressing the joint consideration of these three aspects is presented and research gaps are highlighted. As a preliminary work, some numerical investigations are also presented to highlight the importance of joint consideration of these three aspects. Finally, a conceptual methodology is presented that can lead to further developments in this field. Some potential research areas in this context are identified.

Keywords: production scheduling; maintenance; quality; control chart policy; integrated policies.


Biographical notes: Divya Pandey is currently a Research Scholar at the Indian Institute of Technology, Delhi in the Mechanical Engineering Department. She received her Master of Engineering in Industrial Engineering and Management from Ujjain Engineering College, Ujjain, India. She is
presently working in the field of production scheduling, maintenance planning, and quality engineering.

Makarand S. Kulkarni is currently working as an Assistant Professor at the Indian Institute of Technology Delhi in the Mechanical Engineering Department. He received his PhD in Manufacturing from the Indian Institute of Technology Bombay, India. His research interest includes quality, reliability and maintenance engineering and their integration with operation planning.

Prem Vrat is a Professor of Eminence at the Management Development Institute, Gurgaon, the Former Vice Chancellor of U.P. Technical University, Lucknow, the Founder-Director (December 2001–May 2006) of the Indian Institute of Technology, Roorkee (India) and the Director-in-Charge of IIT Delhi (July–December 2000). He has vast experience in teaching, research and consultancy spanning for more than 41 years. He is on the editorial board of several international journals. He has many awards and prizes to his credit. His areas of interest include productivity management, supply chain management, industrial engineering, operations research and operations management.

1 Introduction

In the current competitive technology driven and highly dynamic industrial environment, firms are facing difficult challenges. Rapidly changing markets, increase in product-variety and concerns for product quality have led the manufacturing companies to automation and modernisation of equipment, which in turn, have raised the cost of manufacturing processes. To achieve operational excellence in this new environment, companies need to focus on shop floor efficiency and effectiveness. For shop floor operations to be efficient and effective, it is necessary to have proper maintenance of production systems, adopt sound production scheduling policies, and maintain high product/process quality.

In any production setting, the scheduler seeks to schedule a set of orders on a number of machines such that the sequence, timing, and machine assignment of these orders are optimal with respect to one or more objectives. A large number of measures for evaluating scheduling performance have been proposed in the literature. These may be grouped into six broad categories (Frauendorfer and Konigsperger, 1996):

1. job-attributed criteria (e.g., job flow time)
2. shop-attributed criteria (e.g., machine utilisation)
3. completion-based criteria (e.g., makespan)
4. due-date-based criteria (e.g., tardiness)
5. financial criteria (e.g., job handling cost)
6. miscellaneous criteria (e.g., labour utilisation).

In general, production scheduling models are often mathematical programming models designed to maximise or minimise one or more of the above measures. Solution methodologies for such models range from traditional integer programming
Joint consideration of production scheduling, maintenance and quality policies

(branch-and-bound) techniques to Lagrangean relaxation and optimisation-based heuristics and meta-heuristics. These models and techniques have been implemented in a wide variety of manufacturing systems. Extensive literature is also available on such models and techniques (Allahverdi et al., 2008; Chen and Sheen, 2007; Gupta and Stafford, 2006; Hejazi and Saghafian, 2005; Li and Ierapetritou, 2008).

In order to keep equipment or a production system in a state in which it can perform its intended functions effectively, appropriate maintenance of equipment is a must. Maintenance activities influence the production schedule and the deterioration pattern of the production process. Proper maintenance is thus necessary to keep a minimum level of operational effectiveness. Extensive literature is available on the use of mathematical modelling for analysing, planning and optimising maintenance actions. Garg and Deshmukh (2006) provide a review of maintenance management issues ranging from various optimisation models, maintenance techniques, maintenance scheduling, and maintenance information systems. Paz and Leigh (1994) identified the area of concern in maintenance scheduling and survey representative work from the academic and practitioner literature. Wang (2002) presented a survey of maintenance policies for deteriorating systems that aim at determining the optimal preventive maintenance (PM) period that optimises maintenance performance measures like maintenance cost rate, discounted cost rate, availability, mean time between failure, failure frequency, etc. PM planning models are typically stochastic, mathematical or simulation models supplemented with optimisation techniques designed to maximise expected equipment availability or minimise expected equipment maintenance costs. The review of analysis and modelling of maintenance operations has been done by several researchers, such as, Dekker (1996), Sherif and Smith (1981), Sherwin (2000), Tu and Li (2004), Valdez-Flores and Feldman (1989), van Dijkhuizen and van Der Heijden (1999), Wang and Pham (1999) and Zhao (2003).

The intense quality driven competition has shifted the focus from product control to process control in manufacturing. Consequently, variations are sought to be eliminated at their source. For controlling and determining the state of production process, various control charts are used. The design of control chart has economic consequences since the cost involved are affected by the choice of the control chart parameters (Rahim and Ben-Daya, 2001). The first model that stimulated most of the work in the area of economic design of $\bar{X}$-control charts was introduced by Duncan (1956). Since then many studies have focused on the economic design of control charts (Hassan et al., 2000; Nenes and Tagaras, 2007; Schiffauerova and Thomas, 2006).

In the past, production scheduling, maintenance and quality control have been treated as separate issues. For example most machine scheduling models assume that machines are available all the time. However, in real life situations, machines do fail or need to be maintained and hence may become unavailable during certain periods. Similarly most of the maintenance models ignore the production requirements. However, maintenance effectiveness cannot be measured in a totally meaningful way without taking into account whether the maintenance function is meeting the production requirements (Paz and Leigh, 1994). On the other hand delaying the maintenance to meet production requirement may increase the process variability which in turn may cause higher rejections. Ollila and Malmipuro (1999) observed that, maintenance has a major impact on efficiency and quality along with equipment availability. In a case study carried out in five Finnish industries, they showed that well functioning machinery is a prerequisite to
quality products. They also showed that lack of proper maintenance is usually among the three most important causes of quality deficiencies in the process industries. Thus, it is hypothesised here that these three shop floor level operational policies also have some interaction effect on each other and hence a joint consideration of various policy options pertaining to quality, maintenance, and scheduling provides an important research area for investigation. This interdependency of production scheduling, maintenance and quality planning has resulted in a considerable amount of interest in developing models that take into consideration more than one of these three aspects.

The purpose of this paper is to review the literature which deals with the joint consideration of more than one of these three aspects of production process. Gaps are highlighted and numerical illustration is presented to highlight the importance of such joint considerations. Finally a conceptual framework is proposed that will lead to further developments in this field.

2 Integrated approaches for production scheduling, maintenance and quality control: a review

Having hypothesized the interaction effect between production scheduling, maintenance, and quality, it is necessary to search for any work that reports the joint consideration of these shop floor policies. Available literature can be broadly classified under the following two headings.

2.1 Integrated approaches to production scheduling and maintenance

PM activities take time that could otherwise be used for production, but delaying PM for production may increase the probability of machine failure resulting in unscheduled downtime. Therefore, integrating PM with production scheduling may lead to a better plan. A few studies have attempted to combine and solve production and maintenance scheduling problems simultaneously. The problem of integration has been generally approached in the literature in two different ways. Some authors have approached this problem by determining the optimal PM schedule in the production system and others by taking maintenance as a constraint to the production system (Allaoui et al., 2008). Dedopoulos and Shah (1995) and Sammarti et al. (1995, 1997) focus on chemical process plants and provide results describing the effects of equipment failures on production schedule robustness.

There are only a few studies that explicitly try to integrate production scheduling and PM decisions by optimising them simultaneously. Ashayeri et al. (1996) use a discrete-time multi-machine model, but they consider lot sizing rather than scheduling of jobs. In addition, they consider discrete probabilities of machine failure rather than a continuous probability distribution for the time to machine failure. Graves and Lee (1999) formulated a single-machine scheduling problem with total weighted completion time as the objective function, but they schedule only one maintenance activity during the planning horizon. Lee and Chen (2000) extended the work of Graves and Lee (1999) to parallel machines, but still with only one maintenance action. Qi et al. (1999) considered a single-machine problem with the possibility of multiple maintenance actions, but they did not explicitly model the risk of not performing maintenance. Xu et al. (2008) introduced and studied a parallel machine scheduling problem with almost periodic
Joint consideration of production scheduling, maintenance and quality policies

maintenance activities to minimise the completion time of the last finished maintenance. They said that the maintenance of a machine is $\epsilon$-almost periodic if the difference of the time between any two consecutive maintenance activities of the machine is within $\epsilon$.

Cassady and Kutanoglu (2003) developed an integrated model for a single machine problem with an objective of minimising expected total weighted tardiness. Their model allows multiple maintenance activities and explicitly captures the risk of not performing maintenance. They also showed that depending on the nature of system, an average saving of 30% might result by integrating production scheduling and PM planning. Kaabi et al. (2002) studied the problem of scheduling and maintenance of machines simultaneously. The objective of their paper was to minimise total tardiness of jobs. Four heuristics were proposed to determine a schedule that minimises the objective function. Liao and Chen (2003) also examined a single-machine scheduling problem involving periodic maintenance to create an efficient heuristic for obtaining a near optimum solution for large sized problems. Cassady and Kutanoglu (2005) proposed an integrated mathematical model that coordinates PM planning decisions with single-machine scheduling decisions so that the total expected weighted completion time of jobs is minimised. Total enumeration approach is used to solve the integrated model and for solving larger problem, a heuristic approach is proposed. As the analysis is based on minimising total weighted completion time, both scheduling and maintenance problems favour processing shorter jobs in the beginning of the schedule. The study showed an improvement of approximately 2% through integration of two decision-making processes. Leng et al. (2006) and Sortrakul and Cassady (2007) further extended the work of Cassady and Kutanoglu (2003) and proposed chaotic partial swarm optimisation (CPSO) heuristic and GA-based heuristics respectively to solve the integrated mathematical model for single machine production scheduling and PM planning as a multi-objective optimisation problem.

Sloan and Shanthikumar (2000) proposed a combined model and its application to the semiconductor manufacturing sector for the single machine case. Jeong et al. (2007) presented an integrated decision-support system for diagnosis, maintenance planning, and scheduling of electronics manufacturing systems to minimise makespan or total completion time. Chen and Liao (2005) considered the maintenance schedule and job production schedule under the resumable case applied in process industry where the completion time of maintenance must be within the completion time of production schedule. If the job continues processing after the machine becomes available again without any loss in time or penalty for resumption, the problem is called ‘resumable’. On the other hand, the problem is called ‘non-resumable’ if the job has to restart from the beginning (all prior processing is wasted) when the machine becomes available (Lee, 1999). A more complex hybrid flow shop problem is tackled in Allaoui and Artiba (2004) but in their work, PM is considered as a constraint to the scheduling problem. Chen (2006) addressed the resumable and non-resumable cases for the single machine and parallel machine scheduling problems, where machines are flexibly maintained and total tardiness is used as a performance measure. Batun and Azizoğlu (2009) proposed a branch and bound algorithm to study the single machine total flow time problem in which the jobs are non-resumable and the machine is subject to PM activities of known starting times and durations. Ji et al. (2007) have considered a single machine scheduling problem with several periodic maintenance activities and the objective is to find a schedule that
minimises the makespan, subjected to periodic maintenance and non-resumable jobs. Ruiz et al. (2007) proposed tools in the form of adaptation of heuristic and meta-heuristic methods to implicitly consider PM operations in permutation flow shop sequencing problems so that the makespan of the jobs is minimised. Yulan et al. (2008) studied the joint determination of PM planning and production scheduling for a single machine with multiple objectives by simultaneously minimising the maintenance cost, makespan, total weighted completion time of jobs, total weighted tardiness, and maximising machine availability. They used multi-objective genetic algorithm (GA) to solve the joint optimisation problem. Allaoui et al. (2008) have studied the problem of jointly scheduling \(n\) immediately available jobs and the PM in a two-machine flow shop to minimise the makespan in the non-resumable case and proved that this problem is NP-hard. Yanga et al. (2008) consider a two-machine flow shops where maintenance activities must be done after completing a fixed number of jobs. The durations of these maintenance activities are constant. Naderi et al. (2008) investigated flexible flow line problems with sequence dependent setup times and different PM policies to optimise the makespan. They proposed a variable neighbourhood search (VNS) technique to optimise the production and maintenance schedule. Naderi et al. (2009) investigated job shop scheduling with sequence-dependent setup times and PM policies. The optimisation criterion is the minimisation of makespan. Berrichi et al. (2009) have proposed a bi-objective integrated model to solve the joint production and maintenance scheduling problem in parallel machine case. Zandieh and Gholami (2008) have studied a hybrid flow shop scheduling problems in which there are sequence-dependent setup times and machine suffer stochastic breakdowns, to optimise objective based on the expected makespan using an immune algorithm (IA).

2.2 Integrated approaches to maintenance and quality

While excessive maintenance results in unnecessary costs, inadequately maintained equipment may produce defective products resulting in large amount of rework and scrap costs. This has attracted attention of researchers for the joint consideration of maintenance and quality policies. Based on above arguments Ben-Daya and Duffuaa (1995) in their work have emphasised the need to link quality control with PM. Murthy and Djamaludin (1988) examined the total framework for controlling output quality in a multistage production system and proposed a fairly general dynamic model formulation for obtaining effective control strategies. The use of maintenance actions to control machine state deterioration is an important feature of the model formulation. Rahim (1993) jointly determined the optimal design parameters on an \(\bar{X}\)-control chart and PM time for a production system with an increasing failure rate. Ben-Daya (1999) Ben-Daya and Rahim (2000) and Rahim (1994) investigated integration of \(\bar{X}\)-chart and PM, while the process deteriorates during in-control period follows a general probability distribution with increasing hazard rate. Cassady et al. (2000) studied an \(\bar{X}\)-chart in conjunction with an age replacement PM policy. Yeung et al. (2008) modified the model suggested by Cassady et al. (2000) of using an \(\bar{X}\)-chart to monitor the output of the production process and to determine when to perform corrective, condition-based maintenance so as to optimise the sample, control chart parameters and interval for performing PM. Rahim and Ben-Daya (2001) provided an overview of the literature dealing with integrated models for production, quality and
Joint consideration of production scheduling, maintenance and quality policies

Radhoui et al. (2009) developed a joint quality control and PM policy for a randomly failing production system producing conforming and non-conforming units. They developed a mathematical model and combined with simulation in order to determine simultaneously the optimal rate of non-conforming units observed on each lot and the optimal size of a buffer stock which minimises the expected total cost per time unit including inventory cost, maintenance cost, and quality cost. Al fares et al. (2005) presented a model and a solution algorithm for incorporating quality and maintenance aspects into a production-inventory system for deteriorating items.

Kuo (2006) studied a joint machine maintenance and product quality control problem of finite horizon discrete time Markovian deteriorating state for unobservable batch production systems. Linderman et al. (2005) developed a generalised analytical model to determine the optimal policy to coordinate statistical process control (SPC) and planned maintenance to minimise total expected cost. Panagiotidou and Tagaras (2007) analysed an economic model for the optimisation of PM in a production process with two quality states. Panagiotidou and Tagaras (2008) developed an economic model for optimisation of maintenance policy, including both perfect and imperfect maintenance actions, which is appropriate for a production process (equipment) with two operating states, namely an ‘in-control’ state and an ‘out-of-control’ state in SPC, and a failure state. Chiu and Huang (1996) developed a model that introduces PM into the economic design of control charts. They assumed non-uniform distribution for the in-control period with an increasing hazard rate and fixed sampling intervals. In addition, they assumed that system will become as good as new after PM action. Zhou and Zhu (2008) examined the integration of economic design of control chart and maintenance management and developed a mathematical model to analyse the cost of the integrated model using the grid-search approach to find the optimal values of policy variables ($n, h, L, k$) that minimise hourly cost. Recently, Panagiotidou and Nenes (2009), proposed a model for the integration of quality and maintenance procedures using Shewhart’s control chart for variables.

3 Research gaps

It is clear from the literature review presented in the preceding section that the joint consideration of scheduling, maintenance and quality are gaining increasing attention from the researchers in recent years. While available approaches are mostly limited to simple problems like single machine, single product, single quality characteristic, etc., multiple machines and multiple products, each having multiple quality characteristic, are quite often are more relevant in actual production system. Further, nature and extent of integration required in case of flow shop need not to be optimal in other flow patterns like, job shop, hybrid shop, etc. In such situations, optimal mix of integrated policies needs to be evaluated for different production systems based on the flow pattern of the jobs. While literatures are available those integrate maintenance with scheduling and maintenance with quality, integration of all the three areas i.e., production scheduling, maintenance and quality control is conspicuously non-existent and hence presents good scope for further research.
Further, the demand of the jobs, variety of products, and availability of resources, production rates, etc. may vary with time. All these factors make the actual production systems dynamic in nature and in turn increase the complexity of mathematical modelling. Many of the scheduling problems are NP-hard in nature (Pinedo, 2002); the integration of scheduling with other shop floor policies in such a dynamic production environment will further increase the complexity of mathematical modelling, thereby precluding any possibility of an analytical solution. In such situations simulation modelling may provide good scope for further research in this area. It will further facilitate in making “what-if” kind of experimentation and analysis for checking the robustness of the integrated approaches.

To achieve higher levels of efficiencies through sound decision-making and proper trade-offs, a good understanding of the inter-dependencies among production scheduling, maintenance scheduling, and quality control in different shop floor environments is required. Even though the link between these areas is not completely missing in the literature, more investigation is needed in this direction. This paper attempts to address this issue and proposes a conceptual model framework.

4 Preliminary investigations

Recognising the need for integration of production scheduling, maintenance and quality policies, some preliminary work have been done by the authors to show the importance of such joint considerations. The same is illustrated with the help of a numerical example in this section.

Consider a single machine that processes three batches having the processing time, setup time, penalty cost, due dates, and other production parameter as given in Table 1.

<table>
<thead>
<tr>
<th>Batch (I)</th>
<th>Processing time in minutes (II)</th>
<th>Batch size (III)</th>
<th>Setup time in hrs (IV)</th>
<th>Total processing time (hrs) $V = (II \times III)/60 + IV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>500</td>
<td>3</td>
<td>53.0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>500</td>
<td>1</td>
<td>26.0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>500</td>
<td>2</td>
<td>18.66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Batch (I)</th>
<th>Release time (VI)</th>
<th>Due date (hrs) (VII)</th>
<th>Penalty cost/batch (P) (in hrs) (VIII)</th>
<th>Carrying cost/job/hrs (IX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>100</td>
<td>75</td>
<td>1.71</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>50</td>
<td>50</td>
<td>1.71</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>40</td>
<td>45</td>
<td>1.71</td>
</tr>
</tbody>
</table>

Let the machine have a two parameter Weibull time to failure distribution with scale and shape parameter $\eta$ and $\beta$ as 1000 and 2 respectively. Machine considered here is expected to operate for three shifts of seven hours each for six days in a week. Other parameters related to machine failure and repair are shown in Table 2.
Joint consideration of production scheduling, maintenance and quality policies

Table 2  
Machine failure/repairs parameter for the illustrative example

<table>
<thead>
<tr>
<th>Mean time for carrying out PM (MTTR)_{PM} (hrs)</th>
<th>Mean time for carrying out CM (MTTR)_{CM} (hrs)</th>
<th>RF_{PM}</th>
<th>RF_{CM}</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>12</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>Cost of rejection per job (C_{Rej}) (in Rs)</td>
<td>Cost of lost production per job (C_{lp}) (in Rs)</td>
<td>Labour cost (LC) (in Rs)</td>
<td>Fixed cost per CM (C_{FCPCM}) (in Rs)</td>
</tr>
<tr>
<td>15000</td>
<td>1000</td>
<td>500</td>
<td>20,000</td>
</tr>
</tbody>
</table>

Note: Restoration factor (RF) of q sets the age of the component after maintenance action to (1 – q) fraction of the age of the component at the time of the maintenance action.

In this paper two control chart policies will be considered. The control chart parameter i.e., the sample size (n), time between sample (TBS), assuming the ±3σ control limits (CL) corresponding to these policies are shown in Table 3.

Table 3  
Control chart policies

<table>
<thead>
<tr>
<th>Control chart policy</th>
<th>TBS (hrs)</th>
<th>Sample size (n)</th>
<th>CL (sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Policy 1</td>
<td>16</td>
<td>4</td>
<td>± 3</td>
</tr>
<tr>
<td>Policy 2</td>
<td>16</td>
<td>6</td>
<td>± 3</td>
</tr>
</tbody>
</table>

4.1 Obtaining optimal production schedule independently

Consider a single machine that is required to process three batches of batch size 500 each. Following assumptions are made to solve the problem:

1 Job cannot be pre-empted by another job.
2 No failure of machine during the schedule.
3 Raw material for all the batches are released at starting of the schedule.
4 All jobs in a batch are completed together upon the completion of the last job in the batch. The batch processing time is equal to the sum of the processing times of its jobs.

It may be noted that these are generally the assumptions made for many scheduling/sequencing problems for which models have been attempted in the past.

The objective is to obtain the batch sequence that minimises the cost per unit time of the schedule (CPUT)\_S. (CPUT)\_S can be calculated as:

\[
(CPUT)_S = \frac{\text{Total penalty cost due to batch delay} + \text{Total raw material inventory carrying cost}}{\text{Schedule completion time}}
\] (1)
Penalty cost is incurred only when a batch is delayed beyond its due date. Penalty cost for a batch can be calculated as:

\[
\text{Batch penalty cost} = (\text{Batch completion time} - \text{Batch due date}) \cdot P_i \tag{2}
\]

As it is assumed here that the raw material for all the batches are released at the starting of the schedule, raw material for a batch is carried until it starts processing, i.e., for the duration of the processing and setup time of all the previous batches (if any) and the setup time of the current batch. Hence the inventory carrying cost is calculated based on the whole batch size for this period. Secondly, during the processing of a batch, raw material of the batch depletes at a constant rate and therefore the inventory carrying cost is calculated for this period also based on the average inventory (half the batch size).

To obtain the optimal production schedule a total enumeration method is used. In the present problem for the three batches, a total of 3! batch sequences are possible. These batch sequences are shown in Table 4. Table 4 also shows the \((CPUT)_S\) value for all the six possible sequences. It is clear from Table 4 that sequence \([B2-B3-B1]\) gives minimum cost per unit time of the production schedule \((CPUT)_S\) and so the same is selected as optimal sequence. Since this analysis ignores the possibility of machine failure, this sequence is referred as ‘considering scheduling only’ batch sequence.

### Table 4

<table>
<thead>
<tr>
<th>Batch sequence</th>
<th>Completion time</th>
<th>Tardiness</th>
<th>Penalty cost</th>
<th>Inventory cost</th>
<th>((CPUT)_S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>([B1-B2-B3])</td>
<td>53 79 98</td>
<td>0 29 58</td>
<td>4045</td>
<td>11407</td>
<td>158</td>
</tr>
<tr>
<td>([B1-B3-B2])</td>
<td>53 98 72</td>
<td>0 48 32</td>
<td>3808</td>
<td>12248</td>
<td>164</td>
</tr>
<tr>
<td>([B2-B1-B3])</td>
<td>79 26 98</td>
<td>0 0 58</td>
<td>2595</td>
<td>9654</td>
<td>125</td>
</tr>
<tr>
<td>([B2-B3-B1])</td>
<td>98 26 45</td>
<td>0 0 5</td>
<td>210</td>
<td>8742</td>
<td>92</td>
</tr>
<tr>
<td>([B3-B1-B2])</td>
<td>72 98 19</td>
<td>12 68 0</td>
<td>7484</td>
<td>11336</td>
<td>193</td>
</tr>
<tr>
<td>([B3-B2-B1])</td>
<td>98 45 19</td>
<td>38 15 0</td>
<td>3558</td>
<td>9583</td>
<td>135</td>
</tr>
</tbody>
</table>

### 4.2 Obtaining optimal maintenance schedule against two different control charts policies

Maintenance models usually assume that the operating condition of the equipment remains stable throughout production and no deterioration mechanism exist other than complete failure. Sometimes, though, the equipment may deteriorate to a less desirable working condition before failing altogether. This means that if machine failure occurs it is not necessary that it always stops the machine immediately but it may also shift the process mean and thereby affect quality of the product being manufactured on the machine. Say for example in case of a grinding machine if work head belt is broken, then it will stop the machine completely while if loosening of ball screw chuck nut occurs then it does not stop the machine completely but may shift the process mean and in turn may result into ovality of the component produced. Similarly, if we consider the cutting process of metal frames using power saws, the cutting disc loses its balance and consequently looses its ability to produce perfectly flat bur-free cuts on the metal parts. Machine or equipment failures may also affects the \(\sigma\) of the process. However in this
preliminary work to illustrate the integration of maintenance and quality control policy it is assumed that the manufacturing process is such that the process standard deviation does not change due to machine failure and change occurs only in the process mean.

Let the machine fail with two competing failure modes FM1 and FM2. FM1 mode denotes a failure that results into immediate stoppage of the machine and FM2 denotes a failure where machine continues to operate but with degraded performance with respect to quality of the products produced. Let the probability of occurrence of failure mode FM1 ($P_{FM1}$) be 0.6 and FM2 ($P_{FM2}$) be 0.4 respectively. Let the expected shift ($\delta$) in the process due to FM2 be 0.6 i.e., $\mu' = \mu + \delta \sigma_p$. Where $\mu$ is the original process mean, $\sigma_p$ is the process standard deviation and $\mu'$ is the shifted process mean.

As process shifts, more number of products gets rejected as compared to normal operation and the failure is detected after a time lag. A $\bar{X}$-control chart is used as a mechanism to detect process shift and there is a single quality characteristic at this stage of manufacturing. Whenever a quality shift is detected by the control chart, maintenance action is performed in order to restore the equipment to the in-control state. The optimum value of maintenance interval considering the control chart policy is such that cost per unit time of maintenance interval ($CPUT_{MQ}$) is minimum. The suffix ($MQ$) stands for maintenance schedule obtained by considering control chart policy. Two control chart policies as shown in Table 3 are considered in this paper.

The ($CPUT_{MQ}$) for the evaluation period is represented as:

$$
CPUT_{MQ} = \frac{E[C_{PM}] \cdot E[N_{PM}] + (E[C_{CM}]_{FM1} \cdot P_{FM1} + E[C_{CM}]_{FM2} \cdot P_{FM2}) \cdot E[N_f]}{\text{Evaluation time period (T_{eval})}}
$$

(3)

where the expected number of failures $E[N_f]$ and expected number of PM actions $E[N_{PM}]$ can be obtained by simulating the machine failures corresponding to given $\eta$ and $\beta$. In the present study Blocksim7 (Reliasoft, 2009) is used for simulation.

The expected cost per PM action of component will be:

$$
E[C_{PM}] = MTTR_{PM} \cdot [PR \cdot C_{ip} + LC] + C_{FCPPM}
$$

(4)

It is assumed that the production rate (PR), shift in process mean and cost of rejection is the average of the corresponding values for the three batches. The evaluation time period is assumed to be one year.

Since FM1 results into immediate failure followed by detection and repair/replacement action, the expected corrective maintenance cost due to FM1 is:

$$
E[C_{CM}]_{FM1} = MTTR_{CM} \cdot [PR \cdot C_{ip} + LC] + C_{FCPCM}
$$

(5)

The expected cost of corrective maintenance action due to failure mode FM2 of the component that includes the rejection cost and the cost of down time will be:

$$
E[C_{CM}]_{FM2} = E[C_{rej}] + MTTR_{CM} \cdot [PR \cdot C_{ip} + LC] + C_{Re}_{p}
$$

(6)

where the expected number of defective units $E[N_{defective\ units}]$ can be calculated as shown in Appendix A and the expected cost of rejection $E[C_{rej}]$ can be obtained as:
\[ E[\text{Re}_j] = C_{\text{Re}_j} \cdot E[N_{\text{rejections}_j}] \]  

Table 5 shows the values of expected number of rejections per failure calculated using equation (10) for sample size 4 and 6 with fixed values of other control chart parameter and optimal maintenance schedule for the two different control chart policy as calculated from the equation (3).

**Table 5** Optimal maintenance schedule for different control chart policy

<table>
<thead>
<tr>
<th>Control chart policy</th>
<th>PR jobs/hr</th>
<th>( \delta )</th>
<th>Control chart parameter</th>
<th>( LCL )</th>
<th>( UCL )</th>
<th>( n )</th>
<th>( TBS )</th>
<th>( E[N_{\text{rejections}}] )</th>
<th>( \beta' )</th>
<th>( 1-R_{\delta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>20</td>
<td>0.6</td>
<td>( -3\sigma )</td>
<td>3\sigma</td>
<td>4</td>
<td>16</td>
<td>73</td>
<td>0.964</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>20</td>
<td>0.6</td>
<td>( -3\sigma )</td>
<td>3\sigma</td>
<td>6</td>
<td>16</td>
<td>39</td>
<td>0.933</td>
<td>0.008</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Control chart policy</th>
<th>Percentage defective</th>
<th>((\text{CPUT})_{\text{MQ}}) cost unit/hr</th>
<th>Optimal maintenance schedule (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>23.125%</td>
<td>642</td>
<td>500</td>
</tr>
<tr>
<td>P2</td>
<td>12.187%</td>
<td>520</td>
<td>700</td>
</tr>
</tbody>
</table>

4.3 Superimposing production and maintenance schedule

When the optimal schedule obtained in Section 4.1 is implemented on shop floor it may be interfered by maintenance department at optimal maintenance schedule interval obtained in Section 4.2. In order to implement both the policies, it is required to superimpose the maintenance interval on the optimal batch sequence. Assuming that the machine can not be stopped for PM until all the jobs in a batch are completed, production manager has four possible alternatives for superimposing the PM schedule. These are represented through a Gantt chart in Figure 1.

**Figure 1** Gantt chart of production horizon showing different alternatives for superimposing PM schedule (see online version for colours)

The objective of combining these two policies is to determine the optimal production schedule at which the CPUT of joint consideration is minimised. However, the problem of superimposition is complicated by the fact that tardiness values for the jobs are stochastic, since the machine may or may not fail during each job and PM decisions affect the probability of machine failure.

The CPUT for joint consideration of production and maintenance schedule can be expressed as:
Joint consideration of production scheduling, maintenance and quality policies

\[(CPUT)_{SM(Q/CPUT)} = \frac{\text{Total penalty cost due to batch and maintenance delay} + \text{Total raw material inventory carrying cost}}{\text{Schedule completion time}} \]  \hspace{1cm} (8)

The suffix \(S + (M/Q)\) indicates that the maintenance schedule obtained by considering the control chart policy is superimposed on the optimal production schedule obtained independently. The calculation of penalty cost due to batch delay is similar to that of Cassady and Kuatanoglu (2003) model of total expected weighted tardiness. However, they considered perfect PM while in the present study the model is slightly modified to include imperfect PM as shown in Appendix B. The penalty cost due to maintenance delay can be obtained from \(M/Q\) model. Table 6 shows the values of \((CPUT)_{S+(M/Q)}\) for all the possible four locations of PM.

### Table 6

<table>
<thead>
<tr>
<th>Batch sequence</th>
<th>Location of PM</th>
<th>((CPUT)_{S+(M/Q)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[B2-B3-B1]</td>
<td>PM is performed before first batch (in this case it is batch 2 i.e., B2)</td>
<td>131</td>
</tr>
<tr>
<td>[B2-B3-B1]</td>
<td>PM is performed before second batch</td>
<td>263</td>
</tr>
<tr>
<td>[B2-B3-B1]</td>
<td>PM is performed before third batch</td>
<td>345</td>
</tr>
<tr>
<td>[B2-B3-B1]</td>
<td>PM is performed after third batch (no PM)</td>
<td>837</td>
</tr>
</tbody>
</table>

It is clear from Table 6 that the optimal solution of the superimpose problem is [B2-B3-B1] with PM action performed at the starting of first batch i.e., in this case it is batch B2 (marked italic in Table 6).

### 4.4 Integrating production and maintenance schedule

The integrated model for production and maintenance schedule considering the control chart parameters differs from superimposition model, in the sense that integrated model does not stick to the optimal job sequence obtained from Section 4.1, rather it evaluates all the possible four locations of PM with all 3! possible batch sequences. Thus in this problem, a total of 24 sequences are evaluated and the integrated solution is one that gives minimum \((CPUT)_{S+(M/Q)}\). It is again calculated as:

\[(CPUT)_{S+(M/Q)} = \frac{\text{Total penalty cost due to batch and maintenance delay} + \text{Total raw material inventory carrying cost}}{\text{Schedule completion time}} \]  \hspace{1cm} (9)

The suffix \(S^{*}(M/Q)\) indicates that the optimal production schedule is obtained by integrating production and maintenance schedule for a given control chart policy. Table 7 shows the calculation of \((CPUT)_{S^{*}(M/Q)}\) for all the possible 24 iterations.

From the above analysis at different locations of PM interval, the batch sequence-PM decisions with the overall minimum \((CPUT)_{S^{*}(M/Q)}\) value are identified as the global
optimal solution. The optimal solution for this example is to use the batch sequence [B3-B2-B1] with PM performed prior to first batch i.e., in this case it is batch B3 (marked italic in Table 7).

Table 7  \((CPUT)_{S^M(Q)}\) for integrated model

<table>
<thead>
<tr>
<th>Batch sequence</th>
<th>((CPUT)_{S^M(Q)}) at PM performed before first batch</th>
<th>((CPUT)_{S^M(Q)}) at PM performed before second batch</th>
</tr>
</thead>
<tbody>
<tr>
<td>[B1-B2-B3]</td>
<td>139</td>
<td>582</td>
</tr>
<tr>
<td>[B1-B3-B2]</td>
<td>131</td>
<td>587</td>
</tr>
<tr>
<td>[B2-B1-B3]</td>
<td>139</td>
<td>783</td>
</tr>
<tr>
<td>[B2-B3-B1]</td>
<td>131</td>
<td>263</td>
</tr>
<tr>
<td>[B3-B1-B2]</td>
<td>126</td>
<td>269</td>
</tr>
<tr>
<td>[B3-B2-B1]</td>
<td>122</td>
<td>236</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Batch sequence</th>
<th>((CPUT)_{S^M(Q)}) at PM performed before third batch</th>
<th>((CPUT)_{S^M(Q)}) at PM performed after third batch</th>
</tr>
</thead>
<tbody>
<tr>
<td>[B1-B3-B2]</td>
<td>754</td>
<td>833</td>
</tr>
<tr>
<td>[B2-B1-B3]</td>
<td>783</td>
<td>848</td>
</tr>
<tr>
<td>[B2-B3-B1]</td>
<td>345</td>
<td>837</td>
</tr>
<tr>
<td>[B3-B1-B2]</td>
<td>745</td>
<td>830</td>
</tr>
<tr>
<td>[B3-B2-B1]</td>
<td>349</td>
<td>825</td>
</tr>
</tbody>
</table>

4.5 Observations

- An important feature in the use of control charts is the design of the control chart i.e., the selection of the sample size, establishing CL, and the frequency of the sampling. In our example we specified 3 sigma CL, the sampling frequency to be every 16 hours, production rate of 20 jobs/hr and varying sample size of 4 and 6. As shown in Table 5 the percentage defectives obtained with sample size of 4 are 23.125% while with sample size of 6 are 12.187%. This means that if we design a control chart for sample size of 6 then we can reduce 10.937% rejection as compared to case with sample size of 4. The result shows that increasing the sample size will decrease the probability of type II error, thus enhancing the chart’s ability to detect an out-of-control state earlier. If the process being out of control is detected at an early stage the maintenance action can be then carried out to restore the process to in-control-state at the early stage. This early detection and subsequent early restoration will prevent a larger number of defectives being produced. Thus the impact of sample size is in early detection which if corrected will yield lower rejections.

- Since the integrated model aims at minimisation of total \((CPUT)_{M/Q}\) which include the cost of PM, labour cost, loss production cost, and cost of failure including cost of rejection, a smaller sample size \((n = 4)\) will lead to higher rejections necessitate more frequent PM as an optimal policy. With higher sample size \((n = 6)\), the frequency of
PM under optimal condition can be relatively less as fewer rejects are produced. Since the objective is minimisation of cost of maintenance and quality policies, the model clearly brings out the interplay of these issues. However, more extensive

*Enrichment of this model is required in future to include many other aspects of quality and maintenance.*

- In Table 6, the minimum obtained from the superimposition model with PM schedule before the batch B2 is 131. This shows a saving of 84% over the superimposed solution with no PM schedule i.e., \((CPUT)_{S+(M/Q)}\) value of 837.

- From Tables 6 and 7 it is clear that optimal batch sequence [B3-B2-B1] obtained from integrated model is not the same as that obtained from superimposition model [B2-B3-B1] for the present example. The optimal solution obtained from integrated model shows a saving of 7% in CPUT value over the optimal solution obtained from superimposition model. In addition if the optimal \((CPUT)_{S+(M/Q)}\) value of integrated solution is compared with the optimal \((CPUT)_S\) of single scheduling with no PM obtained in Section 4.1, then results show a saving of 85%.

Thus the simple example presented in this section clearly shows the importance of joint consideration of production scheduling, maintenance scheduling and quality control policy in improving the operating performance of a production system. Thus, there is a strong case for joint consideration of these three aspects of production system. Hence a conceptual model framework to integrate all the three aspects is proposed and presented in the following section.

### 5 A conceptual framework for integrated model

The example presented in Section 4 highlights the importance of the joint consideration of production scheduling, maintenance and quality control, hence a more generalised methodology is needed to bridge the gaps identified in Section 3. Figure 2 shows a conceptual framework for the integrated model. It is divided into three categories:

1. shop floor characteristics
2. integration framework
3. selection of optimal level of integration.

Integration framework is further divided into three categories:

a. no integration
b. partial integration
c. full integration.

‘No integration’ case is the conventional view, where production scheduling, maintenance scheduling and quality control policies are optimised separately. However, as mentioned in the illustrative example, when these individually optimal solutions are implemented on the shop floor it is necessary to superimpose them. Thus the conventional view is in fact the superimpose case represented by \([S + M + Q]\).
Figure 2: Conceptual framework for integrated model framework

A) No Integration
- Step A1: Obtaining optimal Production Schedule (PS) independently
- Step A2: Obtaining optimal Preventive Maintenance (PM) schedule independently
- Step A3: Obtaining optimal Quality Control chart Parameters (QCP) independently
- Step A4: Superimposing PM schedule and QCP in to optimal production schedule

B) Partial Integration
- Step B11: Obtaining optimal Production schedule and PM schedule jointly (PS+PM)
- Step B12: Obtaining optimal Quality Control chart Parameters (QCP) independently
- Step B13: Superimposing quality control chart parameter in to optimal production schedule and PM interval obtained from step B1 in to production schedule obtained from step B4 PS+PM+QCP

C) Full Integration
- Step B21: Obtaining optimal PM schedule and quality control chart parameters jointly (PM+QCP)
- Step B22: Obtaining optimal Production Schedule (PS) independently
- Step B23: Superimposing quality control chart parameter and PM interval obtained from step B4 in to production schedule obtained from step B3 PS+PM+QCP
- Step B34: Obtaining optimal production schedule, PM interval and quality control chart parameter [PS+PM+QCP] jointly

Selecting the optimal level of integration based on shop floor characteristics
By partial integration we mean the joint optimisation of any of the two shop floor aspects i.e., maintenance and scheduling (M*S) or maintenance and quality (M*Q) and then superimposing the remaining third one on the integrated solution, i.e., (M*Q) + S or S + (M*Q). Note that the (M*Q) differs from the (M/Q) used in illustrative example in the sense that it not only considers the control chart policy while obtaining the maintenance schedule but also aims at obtaining the optimal control chart policy with the maintenance schedule. Thus the CPUT for (M*Q) will include: cost of corrective maintenance, cost of PM, cost due to quality loss, cost of sampling, cost of inspection, cost of false alarm, etc.

By full integration we mean joint consideration of all the three shop floor aspects i.e., production scheduling, maintenance scheduling and quality control charts. It is represented by [S*M*Q]. The objective of such consideration will be to obtain the production schedule, maintenance schedule and control chart parameters that minimise the CPUT. The CPUT, apart from above costs, also includes the cost of schedule delay and inventory carrying cost as shown in the illustrative example.

This integrated framework needs to be evaluated for different shop floor characteristic, demand patterns, variety of products, availability of resources, production rates, etc. for different job flow pattern to obtain the level of integration required under these shop flow characteristics.

6 Potential areas for further research

The work presented in this paper is limited to the use of a single machine and can be extended in several directions, some of which are indicated below:

- It will be interesting to apply the proposed methodology to different shopfloor environments, like flow-shop, open-shop, job-shop, etc., which contain multiple machines with different flow patterns and sequence dependent/independent setup times for jobs.
- The present study assumes three batches of jobs. However, this can be extended to more number of batches, which will increases the complexity of the problem. To solve such problems, different metheuristics like GA, particle swarm optimisation (PSO), simulated annealing (SA), tabu search, etc. can be used and their performance may be compared.
- Application of other approaches to determine cost consequences of a shift in process average may be tried. For example, Taguchi loss function approach could be used to quantify loss due to process shift.
- The present study considers a single quality characteristic. An extension to this could be achieved by considering multiple quality characteristics.
- The illustrative problem solved can be further extended in future researches to incorporate the effect of change in \( \sigma \), thereby making the model more realistic.

Thus the proposed research area opens up a number of interesting avenues for future research.
7 Concluding remarks

In this paper, an attempt has been made to review the literature dealing with integration of more than one of the three aspect of production planning functions i.e., production scheduling, maintenance scheduling and quality control. Potential of such joint consideration is also demonstrated through a numerical example. Both the literature review and the numerical investigation presented in this paper lay emphasis for more work that is required towards the integration of these shop floor aspects. As a direction for future research, it could be interesting to work on real shop floor environment, where multiple machines with multiple components in different manufacturing environments are used to process multiple products. Finally, the conceptual methodology and future scope provided in this paper is expected to provide further directions for the researchers to work in this area.

Acknowledgements

The authors thank the anonymous referees for constructive suggestions leading to enhancement in the value of this paper.

References


Joint consideration of production scheduling, maintenance and quality policies


Appendix A

The expected number of defective units produced during out of control state by a $\bar{X}$-chart with the usual $3\sigma$ limits is expressed as:

$$E[N_{\text{rejections}}] = \frac{TBS}{1 - \beta'} \cdot PR \cdot [1 - R_\delta]$$

(10)

where $[1 - R_\delta]$ is the proportion of non-conforming units due to shift in process mean and $\beta'$ is the type II error.

Thus the probability of not detecting this shift on the first subsequent sample or the type II error ($\beta'$-risk) for $\pm 3\sigma$ and $\delta$ shift is expressed as (Montgomery, 2001):

$$\beta' = F(3 - \delta \sqrt{n}) - F(-3 - \delta \sqrt{n})$$

(11)

In the present study it is assumed that the process capability of the in-control process is 1 (i.e., the upper and lower specification limits would be at $\pm 3\sigma_p$). Thus the proportion of conforming units ‘$R_\delta$’ due to shift $\delta$ will be:

$$R_\delta = F(3 - \delta) - F(-3 - \delta)$$

(12)

where $F$ denotes the standard normal cumulative distribution function and $PR$ is the average production rate of the machine.

Appendix B

Let $a_{[i]}$ denote the virtual age of the machine prior to making sequencing and PM decisions. Let $a_{[-i]}$ denote the age of the machine immediately prior to performing the $i$th job in the sequence, let $a_{[i]}$ denote the age of the machine after $i$th job in the sequence, and let

$$y_{[i]} = \begin{cases} 
1 & \text{if PM is performed prior to the } i\text{th job} \\
0 & \text{otherwise}
\end{cases}
$$

for $i = 1, 2, \ldots, n$
Joint consideration of production scheduling, maintenance and quality policies

Since PM restores the machine with restoration factor \( R_f \) and the repair is minimal,

\[
\overline{a}_{i-1} = a_{i-1} (1 - y_{i-1} \cdot R_f)
\]

\[a_i = \overline{a}_{i-1} + p_i\] (13)

Let ‘\( T \)’ denote the time to failure for a machine, and let \( F(t) \) denote the cumulative distribution function of ‘\( T \)’. Assuming a Weibull distribution for time-to-failure, \( F(t) \) can be written as,

\[F(t) = 1 - \exp \left[ - \left( \frac{t}{\eta} \right)^{\beta} \right],\]

and

\[\overline{F}(t) = 1 - F(t)\] (14)

The probability that the machine fails while \( i \)th job is being processed can be determined using the Weibull probability distribution as follows,

\[\phi_i = F\left(p_i + \overline{a}_{i-1} \cdot \frac{1}{\beta} \right) = 1 - \exp \left[ - \left( \frac{p_i + \overline{a}_{i-1}}{\eta} \right)^{\beta} \right] i = 1, 2, ..., n\] (15)

\[\bar{\phi}_i = 1 - \phi_i \] (16)

The completion time for a job is a discrete random variable that depends on:

1. the age of the machine prior to decision making
2. the processing time for jobs
3. the time to complete PM and the PM decisions
4. the repair time and the probability of machine failure during batches.

Let \( C_{[i]} \) denote the completion time for the first job. Then

\[C_{[i]} = (MTTR)_{PM} \cdot \sum_{i=1}^{i} y_{i} + \sum_{i=1}^{i} p_{i} + M_{[i]} \] \[i = 1, 2, ..., n\] (17)

where \( M_{[i]} \) is a discrete random variable having the following probability mass function

\[\pi_{[i]} = \Pr\{M_{[i]} = q \cdot (MTTR)_{CM}\} = \sum_{N_{r}} \prod_{l=N_{r}} \prod_{i=N_{r}} \phi_{[i]} \]

\[q = 0, 1, ..., i, \quad \text{for all } i = 1, 2, ..., n.\]

Let \( \Theta_{[i]} \) denote the tardiness of the \( i \)th job, \( i = 1, 2, ..., n. \) Note that \( \Theta_{[i]} \) has \( i + 1 \) possible values,
\[ \theta_{[i,a]} = \max \left( 0, C_{[i,a]} - d_{[i]} \right) \quad q = 0,1,...,i \] (21)

Thus the expected tardiness of the \( i \)th job in the sequence is

\[ E(\Theta_{[i]}) = \sum_{q=0}^{i} \theta_{[i,a]} \pi_{[i,a]} \] (22)

Thus the total penalty cost incurred due to batch tardiness is given as

\[ (TPC)_{batch\ tardiness} = \sum_{a=1}^{m} \sum_{i=1}^{n} P_{[i]} E(\Theta_{[i]}) \] (23)