A methodology for joint optimization for maintenance planning, process quality and production scheduling

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Abstract

Performance of a manufacturing system depends significantly on the shop floor performance. Traditionally, shop floor operational policies concerning maintenance scheduling, quality control and production scheduling have been considered and optimized independently. However, these three aspects of operations planning do have an interaction effect on each other and hence need to be considered jointly for improving the system performance. In this paper, a model is developed for joint optimization of these three aspects in a manufacturing system. First, a model has been developed for integrating maintenance scheduling and process quality control policy decisions. It provided an optimal preventive maintenance interval and control chart parameters that minimize expected cost per unit time. Subsequently, the optimal preventive maintenance interval is integrated with the production schedule in order to determine the optimal batch sequence that will minimize penalty-cost incurred due to schedule delay. An example is presented to illustrate the proposed model. It also compares the system performance employing the proposed integrated approach with that obtained by considering maintenance, quality and production scheduling independently. Substantial economic benefits are seen in the joint optimization.

1. Introduction

Production scheduling, maintenance scheduling and process quality is some of the key operational policies, affecting the performance of any manufacturing system. Despite the possible interaction effect between these decisions, not many scientific, model-based approaches are available to optimize them simultaneously. For instance, most production scheduling models do not consider the effect of machine unavailability due to failure or maintenance activity. Similarly, maintenance planning models seldom consider the impact of maintenance on due dates to meet customer requirements. However, maintenance effectiveness cannot be measured in a meaningful way without taking into account whether the maintenance function is meeting the production requirements (Paz & Leigh, 1994). On the other hand, delaying the maintenance to meet production requirement may increase the process variability and risk of machine failure, which in turn may cause higher rejections or downtime losses. Ollila and Malmipuro (1999) observed that maintenance has a major impact on efficiency and quality along-with equipment availability. In a case study carried out in five Finnish industries, they showed that well functioning machinery is a prerequisite to quality products. They also showed that lack of proper maintenance is usually among the three most important causes of quality deficiencies. Thus these three shop floor level operational policies also have some interaction effect on each other and hence a joint consideration of various policy options pertaining to quality, maintenance, and scheduling provides an important research area for investigation. Seen in this light, the present paper provides a methodology for joint consideration of these three shop floor policies. It first develops an approach to integrate maintenance planning and process quality policy. Specifically, the aim is to determine optimal preventive maintenance interval and control chart parameters that minimize the expected cost per unit time of joint policy. Subsequently, an approach is proposed in which the optimal preventive maintenance interval obtained from above mentioned joint maintenance and quality model is integrated with production schedule decisions. It provides the optimal batch sequence that will minimize the penalty-cost associated with schedule delay. Finally, it compares the performance of the proposed integrated approach with the methodology that treats these issues independently.

The paper is organized as follows: in Section 2, a brief review of relevant literature is presented. In Section 3, the problem statement is discussed in detail. Section 4 provides a solution methodology. It is divided into two parts. First part provides a mathematical model for optimization of integrated maintenance scheduling and quality control chart policy and a numerical example is presented for illustration. Second part provides an integrated model of maintenance scheduling and production scheduling, also illustrated
through an example. Comparison of the result obtained from integrated model with those obtained from the independent models is also given. In the concluding section some possible extensions of the proposed approach are given.

2. Literature review

Since the 1980s, research on production scheduling with machine failures or varying machine capacity started appearing in literature. Most of the research has taken (Pinedo & Rammouz, 1988; Adiri, Frostig, & Rinnooy Kan, 1991; Hirayama & Kijima, 1992; Federguen & Mosheiov, 1997; Leung & Pinedo, 2004) a passive approach toward machine unavailability and focused on how to adjust the production schedule to account for the time when machines are unavailable. It is well known that carrying out preventive maintenance on a machine with increasing failure rate can significantly reduce the occurrence of machine failures. If optimally scheduled, this can also increase the machine availability.

Grave and Lee (1999) and Lee and Chen (2000) developed approaches that schedule jobs simultaneously with a single preventive maintenance. They assume that each machine is maintained only once during the planning horizon. Their approaches for production scheduling and preventive maintenance scheduling consist of two stages:

1. Determine the interval during which a machine needs to be maintained only once to increase its availability.
2. Within the interval found in (1), simultaneously schedule the jobs and the single preventive maintenance action.

Cassady and Kutanoglu (2003) compared the optimal values of total weighted tardiness under production scheduling integrated with preventive maintenance planning with that obtained under separate production scheduling and preventive maintenance planning. They assumed that the uptime of a machine follows a Weibull distribution; the machine is minimally repaired when it fails; and the preventive maintenance restores the machine to a state as good as new. Their results indicate that there is an average of 30% reduction in the expected total weighted tardiness when the production scheduling and preventive maintenance planning are integrated. Leng, Ren, and Gao (2006) and Sortrakul and Cassady (2007) extended the work of Cassady and Kutanoglu (2003) and proposed a Chaotic Partial Swarm Optimization (CPSO) heuristic and GA-based heuristics respectively to solve the integrated mathematical model for single machine production scheduling and PM planning as a multi-objective optimization problem.

Similarly, an increasing number of practitioners and researchers have recognized that there is a strong relationship between product quality, process quality and equipment maintenance (Ben-Daya & Duffuaa, 1995), and integration of these may be beneficial to the organization. But research in this field is still quite limited. Rahim (1993) jointly determined the optimal design parameters on an x-bar control chart and preventive maintenance (PM) time for a production system with an increasing failure rate. Ben-Daya (1999), Ben-Daya and Rahim (2000) and Rahim (1994) investigated the integration of x-bar chart and PM, when the deterioration process during in-control period follows a general probability distribution with increasing hazard rate. Cassady, Bowden, Liew, and Pohl (2000) studied an x-bar chart in conjunction with an age based preventive maintenance policy. Rahim and Ben-Daya (2001) provided an overview of the literature dealing with integrated models for production scheduling, quality control and maintenance policy. Recently, Linderman, McKone-Sweet, and Anderson (2005) developed a generalized analytic model to determine the optimal policy to coordinate Statistical Process Control (SPC) and planned maintenance to minimize the expected total cost. Panagiotidou and Tagaras (2007) have proposed an economic model for the optimization of preventive maintenance interval for a production process with two quality states.

While some literature is available for integrating maintenance with scheduling and maintenance with quality, integration of all the three areas i.e. production scheduling, maintenance and quality control has started getting attention of the research community only
recently (Rahim & Ben-Daya, 2001). Hence it presents a good opportunity for further research. This paper is an attempt in that direction.

2.1. Gaps from the related research

Following are the gaps observed in the literature on this subject:

- Most of the integrated models focus on process quality problems and completely ignore the possibility of an equipment failure in terms of immediate stoppage of machine or improper functioning of the equipment that result in a poor product quality and calls for a maintenance action.
- Majority of the integrated models focus on the investigation of preventive replacement or a perfect PM policy that restores the equipment to an as-good-as-new state. Recently very few papers have appeared that incorporate the aspect of imperfect PM.
- Most of the maintenance optimization models available in literature are, in general, considers a fixed value of the cost of corrective maintenance. However, as mentioned earlier, the failure of machine also includes performance degradation in terms of poor quality leading to rejection of product being manufactured by the machine. Thus the cost of corrective maintenance not only includes down time losses and repair/replacement cost, but may also include cost of rejection. The cost of rejection may vary depending upon the type of the production system. If the cost of rejection is very high in a particular production system, then the cost of corrective maintenance is higher compared to preventive maintenance, which will reduce the preventive maintenance period.

3. Statement of the problem

Consider a production system consisting of a single machine producing products of the same type with constant production rate (PR, items per hour) on a continuous basis (three shifts of 7 h each, 6 days-a-week). Further, consider a single component operating as a part of machine with time-to-failure following a two parameter Weibull distribution. Let the shape and scale parameters of the distribution be $B$ and $\eta$ respectively. In this paper, machine failure is considered in terms of failure mode (FM). It is assumed that whenever machine fails it leads to one of the following two consequences.

1. Failure Mode I (FM1): leads to immediate breakdown of the machine.
2. Failure Mode II (FM2): leads to reduction in process quality by shifting the process mean.

Failure Mode I (FM1) is detected immediately as it brings the machine instantly to breakdown state. However, Failure Mode II (FM2) is detected after a time lag through control chart mechanism. It is assumed that whenever the failure is detected, corrective actions are taken to restore the machine back to the operating conditions. Thus, (FM1) results in an expected corrective maintenance cost ($E[CFM_{FM1}]$) comprising of cost of down time, cost of maintenance labor, and fixed cost of repair/restoration. However, Failure Mode II (FM2) affects the functionality of the machine and causes the process to shift, resulting in an increase in the rejection rate, till it is detected. Thus (FM2) in addition; also incurs the cost of lost quality. Preventive maintenance (PM) is carried out to reduce the unplanned down time cost. However, PM also consumes time and resources, which could otherwise be used for production. Preventive maintenance optimization is therefore done to strike a balance between cost of failure and cost of preventive maintenance.

Let us suppose the process quality can be evaluated by measuring one key quality characteristic of the finished product. Let $x$ denote the measurement of this characteristic for a given product. It is assumed that $x$ is a normal random variable having mean $\mu$ and a standard deviation $\sigma$. The value of $\mu$ is referred to as the process mean, and the value of $\sigma$ is referred to as the process standard deviation. When the process is in-control, the process mean is at its target value. The process mean can instantaneously shift, due to machine failure or due to some external causes $E'$ like environmental affects, operators’ mistake, use of wrong tool, etc. The process is also restored if an external cause $E'$ is detected. After a shift the process is said to be out-of-control and the new process mean is given by: $\mu = \mu_0 + \delta \mu_0$, where $\delta$ is some non-zero real number. Usually, the failure which causes this shift is relatively subtle. Therefore, the cause of failure cannot be identified without shutting down the process and performing a close inspection of the equipment. The process also has to be restored similarly if an external cause $E'$ is detected.

Since (FM2) and $E'$ cannot be directly detected, a control chart is used to monitor the quality characteristic $x$. Hence, the time to detect (FM2) and $E'$ depends on the power of the control chart. The parameters of the chart are: ($h$) the time (in hours) between samples, ($n$) the sample size, and ($k$) the number of standard deviations of the sample distribution between the center line of the control chart and the control limits. The resulting upper and lower control limits for the $k$-chart are given by:

$$UCL = \mu_0 + k \frac{\sigma}{\sqrt{n}}, \quad LCL = \mu_0 - k \frac{\sigma}{\sqrt{n}}$$

There are costs involved in control chart design. These include cost of sampling, cost of false alarm, cost of process shift due to machine failure or external events, etc. Thus an economic design of control chart is done to obtain the optimal values of control chart design parameters ($n$, $h$, and $k$).

However, it is clear from the above description that, machine failure and maintenance may affect the process quality. Therefore preventive maintenance optimization and economic design of the control chart must be done simultaneously.

The first problem addressed in this paper is to jointly optimize the preventive maintenance schedule and control chart policy. Specifically, the objective is to obtain optimal values of $h$, $n$, $k$ and $t_{PM}$ (preventive maintenance interval) that minimize the total expected cost associated with poor quality, inspection/sampling, corrective/preventive maintenance and process downtime.

The second problem is to integrate the optimal preventive maintenance interval obtained from the joint maintenance and quality model with the production-schedule to determine the optimal job-sequence that will minimize the penalty-cost due to schedule delay.

4. Methodology for joint optimization of maintenance and quality policy

In order to demonstrate the benefits of combining preventive maintenance and Statistical Process Control, a cost model has been developed that captures the costs associated with the manufacturing process which are affected by quality control policies and maintenance planning. These costs comprise of cost of poor quality, cost of sampling/inspection, cost of preventive maintenance, cost of downtime and fixed cost of repair/restoration.

A block replacement policy is considered in this paper. Preventive maintenance is treated as imperfect. Imperfect maintenance makes the system better than what it was before maintenance but not as good as new. In this paper the Duncan’s model (Duncan, 1956) of economic design of control chart is modified to capture
the cost of process shift incurred due to external reasons ‘E’ and FM2.

Due to the stochastic nature of the quality characteristic and machine failures, the actual cost of implementing a specific block-replacement imperfect preventive maintenance policy and x chart is a random variable. Therefore, the performance of the manufacturing process is measured by C(h, n, k, tPM), the expected cost per unit time. These four variables are therefore the decision variables. All other parameters (P, fP, h, k, hCM, T0, Treset, Ts, tPM, hCM, k, hCM, T0, Treset, T, tPM, MTPM, B, η) are assumed to be known and treated as input constants.

4.1. Development of integrated cost model

In this section we develop an integrated cost model for joint optimization of optimal preventive maintenance interval and design parameters of control chart.

The total expected cost in the model includes:

1. The expected cost for minimal corrective maintenance due to FM1, (E[C_CM|FM1]).
2. Expected cost per preventive maintenance, (E[C_CM]).
3. The expected total cost of quality loss due to process failure, 
   (E[TCQ_process-failure]).

The expected total cost per unit time of integrated preventive maintenance and process quality control chart policy [ECPUT]tPM-Q is the ratio of the sum of the expected total cost of the process quality control (E[TCQ_process-failure]), expected total cost of the preventive maintenance (E[C_CM]) and expected total cost of machine failure E[C_CM|FM1] to the evaluation time. Therefore the expected total cost per unit time for the integrated model can be written as:

\[ E_{[ECPUT]tPM-Q} = \frac{E[C_CM|FM1] + E[C_CM] + E[TCQ_{process-failure}]}{T_{eval}} \]  \hspace{1cm} (1)

Optimal preventive maintenance interval (tPM) and process control chart design parameters (n, h, k) are obtained by minimizing [ECPUT]tPM-Q.

Models for each of the ingredient costs in [ECPUT]tPM-Q model are developed in the following sub-sections.

4.1.1. Expected cost model for corrective maintenance due to FM1

Failure Mode 1 (FM1) is detected immediately, hence it incurs only the down time cost during repairing/restoring the machine plus the labor and material cost. Thus corrective maintenance cost due to FM1 during the given period of time can be expressed as:

\[ E[C_CM|FM1] = \{MT_{CM} \cdot [PR \cdot C_P + LC] + C_{CPCM}\} \times P_{FM1} \times N_f \]  \hspace{1cm} (2)

where PR = production rate (jobs/h), C_P = cost of lost production (Rs/job), LC = labor cost (Rs/h of preventing machine), MT_{CM} = mean time to corrective maintenance (h) and C_{CPCM} is the fixed cost of the corrective maintenance. It includes the cost of material, lubricant, maintenance equipment, etc. For example, if maintenance involves only the replacement of the component, then in such cases it may be taken as equal to the cost of the component replaced.

In the present study, a simulation based approach (using ReliaSoft’s BlockSim (Reliasoft, 2009)) is used for obtaining the expected number of corrective maintenance as a function of PM interval and restoration factors for a given η and B during a given evaluation period.

4.1.2. Cost per preventive maintenance

Cost per imperfect preventive maintenance action of component incurs only the down time cost during repairing/restoring the machine plus the labor and material cost and it can be expressed as:

\[ E[C_CM] = \{MT_{PM} \cdot [PR \cdot C_P + LC] + C_{CPPM}\} \times T_{eval} / tPM \]  \hspace{1cm} (3)

where C_{CPPM} is a fixed cost per preventive maintenance (Rs/preventive component), MT_{PM} = mean time to preventive maintenance (h), and \( T_{eval} / tPM \) the number of preventive maintenances (N_{PM}) is rounded off to the lower whole value.

4.1.3. Model for expected total cost of quality loss due to process failure, (E[TCQ_{process-failure}])

4.1.3.1. Process cycle length. The cycle length consists of the in-control time, the out-of-control time and process resetting or machine restoration time. Assume that the in-control time is a negative exponential distribution with mean 1/λ. The expected in-control time consists of the mean time to failure and the expected amount of time for investigating false alarms (Lorenzen & Vance, 1986),

\[ E[T_i] = 1/λ + T_0 \times S / ARL_1 \]  \hspace{1cm} (4)

where T_0 is the expected search time for a false alarm and if the sampled statistics is independent, then ARL_1 = 1/α, and the symbol α is used to represent the probability of a Type I error and given as: \( α = 2R - k \).

The expected number of samples while the process is in-control (S) with a process failure rate of (λ) can be calculated as (Lorenzen & Vance, 1986):

\[ S = \sum_{i=0}^{\infty} iPr \{ \text{assignable cause occurs between the } i \text{th and } (i+1)\text{th samples} \} \]

\[ = \sum_{i=0}^{\infty} i[1 - e^{-λ(h+i)} - e^{-λ(h+i+1)}] \]

\[ = e^{-λh} / (1 - e^{-λh}) \]

In this paper we consider machine failure in terms of machine operating with degraded functionality (increased rejection rate) and the sudden breakdown which ceases the machine operation.

The probability of occurrence of machine failures can be captured and the sudden breakdown which ceases the machine operation. Operating with degraded functionality (increased rejection rate) can be expressed using the following parameters:

- The time between occurrence of an assignable cause and the next sample.
- The expected time to trigger an out-of-control signal.
- The expected time to validate the assignable cause.
- The expected time to reset the process if failure is due to external reason ‘E’.
- The overall process failure rate λ due to (FM2) and ‘E’ is λ = λ1 + λ2.

where: \( λ_2 = N_r / T_{eval} \) and \( λ_1 = \text{Mean Time between process failure} \)

\( N_r \) is the number of failures during \( T_{eval} \).

The out-of-control time consists of the expected time of the following events:

1. The time between occurrence of an assignable cause and the next sample.
2. The expected time to trigger an out-of-control signal.
3. The expected time to plot and chart a sample.
4. The expected time to validate the assignable cause.
5. The expected time to reset the process if failure is due to external reasons or the expected time to restore the machine if failure is due to (FM2).

This can be mathematically expressed using the following mathematical form.

\[ \text{Out-of-control time} = \left\{ h \times \left[ ARL_{2M} / λ_2 + ARL_{2E} / λ_1 \right] - τ \right\} + n \times T_s + T_1 + \left( T_{reset} \times λ_1 / MT_{CM} \times T_2 / λ_2 \right) \]  \hspace{1cm} (5)
where ARL2 is the average run length when the process has shifted to an out-of-control state. The expected process cycle length is the sum of the Eqs. (4) and (5) and given as:

$$E[T_{cycle}] = \frac{1}{\lambda} + T_0 \times \frac{S}{\text{ARL}_1} + \left( h \times \left( \text{ARL}_2/c_2 + \text{ARL}_2/c_2 \right) \right) - \tau + nT_s + T_1 + \left( T_{reset} + MTCM \times \frac{t_2}{\lambda} + \frac{t_1}{\lambda} \right)$$

where $T_{reset}$ is the time to perform the resetting of the process which moves to out-of-control condition due to external reason and $T_1$ is the expected time to determine the assignable cause.

4.1.3.2. Process quality cost. The process quality cost consists of three main components: the cost of rejection incurred while operating the process in 'in-control' ($C_I$) and 'out-of-control' ($C_o$), the cost of sampling, the cost of evaluating the alarms-both false and assignable and the cost of resetting or restoring (through corrective maintenance) the process.

Let $C_f$ be the cost of false alarm. This includes the cost of searching and testing for the cause. Then the expected cost for false alarm is given as:

$$E[C_f] = C_f \times (S/\text{ARL}_1) \times T_0$$

(7)

Let $C_p$ be the fixed cost per sample and $C_v$ be the variable cost per job. Thus the expected cost per cycle for sampling is the sum of the fixed cost per sample and variable cost per job and is given as:

$$E[C_s] = \left( C_p + C_v \cdot n \right) \times \left( \frac{1}{\lambda} + T_0 \times \frac{S}{\text{ARL}_1} + \left( h \times \left( \text{ARL}_2/c_2 + \text{ARL}_2/c_2 \right) \right) - \tau + nT_s \right)$$

where $F$ denotes the standard normal cumulative distribution function. This reduces to $\beta = F(k - \delta \sqrt{n}) - F(-k - \delta \sqrt{n})$. In the present study it is assumed that the process capability of the in-control process is 1 (i.e. the upper and lower specification limits would be at $\pm 3\sigma_p$). Thus the proportion of non-conforming units 'R's due to shift $\delta$ will be: $R_s = 1 - F(3 - \delta) - F(-3 - \delta)$ and the proportion of non-conforming units 'R' when the process is in control is given as: $R_c = 1 - F(k - \delta)$ (Montgomery, 2004).

It is assumed that the process is stopped during search and repair. Let $C_{reset}$ be the cost for finding and repairing the assignable cause plus the downtime cost as the process is stopped. Thus, expected cost of finding and repairing for a valid alarm for assignable cause due to external failure is given by:

$$E[C_{reset}] = [C_{reset} \times T_{reset}] \times \left( \lambda_1/\lambda \right)$$

(13)

The expected cost of finding and taking a corrective action for a valid alarm due to failure mode $FM_2$ is given by:

$$E[(C_{CM})_{FM_2}] = [(MT_{CM}) \cdot (P \cdot C_p + L_C) + C_{FCPCM}] \times \left( \lambda_1/\lambda \right)$$

(14)

Adding Eqs. 7, 8, 10, 11, (13), and (14) gives the expected cost of process failure per cycle as:

$$E[C_{process}] = E[C_I] + E[C_f] + E[C_{CM}] + E[C_{CM}/C] + E[C_{reset}] + E[(C_{CM})_{FM_2}]$$

(15)

Hence the expected process quality control cost for the evaluation period is given as:

$$E[TCQ_{process-failure}] = E[C_{process}] \times M$$

(16)

where $M$ is calculated as: $\frac{T_{eval}}{T_{pm}}$

4.2. Development of stand-alone models

To evaluate the effectiveness of the integrated model, it is important to compare the results obtained from the integrated model with stand alone model for maintenance and quality. In the following sub-sections these stand alone models are discussed.

4.2.1. Maintenance models

In this model, we assume that only planned maintenance is considered and ignore the probability of quality degradation due to machine failure. Therefore, the expected cost per corrective maintenance action can be expressed as:

$$E[C_{CM}] = \{MT_{CM} \cdot (P \cdot C_p + L_C) + C_{FCPCM} \} \times N_f$$

(17)

The expected cost per preventive maintenance action can be expressed as:

$$E[C_{PM}] = \{MT_{PM} \cdot (P \cdot C_p + L_C) + C_{FCM} \} \times T_{eval} \times T_{eval}$$

(18)

Thus expected cost per unit time for the planning period is given as:

$$E[CPU_{M}] = \frac{E[C_{CM}] + E[C_{PM}]}{T_{eval} \times T_{eval}}$$

(19)

Optimal preventive maintenance interval is determined by minimizing the $E[CPU_{M}]$ (Eq. (19)).
4.2.2. Statistical Process Control (SPC) model

This model has been investigated a lot in the literature. The expected cycle length and the expected cost of control chart are given as (Lorenzen & Vance, 1986):

\[
E[T_{cycle}]_{SPC} = 1/\bar{S} + T_0 + S/ARL_1 + \{h \times (ARL_2)\} - \tau + nT_s + T_{reset}
\]

\[
E[C_{SPC}] = C_f \left( \frac{S}{ARL_1} \right) \cdot T_0 + (C_f + C_V \cdot n) \cdot \left( T_0 \cdot \frac{S}{ARL_1} \right) + h \cdot (ARL_2) - \tau + n \cdot T_s
\]

\[
+ (\alpha \cdot PR \cdot C_{Rj}) \cdot \left( \frac{1}{\bar{S}} + T_0 \cdot \frac{S}{ARL_1} \right)
\]

\[
+ \left( PR \cdot \frac{(R_j)_{\bar{S}}}{1 - R_j} \cdot C_{Rj} \right) \cdot (h \cdot (ARL_2) - \tau + n \cdot T_s)
\]

\[
+ (C_{reset} \cdot T_{reset})
\]

(20)

(21)

where \( S \) is expected number of samples when the process is in-control while using SPC in isolation. Therefore the expected total cost per unit time for the SPC model is given as:

\[
E[CPUT]_{SPC} = E[C_{SPC}] / E[T_{cycle}]_{SPC}
\]

(22)

Optimal values of control chart variables \((n, h, k)\) are determined by minimizing the \(E[CPUT]_{SPC}\) (Eq. (22)).

4.3. Numerical Illustration

Consider a single machine whose failure is assumed to follow a two parameter Weibull distribution with \( \eta = 1000 \) and \( B = 2.5 \) as the characteristic life and shape parameter respectively. Machine considered here is expected to operate for three shifts of seven hours each for 6 days in a week. Time to carryout preventive maintenance action (MTPM) = 3 time units with restoration factor (RFCM) = 0.6 (it implies 60% restoration of life and sets the age of the block to 40% of the age of the block at the time of the maintenance action) and time to corrective maintenance (MTCM) = 12 time units with restoration factor (RFCM) = 0 (repair is minimal, i.e., the age of a repaired machine is the same as its age when it failed). The manufacturer has used control chart to monitor the manufacturing process producing that product. Assuming that the process is characterized by an in-control state with process standard deviation of \( \sigma = 0.01 \) and a single assignable cause due to external failure is of magnitude \( \delta_e = 1 \) and let deviation due to machine failure be \( \delta_{MIC} = 0.8 \), which occurs randomly and results in a shift of process mean from \( \mu_0 \) to \((\mu_0 + \delta \sigma)\). The initial values of relevant parameters are given in Table 1.

5. Batch sequencing model

Consider a single machine that processes three batches having the processing time, setup time, penalty cost, due dates, and other production parameter as given in Table 4.

Following assumptions are made to solve the problem:

1. A job cannot be preempted by another job.
2. There are no failures of machine during the schedule.
3. Raw material for all the batches is released at starting of the schedule.
4. The batch processing time is equal to the sum of the processing times of its jobs and the setup time.

The objective is to obtain the batch sequence that minimizes the Cost Per Unit Time of the schedule \( (CPUT)_b \). \((CPUT)_b \) can be calculated as:

\[
(CPUT)_b = \frac{Total \ Penalty \ cost \ due \ to \ batch \ delay + Total \ raw \ material \ inventory \ carrying \ cost}{Schedule \ completion \ time}
\]

It is assumed that the cost of per piece of camshaft is Rs. 500 and the profit margin for each piece is Rs. 40. But if the defect is detected in the product at the end of the last process or at the customer end then it will cost more to the manufacturer in terms of consequential cost because of loss of goodwill, or returning of complete product lot to the manufacturer and that may be 10 times more than the cost of good quality product. Thus we assume cost of rejection as high as Rs. 5000/piece.

Penalty cost is incurred only when a batch is delayed beyond its due date. Penalty cost for a batch can be calculated as:

\[
Batch \ penalty \ cost = (Batch \ completion \ time - batch \ due \ date) \cdot P_i
\]

(24)

Since it is assumed here that the raw materials for all the batches are released at the starting of the schedule, raw material inventory
Total penalty cost due to batch and maintenance delay can be calculated as follows:

\[
S_{\text{PM}} = \text{Total penalty cost due to batch and maintenance delay} + \text{Total raw material inventory carrying cost}
\]

\[
\text{Schedule completion time}
\]

The suffix \( S \times (M \times Q) \) indicates that the preventive maintenance schedule obtained through the integrated model of maintenance and process quality control is integrated on the entire feasible production schedule obtained independently. The total penalty cost due to batch and maintenance delay can be calculated as follows (details can be seen in Pandey, Kulkarni, & Vrat, 2010):

The probability that the machine fails while a job is getting processed can be determined using the Weibull probability distribution as follows:

\[
\phi_k = F\left(\frac{p_k - a_k}{\theta_k + a_k} \bigg| a_k \right) = 1 - \exp \left( - \left( \frac{p_k - a_k}{\theta_k + a_k} \right)^\beta + \left( \frac{a_k}{\theta_k} \right)^\beta \right)
\]

\[
\bar{\phi}_k = 1 - \phi_k
\]
Thus the total penalty cost incurred due batch tardiness is given as 

\[
(TPC)_{banchardiness} = \sum_{k=1}^{m} P_{bk} E(\Theta_{bk})
\]  

where \(d_{bk}\) and \(P_{bk}\) are the due date and penalty cost for the \(k\)th batch respectively. Table 6 shows the calculations of \((CPU^3)_{b}=M\) for all the four possible locations of PM for the given batch sequence \([B1-B2-B3].\) For example, in the sequence of \([B1-B2-B3]\) if PM is performed before the first batch \((B1)\), then the value of \((CPU^3)_{b}\) for the given sequence is 183. This process is repeated for each set of PM decisions, and the set with the smallest objective function value is identified as optimal for that job sequence. The optimal decision for this sequence is to perform PM only before the second batch (marked bold in Table 6).

Once this analysis has been performed for all the batch sequences and the integrated solution for the batch sequence and PM decisions with the overall minimum objective function value are identified as the optimal solution. The results for all the six feasible sequences for this example are presented in Table 7. The global optimal solution out of all obtained solutions is to use the batch sequence \([B2-B3-B1]\) with PM performed before second batch i.e. \(B3.\)

Similarly, the objective function value in the case where one considers scheduling only and no PM is obtained for all the six feasible batch sequence as shown in Table 8.

### 5.2. Analysis of results

The objective function value obtained for the case with scheduling only and no PM is 563 (computed from Eq. (25), see Table 8) for batch sequence \([B3-B2-B1]\) which is not the same solution as obtained from the integrated production scheduling and PM model i.e. \([B2-B3-B1].\) The minimum objective function value obtained

### Table 4
Production parameters for the illustrative example.

<table>
<thead>
<tr>
<th>Batch</th>
<th>Processing time (min) per job</th>
<th>Batch size (III)</th>
<th>Setup time (IV)</th>
<th>Total processing time (h)</th>
<th>Release time (V)</th>
<th>Due date (h) (VI)</th>
<th>Penalty cost/h/batch (P) (VII)</th>
<th>Carrying cost/job/h (VIII)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>500</td>
<td>3</td>
<td>53.0</td>
<td>0</td>
<td>100</td>
<td>75</td>
<td>1.71</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>500</td>
<td>1</td>
<td>26.0</td>
<td>0</td>
<td>50</td>
<td>50</td>
<td>1.71</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>500</td>
<td>2</td>
<td>18.66</td>
<td>0</td>
<td>40</td>
<td>45</td>
<td>1.71</td>
</tr>
</tbody>
</table>

### Table 5
\((CPU^3)_{b}\) calculation for all possible sequences.

<table>
<thead>
<tr>
<th>Batch sequence</th>
<th>Completion time</th>
<th>Tardiness</th>
<th>Penalty Cost</th>
<th>Inventory Cost</th>
<th>((CPU^3)_{b})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Batch 1</td>
<td>Batch 2</td>
<td>Batch 3</td>
<td>Batch 1</td>
<td>Batch 2</td>
</tr>
<tr>
<td>[B1-B3-B2]</td>
<td>53</td>
<td>98</td>
<td>72</td>
<td>0</td>
<td>48</td>
</tr>
<tr>
<td>[B2-B1-B3]</td>
<td>79</td>
<td>26</td>
<td>98</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[B2-B3-B1]</td>
<td>98</td>
<td>26</td>
<td>45</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[B3-B1-B2]</td>
<td>72</td>
<td>98</td>
<td>19</td>
<td>0</td>
<td>48</td>
</tr>
<tr>
<td>[B3-B2-B1]</td>
<td>98</td>
<td>45</td>
<td>19</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 6
\((CPU^3)_{b}=M\) at different locations of PM schedule superimposed in production schedule.

<table>
<thead>
<tr>
<th>Batch sequence</th>
<th>Location of PM</th>
<th>((CPU^3)_{b}=M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[B1-B2-B3]</td>
<td>PM is performed before first batch</td>
<td>142</td>
</tr>
<tr>
<td>(in this case it is batch 1 i.e. B1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[B1-B2-B3]</td>
<td>PM is performed before second batch</td>
<td>197</td>
</tr>
<tr>
<td>(in this case it is batch 2 i.e. B2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[B1-B3-B2]</td>
<td>PM is performed before first batch</td>
<td>348</td>
</tr>
<tr>
<td>[B1-B2-B3]</td>
<td>PM is performed after third batch (No PM)</td>
<td>583</td>
</tr>
</tbody>
</table>

Let \(N_k = \{1, 2, \ldots, k\}\), and let \(N_{kq}\) denote a subset of \(N_k\) containing \(q\) elements. Then, \(M_{k|q}\) is a discrete random variable having the following probability mass function

\[
\pi_{k|q} = Pr[M_{k|q} = q \times MT_{CM}] = \sum_{N_{kq}} \prod_{l \in N_{kq}} \varphi_{k|l} \prod_{l \notin N_{kq}} \Omega_{k|l}, \quad q = 0, 1, \ldots, k
\]  

(28)

For all \(k = 1, 2, \ldots, m\), let

\[
C_{k|q} = (MTTR)_{PM} \sum_{k=1}^{m} y_{k|q} + \sum_{k=1}^{m} p_{k|q} + q \times MT_{CM}, \quad q = 1, 2, \ldots, k
\]  

(29)

Let \(\theta_{k|q}\) denote the tardiness of the \(k\)th batch, \(k = 1, 2, \ldots, m\). Note that \(\theta_{k|q}\) has \(k+1\) possible values,

\[
\theta_{k|q} = \max(0, C_{k|q} - d_{k|q}) \quad q = 0, 1, \ldots, k
\]  

(30)

Thus,

\[
E(\theta_{k|q}) = \sum_{q=0}^{k} \theta_{k|q} \cdot \pi_{k|q}
\]  

(31)

Thus the total penalty cost incurred due batch tardiness is given as

\[
(TPC)_{banchardiness} = \sum_{k=1}^{m} P_{bk} E(\theta_{bk})
\]  

(32)
from integrated solution is 108 (computed from Eq. (25), see Table 7) shows a saving of 80%. Thus, the results indicate that integrating production scheduling and PM policies is substantially better than solving the two problems independently.

6. Conclusions and future extensions

This paper proposes a model for integrating PM and process quality control. The model allows joint optimization of quality control charts and preventive maintenance interval \( (n_k, n_{k+1}) \) to minimize the expected total cost per unit time. To examine the effectiveness of the integrated model, two stand-alone models are also developed. Numerical example indicates that the proposed integrated model performs better than the stand alone models. A sensitivity analysis of the model is presented to study the effect of various model parameters on the behavior of the system. This may help the manufacturer to identify the parameters which are more sensitive from those which are not.

Further, the optimal PM interval obtained from the joint maintenance and process quality control model is integrated with the production schedule. For the example studied, integrating the two decision making problems i.e. production scheduling and PM results in an average improvement of approximately 80%. Depending on the nature of the manufacturing system, the average saving may be different but still it can be very substantial. The proposed model can lead to a number of potential extensions as follows:

- The work presented in this paper is limited to a single machine, however it will be interesting to apply the proposed methodology to different shop floor environments, like flow-shop, open shop, job-shop etc., which contain multiple machines and different flow patterns and sequence dependent/independent setup times.

- This study assumes three batches of jobs. However, this can be extended to more number of batches, which will increase the complexity of the problem but take it closer to reality. To solve such problems, different meta-heuristics like Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA), TABU Search, etc. can be used and their performance may be compared.

- Taguchi quality loss function approach could be employed to quantify loss due to process shift.

- As part of future research, the researchers can try using Cumulative Sum (CUSUM) chart instead of X-bar chart for detecting the small shift in the process and the results can be compared. As an extension of this model, a more realistic enrichment can be attempted by either analytical or simulation model in future extensions.

- In this paper minimization of expected cost per unit time is considered as an objective function. As an extension other objective functions like machine availability, process capability, etc. could be used.

**Acknowledgement**

The authors would like to take the opportunity to thanks the anonymous referees for their constructive suggestions which has enhanced the readability of the paper.

**References**


**Table 8**

<table>
<thead>
<tr>
<th>Batch sequence</th>
<th>Location of PM</th>
<th>(\text{CPU}^\text{PM}_\text{t}^\text{om} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[B1-B2-B3]</td>
<td>No preventive maintenance</td>
<td>583</td>
</tr>
<tr>
<td>[B1-B3-B2]</td>
<td>No preventive maintenance</td>
<td>571</td>
</tr>
<tr>
<td>[B2-B1-B3]</td>
<td>No preventive maintenance</td>
<td>585</td>
</tr>
<tr>
<td>[B2-B3-B1]</td>
<td>No preventive maintenance</td>
<td>571</td>
</tr>
<tr>
<td>[B3-B2-B1]</td>
<td>No preventive maintenance</td>
<td>563</td>
</tr>
<tr>
<td>[B3-B1-B2]</td>
<td>No preventive maintenance</td>
<td>568</td>
</tr>
</tbody>
</table>