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A methodology for simultaneous optimisation of design parameters for the preventive maintenance and quality policy incorporating Taguchi loss function

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The performance of a production system depends on the breakdown-free operation of equipment and processes. Maintenance and quality control play an important role in achieving this goal. In addition to deteriorating with time, equipment may experience a quality shift (i.e. process moves to out-of-control state), which is characterised by a higher rejection rate and a higher tendency to fail. This paper develops an integrated model for joint optimisation of preventive maintenance interval and control parameters incorporating the Taguchi loss function. We consider two types of maintenance policies: minimal corrective maintenance that maintains the state of the equipment without affecting the age and imperfect preventive maintenance that upgrades the equipment in between ‘as good as new’ and ‘as bad as old’ condition. The proposed model enables the determination of the optimal value of each of the four decision variables, i.e. sample size (n), sample frequency (h), control limit coefficient (k), and preventive maintenance interval (t{sub}PM) that minimises the expected total cost of the integration per unit time. A numerical example is presented to demonstrate the effect of the cost parameters on the joint economic design of preventive maintenance and process quality control policy. The sensitivity of the various parameters is also examined.

Keywords: preventive maintenance; process quality policy; integrated model; cost minimisation

1. Introduction

The performance of a production system strongly depends on the breakdown-free operation of equipment and processes. The performance can be improved if these breakdowns can be minimised in a cost-effective manner. Maintenance and quality control play important roles in achieving this goal. An appropriate Preventive Maintenance (PM) policy not only reduces the probability of machine failure but also improves the performance of the machine in terms of lower production costs and higher product quality. Similarly, an appropriately designed quality control chart may help in identifying any abnormal behaviour of the process, thereby helping to initiate a restoration action. However, both PM and quality control add costs in terms of down time, repair/replacement, sampling, inspection, etc. Traditionally, these two activities have been optimised independently. However, researchers have shown that a relationship exists between equipment maintenance and process quality (Pandey et al. 2010), and joint consideration of these two shop-floor policies may be more cost-effective in improving the performance of the production system. Recent literature indicates that such joint consideration has started receiving attention from the research community.

The objective of this paper is to present an integrated model that can be used to minimise the expected total cost of process failures, inspection, sampling, and corrective maintenance/preventive maintenance (CM/PM) action by jointly optimising maintenance and quality control chart parameters.

In many preventive maintenance models, the system is assumed to be ‘as good as new’ or ‘as bad as old’ after each PM. A more realistic situation is one in which the failure pattern of a preventively maintained system changes. One way to model this is to consider imperfect maintenance where, after each PM, the failure rate of the system is somewhere between ‘as good as new’ and ‘as bad as old’. It can be assumed that the failure rate of the equipment can be decreased by a certain amount after each PM action. This amounts to a virtual reduction in the age of the equipment. When the PM for a system is modelled using imperfect maintenance, its failure behaviour is assumed to change after each PM. Consequently, this will affect the control chart policies. Apart from imperfect maintenance, the present study also uses Taguchi loss function in a control chart framework (Alexander et al. 1995).

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The basic idea is to perform PM when the amount of deviation in the product characteristic used to measure quality moves from the target value. Therefore, it is possible to reduce the deviation from the target and consequently enhance quality by performing PM. This observation provides the basis for developing models that integrate maintenance and quality policy and allows for the joint optimisation of both.

The main contribution of this paper is to introduce a new integrated approach for optimisation of maintenance and process control policy and a way to categorise the machine and process failure. Whenever a complete failure occurs, corrective maintenance action, which restores the machine, is implemented immediately. In parallel, the process is monitored through a control chart in order to identify the actual operating state. Whenever a quality shift is detected owing to machine degradation (partial failure), a corrective maintenance action is performed to restore an in-control state through a repair action. Whenever quality shift is detected owing to external reasons, a resetting of the process is performed to restore the process to its in-control state. Thus, this type of maintenance action has twin benefits, i.e. they eliminate the quality cost related to out-of-control operation owing to external reasons and machine degradation and also improve the machine’s reliability by protecting it against failures.

In the next section, a brief literature review of integrated approaches to maintenance and process quality control is presented, and the observations made from the literature are summarised. Based on the observations, the problem statement and assumptions are provided in Section 3. Section 4 presents the methodology for joint optimisation of design parameters of preventive maintenance and a quality control chart followed by a solution procedure in Section 5. In Section 6, a numerical example is provided to explain the applicability of the proposed model. Section 7 provides the sensitivity analysis results. Finally, comments on the results are provided in concluding remarks of the paper.

1.1 Notations

\[ ARL_{E} \] average run length during an out-of-control period owing to external reasons
\[ ARL_{M/(E)} \] average run length during an out-of-control period owing to machine failure
\[ ARL_{I} \] average run length during in-control period
\[ k \] control limit coefficient
\[ C_{lp} \] cost of lost production (Rs/job)
\[ C_{Frej} \] cost of rejection while the process moves out-of-control
\[ C_{reset} \] cost of resetting
\[ T_{eval} \] evaluation period
\[ E[C_{CM}]_{FM} \] expected cost of corrective maintenance (CM) owing to failure model
\[ E[C_{PM}] \] expected cost of preventive maintenance (PM)
\[ E[T_{Cycle}] \] expected cycle length
\[ E[T_{I}] \] expected in-control period
\[ E[N_{CM}] \] expected number of corrective maintenance
\[ E[f_{alarm}] \] expected number of false alarms during the process
\[ T_{false} \] expected search time for false alarm
\[ T_{0} \] expected time spent searching for a false alarm
\[ T_{1} \] expected time to determine occurrence of assignable cause
\[ E[T_{restore}] \] expected time to restore the process which may be moved out-of-control owing to machine degradation or external causes
\[ E[T_{CQ}]_{process-failure} \] expected total cost of quality owing to process failure
\[ [ETCPUT]_{\lambda_2} \] expected total cost per unit time of integrated maintenance and quality policy
\[ \lambda_1 \] failure owing to external causes
\[ \lambda_2 \] failure owing to machine degradation
\[ C_{FCPCM} \] fixed cost per CM (Rs/component)
\[ C_{FCPPM} \] fixed cost per PM (Rs/preventing component)
\[ LC \] maintenance personnel cost (Rs/h of preventing machine)
\[ \tau \] mean elapse time from the last sample before the assignable cause to the occurrence of assignable cause when the maintenance and quality policies are integrated
2. Literature review

In this section, literature related to the specific issues of integration of maintenance and quality control is presented. Readers interested in literature related to purely maintenance and design of control charts may refer to Duncan (1956), Alexander et al. (1995), Hassan et al. (2000), Sherwin (2000), Wang (2002), Garg and Deshmukh (2006), Schiffauerova and Thomson (2006), and Nenes and Tagaras (2007).

Ben-Daya and Duffuaa (1995) have emphasised the need to link quality control with preventive maintenance. Rahim (1993) determined the optimal design parameters on an $\bar{X}$-control chart and preventive maintenance time for a production system with an increasing failure rate. Rahim (1994), Ben-Daya (1999), and Ben-Daya and Rahim (2000) attempted the integration of $\bar{X}$-chart and PM, when the process deteriorated during in-control period following a general probability distribution with increasing hazard rate. Cassady et al. (2000) studied $\bar{X}$-chart in conjunction with an age-based preventive maintenance policy. Yeung et al. (2008) modified the model suggested by Cassady et al. (2000) for using an $\bar{X}$-chart to monitor the output of the production process and to determine when to perform corrective, condition-based maintenance so as to optimise the sample size, control chart parameters, and time to interval for performing PM. Rahim and Ben-Daya (2001) provided an overview of the literature dealing with integrated models for production, quality, and maintenance.

Kuo (2006) studied a joint machine maintenance and product quality control problem of finite horizon, discrete time, and markovian deteriorating state for unobservable batch production systems. Linderman et al. (2005) developed a generalised analytical model to determine the optimal policy to coordinate Statistical Process Control (SPC) and planned maintenance to minimise expected total cost. Panagiotidou and Tagaras (2007) developed an economic model for the optimisation of preventive maintenance interval for a production process with two quality states. Panagiotidou and Tagaras (2008) developed an economic model for optimisation of maintenance policy for both perfect and imperfect maintenance actions, appropriate for a production process (equipment) with two operating states (‘in-control’ state and an ‘out-of-control’) in statistical process control. Chiu and Huang (1996) developed a model that introduces preventive maintenance into the economic design of control charts. They assumed non-uniform distribution for the in-control period with an increasing hazard rate and fixed sampling intervals. In addition, they assumed that a system would become as good as new after preventive maintenance action. Zhou and Zhu (2008) attempted the integration of economic design of control chart and maintenance management, and developed a mathematical model to analyse the cost of the integrated model using the grid-search approach to find the optimal values of policy variables $(n, h, L, k)$ that minimise expected hourly cost. Recently, Panagiotidou and Nenes (2009) proposed a model for the integration of quality and maintenance procedures using Shewhart’s control chart for variables. Wang (2010) has developed three models for the maintenance of the manufacturing system monitored by $np$ control charts with respect to the sampling interval.
2.1 Gaps from the related research

The following gaps are observed in the literature on this subject:

1. Most of the integrated models focus on process quality problems and completely ignore the possibility of an equipment failure in terms of machine breakdown or improper functioning of the equipment that result in poor product quality and call for maintenance action.

2. Most of the integrated models focus on the investigation of preventive replacement or perfect PM policy that restores the equipment to an ‘as-good-as-new’ state. Relatively very few papers have appeared that incorporate the aspect of imperfect PM.

3. Most of the integrated models consider the conventional goal-post approach for the economic design of control chart; however any deviation of quality characteristics from the target value is an indirect loss to the customers. Thus, incorporating the quadratic loss function approach with the economic design of maintenance and process quality control chart policy should give better results.

4. As mentioned earlier, the failure of a machine also includes performance degradation in terms of poor quality leading to rejection of the product being manufactured by the machine. Thus, the cost of corrective maintenance not only includes down-time losses and repair/replacement cost but also may include cost of rejection. The cost of rejection may vary depending upon the type of the production system. If the cost of rejection is very high in a particular production system, then the cost of corrective maintenance is higher compared with preventive maintenance, which will reduce the preventive maintenance period. This aspect appears to be neglected in most of the maintenance optimisation models.

The above-mentioned gaps are addressed in this paper.

3. Problem statement and assumptions

We consider a production system consisting of a single machine producing products of the same type with a constant Production Rate (PR, items per hour) on a continuous basis (three shifts of seven hours each, six days a week). Further, we consider a single component operating as a part of machine with time-to-failures following a two-parameter Weibull distribution with the scale and shape parameters as $\eta$ and $\beta$ respectively. The failures of most components in a mechanical system like production machines can be modelled using a two-parameter Weibull distribution. However, the approach proposed in this paper is generic and can be used with other distributions also. Machine failures are divided into two failure modes:

1. Failure mode 1 (FM$_1$): leads to immediate breakdown of the machine.
2. Failure mode 2 (FM$_2$): leads to reduction in process quality owing to shifting of the process mean.

Similar classifications are also used by Lad and Kulkarni (2008). They have defined the failure of a machine tool as any event that either brings the machine down or leads to the machine still running but producing higher rejections. This means that if a failure occurs, it is not necessary that it be detected immediately, and the machine is stopped, but it may also affect the quality of products being manufactured on the machine. For example, in the case of a grinding machine, if the work-head belt is broken, it will stop the machine completely, while if a loosening of the ball screw chuck nut occurs, the machine may continue to run but may result in oval components being produced. It is therefore necessary to consider these types of failures, and the corresponding failure costs, which may be situation-specific, in maintenance-planning decisions. The failures belonging to the second type of failure mode can also be considered as partial failures and are defined by Black and Mejabi (1995) as degradation in the machine performance without complete failure. Thus, the problem is to jointly optimise design parameters for the preventive maintenance and process quality policy.

Let us consider the following assumptions:

1. Corrective maintenance is minimal in nature, i.e. after corrective action, the equipment has the same age as it did at the time of failure.
2. Preventive maintenance is imperfect in nature.
3. A single part gets manufactured on the machine with a single critical to quality (CTQ) characteristic.
4. The production process starts in an in-control state, with the process mean $\mu$ and standard deviation $\sigma$ for the CTQ.
(5) A single assignable cause, which occurs at random, results in an average shift in the process mean from \( \mu_0 \) to \( \mu_1 = \mu_0 + \delta \), while the standard deviation \( \sigma \) remains constant. \( \delta \) is the magnitude of shift.

(6) The process is monitored by an \( \bar{X} \)-control chart.

(7) In this model it is assumed that failure modes \( \text{FM}_1 \) and \( \text{FM}_2 \) are independent and result in failures with an appropriate time to failure distribution. For a given failure, let the probability that it is due to \( \text{FM}_1 \) and \( \text{FM}_2 \) be \( P_{\text{FM}_1} \) and \( P_{\text{FM}_2} \) respectively. Since it is assumed that these are the only failure mode types, \( P_{\text{FM}_1} + P_{\text{FM}_2} = 1 \). These probabilities can be obtained from the failure reports maintained by maintenance personnel from production lines. The failure reports mainly cover the following information: Component ID, Time to failure, Failure mode, Action taken, Failure cost.

3.1 Model description

If \( \text{FM}_1 \) occurs, it immediately stops the machine. Corrective actions are taken to repair the machine to its operating condition. Thus, the expected cost of corrective maintenance \( E[C_{\text{CM}}]_{\text{FM}_1} \) includes the cost of down time, and the cost of repair/restore action. \( \text{FM}_2 \) affects the functionality of the machine and in turn increases the rejection level. In other words, \( \text{FM}_2 \) affects the process rejection rate. It is assumed that whenever \( \text{FM}_2 \) is detected, the process is stopped immediately, and corrective actions are taken to repair the process back to the normal condition. Apart from failures owing to \( \text{FM}_2 \), the process may also deteriorate owing to external causes (E) such as environmental effects, operators’ mistakes, use of wrong tool, etc. The process is reset to the in-control state if an external event ‘E’ is detected. Figure 1 represents the modes of machine and process failure. The time-to-failure of the process is assumed to follow an exponential distribution (Ben-Daya 1999).

The detection of \( \text{FM}_2 \) or an external cause is achieved by monitoring the process. In this paper a control chart mechanism is considered for process monitoring. Let the design parameters of the control chart be sample size \( (n) \), length of the sampling interval \( (h) \), and coefficient \( (k) \) that determines the distance between the centre line and the control limits. Thus, the expected total cost of process failure \( E[TCQ]_{\text{process failure}} \) owing to \( \text{FM}_2 \) and external events considering the cost of down time, cost of rejections owing to process shifts, cost of repair/resetting, cost of sampling/inspection, cost of investigation of false/valid alarm, and cost of deviation from the target value of the CTQ. Apart from the above corrective actions, the machine can undergo preventive maintenance \( (t_{\text{PM}}) \) to minimise the unplanned downtime losses. In this paper, imperfect preventive maintenance has been considered. This means that the PM upgrades the equipment to a state between the as-good-as-new and as-bad-as-old conditions. The frequency of failures can be significantly decreased through PM, i.e. it reduces the occurrence of both \( \text{FM}_1 \) and \( \text{FM}_2 \). Reduction in \( \text{FM}_2 \) reduces the quality costs related to the out-of-control operation. However, PM also consumes some resources and productive machine time that could otherwise be used for production. The expected cost of PM \( E[C_{\text{PM}}] \) comprises the cost of downtime and cost of performing preventive maintenance actions.

![Figure 1. Representation of the machine and process failure mode.](image-url)
4. Optimisation model

The problem is to determine the optimal values of the decision variables \((n, h, k, \text{ and } t_{PM})\) that minimise the expected total cost per unit time of the system \([ETCPUT]_{M_0Q}\). Recall that the age of the equipment after a PM is reduced according to the restoration factor. The expected total cost per unit time of preventive maintenance and control chart policy \([ETCPUT]_{M_0Q}\) is the ratio of the sum of the expected total cost of the process quality control \(E[TCQ]\text{(process-failure)}\), expected total costs of the preventive maintenance \(E[C_{PM}]\) and expected total cost of machine failure \(E[C_{CM}]_{FM_1}\) to the evaluation time. The cost incurred owing to FM2 is included in the cost of process quality control. Therefore, the expected total cost per unit time for the integrated model is given as:

\[
\text{Minimise}\left[E \frac{ETCPUT}{C_{138}M_{C_3Q}}\right]_{M_0Q} = \frac{E[C_{CM}]_{FM_1} + E[C_{PM}] + E[TCQ]\text{process failure}}{T_{eval}}
\]

where \([ETCPUT]_{M_0Q} = f(n, h, k, t_{PM})\) and \(T_{eval}\) is the Planning period for which the analysis is done. Thus, the optimisation problem can be formulated as:

\[
\text{Minimise}\left[E \frac{ETCPUT}{C_{138}M_{C_3Q}}\right]_{M_0Q}
\]

Subject to

\[
\begin{align*}
n_{m-1} & \leq n_m \leq n_{m+1} \\
h_{m-1} & \leq h_m \leq h_{m+1} \\
k_{m-1} & \leq k_m \leq k_{m+1} \\
t_{PM_{m-1}} & \leq t_{PM_m} \leq t_{PM_{m+1}} \\
n, h, k, t_{PM} & \geq 0
\end{align*}
\]

In the next section we develop models for expected cost of preventive maintenance and corrective maintenance owing to FM1 and expected cost of process failure owing to FM2 as well as external causes for the given evaluation period \((T_{eval})\).

4.1 Expected cost model for corrective maintenance owing to FM1 and preventive maintenance

To estimate the expected cost of corrective maintenance owing to FM1 and preventive maintenance, the analyst must have the following information:

- The amount of time that the equipment is expected to be down each time CM/PM is required. This can include the time to perform the maintenance as well as any logistical delays (i.e. waiting for labour and/or materials required).
- The cost of CM/PM including the downtime, labour, materials, and other costs.
- The degree to which the equipment will be restored by CM/PM (e.g. ‘as good as new,’ ‘as bad as old,’ or ‘Imperfect’). This is quantified in terms of a restoration factor. The restoration factor can be determined empirically or based on expert judgement as calculated in Reliasoft (2009) and Lad and Kulkarni (2010) respectively.
- The probability that the equipment will fail owing to a particular failure mode.

The expected cost of minimal corrective maintenance owing to FM1 is given as:

\[
E[C_{CM}]_{FM_1} = MTTR_{CM} \cdot \left( PR \cdot C_{IP} + LC \right) + C_{FCPCM} \times P_{FM_1} \times N_f
\]

where \(MTTR_{CM}[PR \times C_{IP} + LC]\) is the down time cost owing to corrective maintenance.

The expected total cost of preventive maintenance action of component will be:

\[
E[C_{PM}] = MTTR_{PM} \cdot \left( PR \cdot C_{IP} + LC \right) + C_{FCPPM} \times \frac{T_{eval}}{t_{PM}}
\]

where \(MTTR_{PM} \cdot \left( PR \cdot C_{IP} + LC \right)\) is the down time cost owing to preventive maintenance, \(\frac{T_{eval}}{t_{PM}} = N_{PM}\); the number of preventive maintenances \((N_{PM})\) is rounded off to the lower integer value.

In the present study, the expected number of corrective maintenances \(E[N_{CM}]\) has been obtained by simulating machine failures for a given evaluation period as described in Section 5.2.
In this section we first derive the expression for the expected cycle length \( E[T_{Cycle}] \) and then for the expected total cost of process failure \( E[TCQ] \). The expected cycle length has been defined as the expected time between the start of successive in-control periods. There are costs incurred during the in-control period owing to sampling the process, defectives produced, and false alarms. When the process goes out of control, we assume that it cannot return to the in-control state without intervention. Again, there are costs incurred owing to sampling and increased level of defectives produced, as well as cost of searching for the cause, restoring the system, and downtime. Upon restoring the system, one quality cycle is completed and the next cycle begins. The expected cycle length is depicted in Figure 2.

### 4.2 Expected cost model for quality loss owing to process failure \( E[TCQ]_{process\text{-}failure} \)

In this section we first derive the expression for the expected cycle length \( E[T_{Cycle}] \) and then for the expected total cost of process failure \( E[TCQ] \). The expected cycle length has been defined as the expected time between the start of successive in-control periods. There are costs incurred during the in-control period owing to sampling the process, defectives produced, and false alarms. When the process goes out of control, we assume that it cannot return to the in-control state without intervention. Again, there are costs incurred owing to sampling and increased level of defectives produced, as well as cost of searching for the cause, restoring the system, and downtime. Upon restoring the system, one quality cycle is completed and the next cycle begins. The expected cycle length is depicted in Figure 2.

#### 4.2.1 Calculation of process-cycle length

The expected cycle time is the sum of the following: (1) the time until the assignable cause occurs, (2) the time to analyse a sample and chart the result, (3) the time until the chart gives an out-of-control signal, (4) the time to discover and analyse the assignable cause, and (5) the time to reset the process if failure is due to external causes or to repair the process if failure is due to FM\(_2\). It is assumed that the in-control time follows a negative exponential distribution with mean \(1/\lambda\). The value of process failure rate \(\lambda\) is calculated as shown in Section 5.1.

Let the expected number of samples \(S\) taken while in control be calculated as

\[
S = \sum_{i=0}^{\infty} \Pr(\text{assignable cause occurs between the } i\text{th and } (i+1)\text{st samples})
\]

\[
= \sum_{i=0}^{\infty} \left[ e^{-\lambda hi} - e^{-\lambda (i+1)} \right]
\]

\[
= -(1 - e^{-\lambda h}) \frac{d}{d(\lambda h)} \sum_{i=0}^{\infty} e^{-\lambda hi}
\]

\[
= e^{-\lambda h} / (1 - e^{-\lambda h})
\]

(4)

The expected number of false alarms during the process is given by

\[
E[\text{false alarm}] = S / \text{ARL1}
\]

(5)
If the sampled statistics are independent, then $ARL_1 = 1/\alpha$, where, $\alpha = \Pr(\text{out-of-control signal} \mid \text{process is in-control})$ and is expressed as:

$$\alpha = 2F(-k)$$

Thus, the expected time spent searching for a false alarm is:

$$T_{\text{false}} = T_0 \times E[f_{\text{alarm}}]$$

The expected time until the occurrence of an assignable cause, during the in-control period is:

$$= 1/\lambda + T_0 \times E[f_{\text{alarm}}]$$  \hspace{1cm} (6)

Let $\tau$ be the expected time of occurrence of the assignable cause, given that it occurs between the $i$th and $(i+1)$st samples. Then, (Duncan 1956)

$$\tau = \frac{\int_0^{h(i+1)} \lambda(x-hi)e^{-\lambda x} \, dx}{\int_0^{h(i+1)} \lambda e^{-\lambda x} \, dx}$$

$$= [1 - (1 + \lambda h)e^{-\lambda h}]/[\lambda(1 - e^{-\lambda h})]$$

Simplifying the above equation, this becomes:

$$\tau = h/2$$  \hspace{1cm} (7)

$\tau$ is independent of $i$. Thus, the expected time before a sample statistic falls outside the control limit is:

$$= h \times \left( ARL_{2M/c} \times \frac{\lambda_2}{\lambda} + ARL_{2E} \times \frac{\lambda_1}{\lambda} \right) - \tau$$  \hspace{1cm} (8)

where $ARL_2$ is the average run length when the process has shifted to an out-of-control state, and if the sampled statistics are independent; then, $ARL_2 = 1/(1-\beta)$

where $\beta = \Pr(\text{in-control signal} \mid \text{process is out-of-control})$.

$$\beta = P\{LCL \leq \tilde{x} \leq UCL \mid \mu = \mu_1 = \mu_0 + \delta\sigma_p\}$$  \hspace{1cm} (9)

Since $\tilde{x} \sim N(\mu, \sigma_p^2/n)$, and the upper and lower control limits are $UCL = \mu_0 + k\sigma_p/\sqrt{n}$ and $LCL = \mu_0 - k\sigma_p/\sqrt{n}$, we may write Equation (9) as

$$\beta = F\left(\frac{UCL - (\mu_0 + \delta\sigma_p)}{\sigma_p/\sqrt{n}}\right) - F\left(\frac{LCL - (\mu_0 + \delta\sigma_p)}{\sigma_p/\sqrt{n}}\right)$$

where $F$ denotes the standard normal cumulative distribution function. This is reduced to

$$\beta = F(k - \delta\sqrt{n}) - F(-k - \delta\sqrt{n}).$$

For a sample of $n$ units, the time to analyse the sample and chart the result is given by

$$= n \cdot T_s$$  \hspace{1cm} (10)

The expected time from the occurrence of an assignable cause to declaring the process to be in an out-of-control state will be:

$$= h \times \left( ARL_{2M/c} \times \frac{\lambda_2}{\lambda} + ARL_{2E} \times \frac{\lambda_1}{\lambda} \right) - \tau + n \times T_s$$  \hspace{1cm} (11)

Let $T_1$ be the expected time to investigate the assignable cause and $E[T_{\text{restore}}]$ be the expected time to restore the process which may be moved out of control owing to machine degradation or external causes. Once the assignable cause is detected, the restoration of the process depends on the type of failure, i.e. the failure may occur owing to
machine degradation or owing to external causes. Accordingly, the expected time to repair or reset \( E[T_{\text{restore}}] \) the process is considered and can be calculated as:

\[
E[T_{\text{restore}}] = (T_{\text{resetting}} \times \frac{\lambda_1}{\lambda} + MTTR_{CM} \times \frac{\lambda_2}{\lambda})
\]  

The expected time to detect a shift, discover the assignable cause and repair the process is given by

\[
= \left\{ h \times \left( ARL2_{M/c} \frac{\lambda_2}{\lambda} + ARL2_{E} \frac{\lambda_1}{\lambda} \right) \right\} - \tau + n \times T_s + T_1 + E[T_{\text{restore}}]
\]  

Thus, the expected process cycle length can be calculated as:

\[
E[T_{\text{Cycle}}] = \frac{1}{\lambda} + T_{\text{false}} + \left\{ h \times \left( ARL2_{M/c} \frac{\lambda_2}{\lambda} + ARL2_{E} \frac{\lambda_1}{\lambda} \right) \right\} - \tau + nT_s + T_1 + E[T_{\text{restore}}]
\]  

### 4.2.2 Expected process quality control cost \( E[T_{\text{TCQ}}] \) process-failure model

In this section we derive expression for the expected cost of process quality control. It includes the following costs:

1. the expected cost of false alarm;
2. the expected cost of detecting the assignable cause;
3. the cost of sampling;
4. the expected cost of operating while in in-control state;
5. the expected cost of operating while in out-of-control state;
6. the proportion of cost of restoring in the case of process shifts owing to machine degradation and cost of resetting in the case of process shifts owing to external reasons.

Let the cost per unit time for investigating a false alarm be \( C_{\text{false}} \). This includes the cost of searching and testing for the cause. Then, the expected cost for a false alarm can be calculated as:

\[
E[C_{\text{false}}] = C_{\text{false}} \cdot (S/ARL1) \cdot T_0
\]  

Let \( C_F \) be the fixed cost per sample of sampling and \( C_V \) be the variable cost per unit sampled. Thus, the expected cost per cycle for sampling is the sum of the fixed cost per sample and variable cost per unit sampled, and is given as:

\[
E[C_{\text{Sampling}}] = \left( C_F + C_V \cdot n \right) \times (1/\lambda + T_{\text{false}} + \left\{ h \times \left( ARL2_{M/c} \frac{\lambda_2}{\lambda} + ARL2_{E} \frac{\lambda_1}{\lambda} \right) \right\} - \tau + nT_s)
\]  

To calculate the cost of quality loss incurred owing to defecitve produced while the process is in-control and out-of-control, a Taguchi loss function approach has been used. Consider a Critical to Quality (CTQ) with bilateral tolerances of equal value \( \Delta' \). The cost to society for manufacturing a product out of specification is \( A \) Rs/unit, and uniform rejection cost is incurred beyond the control limits. In the present study it is assumed that the process capability of the in-control process is 1 (i.e. the upper and lower specification limits would be at \( \pm 3\sigma_p \)). Thus, the proportion of non-conforming units \( R_s \) owing to shift \( \delta \), when the process moves out of control state, will be:\n
\[
R_s = 1 - F(3 - \delta) - F(-3 - \delta)
\]

and the proportion of non-conforming units \( R' \) when the process is in-control is given as:\n
\[
R' = 1 - F(k) - F(-k)
\]  

The quality loss per unit time incurred while process is in-control state is given as:

\[
[L_{\text{in-control}}] = \left( PR \times \int_{LSL}^{USL} A \frac{1}{\Delta'} \times (y - T)^2 \times f(y) dy \right) + \left( R' \times PR \times C_{\text{false}} \right)
\]  

where \( y \) is a random variable denoting sample means of the quality characteristic, and \( f(y) \) is the normal density function with mean \( \mu \) and standard deviation \( \sigma \). LSL and USL are the lower and upper process specification limits and are given as: \( LSL = \mu - \Delta' \) and \( USL = \mu + \Delta' \).

Thus, the expected process quality loss incurred per cycle in the in-control state is:

\[
E[L_{\text{in-control}}] = [L_{\text{in-control}}] \times (1/\lambda + T_{\text{false}})
\]
The quality loss per unit time incurred while the process is operating in out-of-control state owing to machine failure is given as:

\[
[L_{\text{out-of-control}}]_{MC} = \left( PR \times \int_{\text{LSL}}^{\text{USL}} \frac{A}{(\Delta y)^2} \times ((y' - T)^2 \times f(y')dy \right) \\
+ \left( PR \times \frac{(R_s)_{MC}}{1 - \beta_{MC}} \times C_{\text{Frej}} \right)
\]

(19)

Thus, the expected quality loss incurred per cycle in the out-of-control state owing to machine failure is:

\[
E[L_{\text{out-of-control}}]_{MC} = [L_{\text{out-of-control}}]_{MC} \times \left( h \times \left( ARL_{2MC} \frac{\lambda_2}{\lambda} + ARL_{2E} \frac{\lambda_1}{\lambda} \right) \\
- \tau + n \times T_s + T_1 \times (\lambda_2/\lambda) \right)
\]

(20)

Similarly, the quality loss per unit time incurred while the process is operating in out-of-control state owing to external reasons is given as:

\[
[L_{\text{out-of-control}}]_E = \left( PR \times \int_{\text{LSL}}^{\text{USL}} \frac{A}{(\Delta y)^2} \times ((y' - T)^2 \times f(y')dy \right) \\
+ \left( PR \times \frac{(R_s)_E}{1 - \beta_E} \times C_{\text{Frej}} \right)
\]

(21)

Thus, the expected quality loss cost incurred in out-of-control state owing to external reasons is:

\[
E[L_{\text{out-of-control}}]_E = [L_{\text{out-of-control}}]_E \times \left( h \times \left( ARL_{2MC} \frac{\lambda_2}{\lambda} + ARL_{2E} \frac{\lambda_1}{\lambda} \right) \\
- \tau + n \times T_s + T_1 \times (\lambda_1/\lambda) \right)
\]

(22)

Let \( C_{\text{resetting}} \) be the cost for finding and resetting the assignable cause owing to external reasons, downtime if process ceases functioning, and for finding and resetting the process. The expected value of \( C_{\text{resetting}} \) can be calculated as:

\[
E[C_{\text{resetting}}] = [C_{\text{resetting}} \times T_{\text{resetting}}] \times (\lambda_1/\lambda)
\]

(23)

The expected cost of corrective maintenance action owing to failure mode FM2 of the component and for finding and repairing the assignable cause owing to machine failure is given by:

\[
E[(C_{\text{repair}})_{FM2}] = (MTTR_{CM}) \cdot [PR \cdot C_{ip} + LC] + C_{FPCM} \times (\lambda_2/\lambda)
\]

(24)

The expected cost of corrective maintenance owing to FM2 includes the cost of lost production, labour cost, and fixed cost per corrective maintenance during the time to repair.

Adding Equations (15), (16), (18), (20), (22), (23), and (24) gives the expected cost of process failure per cycle as:

\[
E(C_{\text{process}}) = C_{\text{false}} \cdot (S/ARL1) \cdot T_0 + E[L_{\text{in-control}}] + E[L_{\text{out-of-control}}]_E \\
+ E[L_{\text{out-of-control}}]_{MC} + E[C_{\text{Sampling}}] + E[C_{\text{resetting}}] \\
+ E[(C_{\text{repair}})_{FM2}]
\]

(25)

We assume that process failure is repetitive in nature, i.e. every time when the process moves out-of-control from the in-control state and is again restored, it will take the same expected time (having fixed expected cycle length).

If there are \( M \) process failure cycles in a given evaluation period, the expected process quality control cost for the evaluation period will be:

\[
E[TCQ]_{\text{process failure}} = E(C_{\text{process}}) \times M
\]

(26)
Where

\[ M = \frac{T_{\text{evaluation}}}{E[T_{\text{cycle}}]} \]

5. Solution procedure

In this section, we address two issues:

1. The way the failure rate of the process and machine are considered.
2. The procedure for calculating the number of failures \((N_f)\) of a machine for a given evaluation period as a function of preventive maintenance interval \((t_{PM})\) using a simulation approach.

5.1 Failure rate of machine and process

In this paper, we consider machine failures in terms of a machine operating with a degraded functionality and the sudden breakdown which ceases the machine operation. The probability of occurrence of machine failures is captured from past failure data. Similarly, the process may fail because of machine degradation or some external causes as mentioned above. Let the rate of failure owing to machine degradation be \(\lambda_2\) and that owing to external causes be \(\lambda_1\). Thus, the process failure rate \((\lambda)\) is the sum of failure rates owing to machine degradation and owing to assignable causes. It can be written as:

\[ \lambda = \lambda_1 + \lambda_2 \]

The process failure rate owing to machine degradation can be calculated as:

\[ \lambda_2 = \frac{N_f \times P_{FM}}{T_{\text{eval}}} \]

and the process failure rate owing to assignable causes is calculated as:

\[ \lambda_1 = \frac{1}{\text{Mean Time between process failure}} \]

where \((N_f)\) is the number of machine failures, and \(T_{\text{eval}}\) is the evaluation period.

5.2 Model for number of failures \((N_f)\) of a machine for a given evaluation period as a function of preventive maintenance interval \((t_{PM})\) using a simulation approach

It is known that it is not possible to obtain the number of failures analytically for a short planning period with time to failure following Weibull distribution (Cassady et al. 2005). Further, most of the practical systems undergo an imperfect preventive maintenance, i.e. after preventive maintenance the machine age is restored in between as good as new and as bad as old, which again increases the complexity of the problem. To overcome these limitations, a simulation-based approach is used in this paper to obtain the number of failures for a given preventive maintenance and restoration factor. BLOCKSIM7 software is used for this purpose. It uses the concept of virtual age.

The concept of virtual age was first introduced by Kijima (1989). Under this concept, a unit accumulates age during each period of operation. After each failure or preventive action, repair (corrective or preventive) ‘removes’ some of this accumulated age. Consider a unit of equipment that at any point in time is in one of the two states, functioning or repair (corrective or preventive); and assume that the unit is initially (at time \(t = 0\)) functioning. Let \(X_n\) denote the duration of the period between the completion of \((n-1)\)th repair, and the \(n\)th repair; and let \(V_n\) denote the virtual age of the unit at the completion of the \(n\)th repair. Kijima’s model (Type I) of virtual age is

\[ V_n = V_{n-1} + (1 - a)X_n \] (27)

where \(a\) is some constant such that \(0 \leq a \leq 1\), and \(V_0 = 0\). Thus, ‘\(1 - a\)’ captures the degree of equipment restoration achieved through repair action. It is assumed that preventive action is performed after a fixed time interval \(t_{PM}\). While using the system improvement model shown in Equation (27), different degrees of restoration can be used for corrective and preventive repair.
Assume that the process, in its in-control state, is characterised by a process mean of \( \mu_0 \) and standard deviation of \( \sigma \). Therefore, the process is normal with \( \mu_0 = 24 \text{ mm} \) and \( \sigma = 0.01 \). This is a common assumption in many production processes. The process mean is assumed to shift due to external reasons. It is assumed that the magnitude of the shift due to external reasons is \( \delta_E = 1.5 \), and the magnitude of the shift due to machine failure is \( \delta_{M/C} = 0.6 \). The initial values of necessary parameters are given in Table 2.

To study the effects of some of the model parameters, a sensitivity analysis is performed with the illustrative example shown in Section 6. In Table 3, level 1 is the basic level which was used to solve the example in Section 6. Levels 2 and 3 represent the values of these parameters at +10% and +20% of the basic level respectively.

### Table 1. Simulation and regression result for number of failures as a function of preventive maintenance interval.

<table>
<thead>
<tr>
<th>Preventive maintenance interval</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
<th>3500</th>
<th>4000</th>
<th>4500</th>
<th>5000</th>
<th>5500</th>
<th>6000</th>
<th>6500</th>
<th>7000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of failures through simulation</td>
<td>7</td>
<td>19</td>
<td>29</td>
<td>38</td>
<td>47</td>
<td>52</td>
<td>61</td>
<td>63</td>
<td>65</td>
<td>68</td>
<td>71</td>
<td>76</td>
<td>82</td>
<td>87</td>
</tr>
</tbody>
</table>

Thus, the length of an interval for which the equipment functions depends on the virtual age of the equipment at the beginning of the interval. Note that perfect repair \( a = 0 \), and minimal repair \( a = 1 \) are both special cases of this virtual age model.

For a given time-to-failure distribution, restoration factor, and maintenance policy, the number of failures can be obtained through simulation. However, to obtain the optimal preventive maintenance interval for a machine, the number of failures needs to be estimated every time the preventive maintenance interval changes. Thus, it becomes very time-consuming. In order to reduce the simulation effort, a regression-based approach is used. In the regression analysis, the number of failures is obtained as a function of preventive maintenance interval.

### 6. Numerical example

Equation (1) indicates that optimising the four variables \( n^*, h^* , k^* , t_{PM}^* \) is not a simple process. In this section, we present a numerical example to illustrate the nature of the solution obtained from the economic design of the proposed integrated model.

To illustrate, consider a single component operating as a part of a machine. Machine failure is assumed to follow a two-parameter Weibull distribution with \( \eta = 1000 \) and \( \beta = 2.5 \) as the characteristic life and shape parameter respectively. The machine considered here is expected to operate for three shifts of seven hours each for six days in a week. Time to execute a preventive maintenance action \( (T_{PM}) = 7 \) time units with restoration factor \( (RF_{PM}) = 0.6 \) (it implies 60% restoration of life and sets the age of the block to 40% of the age of the block at the time of the maintenance action) and time to execute corrective maintenance action \( (T_{CM}) = 12 \) time units with restoration factor \( (RF_{CM}) = 0 \) (repair is minimal, i.e. the age of a repaired machine is the same as its age when it failed). The time to failure for the component was obtained through simulation. The ‘Kijmas’ model was used to calculate the virtual age of the component after corrective and preventive actions. Multiple simulation runs were used to determine the expected number of failures in the given evaluation period. A regression model of form \( (N_f = a \times t_{PM}^b) \) was fitted to results obtained from the simulation. Table 1 shows the simulation and regression results. The regression model is given below:

\[
N_f = 0.0437 \times (PM)^{0.8703}
\]

(28)

The \( R^2 \) value of the model is 0.97. This indicates a good fit.

A hypothetical example is illustrated in this section to analyse the proposed integrated model. It is assumed that the \( X \) control chart is used to monitor a CTQ characteristic with a bilateral tolerance limit of equal value \( A = 0.05 \). Assume that the process, in its in-control state, is characterised by a process mean of \( \mu_0 = 24 \text{ mm} \) and a process standard deviation of \( \sigma = 0.01 \) and that the magnitude of the shift owing to external reasons is \( \delta_E = 1.5 \) and owing to machine failure is \( \delta_{M/C} = 0.6 \), which occurs at random and results in a shift of process mean from \( \mu_0 \) to \( \mu_0 + \delta \sigma \). The initial values of necessary parameters are given in Table 2.

Maple 13 has been used to solve the optimisation problem and the numerical results are summarised in Figure 3, which illustrates the effect of each of the four decision variables on the expected total cost of system per unit time \( [ETCPUT]_{M\sigma Q} \). Using the global optimisation technique, the optimal values of decision variables that minimise the expected total cost of system per unit time \( [ETCPUT]_{M\sigma Q} \) are as follows: \( n^* = 14 \), \( h^* = 3.5 \), \( k^* = 9 \), \( t_{PM}^* = 198 \), and the corresponding expected total cost of system per unit time is \( [ETCPUT]_{M\sigma Q} = 376 \).

### 7. Sensitivity analysis

In order to study the effects of some of the model parameters, a sensitivity analysis is performed with the illustrative example shown in Section 6. In Table 3, level 1 is the basic level which was used to solve the example in Section 6. Levels 2 and 3 represent the values of these parameters at +10% and +20% of the basic level respectively.
The optimum value of the decision variables and the corresponding values of the objective function at different levels of the model parameters are shown in Table 4. Case 1 shows the basic model results, and the subsequent two cases show the results at levels 2 and 3 of each of the nine model parameters given in Table 3. Thus, a total of 19 cases are solved to study the sensitivity of model parameters as given in Table 4. The following observations are made from the analysis.

- Cases 1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19 suggest that $T_1$, $T_0$, $T_{\text{resetting}}$, $C_{\text{Frej}}$, $A$, and $C_f$ have little impact on the optimal sample size, sample frequency, width of control limit, PM interval, and expected...
total cost of the system. This suggests that the economic model is quite robust to variation in these parameters.

- Cases 1, 2, 3, 4, and 5 study the relative sensitivity of the magnitude of the shift in the process mean owing to assignable causes attributed to external reasons and machine failure with failure mode 2, i.e. \( \delta_E \) and \( \delta_{M/C} \) respectively. The smaller the shift, the harder it is to detect. Thus, the smallest value of \( \delta_E \) in Table 4 leads to the largest sample size and the longest sampling interval. The percentage increase in the objective function value for a 10 and 20% increase in \( \delta_E \) are 8% and 16.32% respectively. From Table 4, an increase in \( \delta_{M/C} \) results in a decrease in the optimal value of \( n \) and an increase in the values of \( h \) and \( k \). The percentage decrease in the objective function value for a 10 and 20% increase in \( \delta_{M/C} \) are 6.2% and 11.13% respectively. Thus, the results show that the optimum design is most sensitive to errors in estimating the magnitude of the process shift owing to external reasons \( (\delta_E) \) and owing to machine failure \( (\delta_{M/C}) \).

### 8. Conclusion

This paper opens up a promising research area and suggests that maintenance and process quality control decisions, when considered jointly, can lead to better process performance. An integrated model using Taguchi loss function is
developed for the joint optimisation of preventive maintenance interval and quality control policy of the process subject to machine failures and quality shifts. A numerical example is given to validate the model and to examine the effect of $n$, $h$, $k$, and $r_{PM}$ on the total expected cost of the system.

A sensitivity analysis of the model is presented to study the effect of various model parameters on the behaviour of the system. This may help the manufacturer to identify the parameters which are more sensitive from those which are not. Thus, the methodology developed in this paper is a step towards better planning through joint consideration of maintenance and process quality-control policy.

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References


