# Reversible reactions and relaxation kinetics 

CML103

Class 5
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## All chemical reactions are equilibrium reactions

$\mathrm{A} \underset{k_{-1}}{\stackrel{k_{1}}{\rightleftharpoons}} \mathrm{P}$

## In equilibria concentrations approach

 equilibrium values over time

$\frac{\mathrm{d}[\mathrm{A}]}{\mathrm{d} t}=$ Rate of formation of A - Rate of consumption of A
$\frac{\mathrm{d}[\mathrm{A}]}{\mathrm{d} t}=\operatorname{Rate}_{1}+\operatorname{Rate}_{2}+\operatorname{Rate}_{3}-\operatorname{Rate}_{4}-$ Rate $_{5}$

$$
\mathrm{A} \underset{k_{-1}}{\stackrel{k_{1}}{k_{-1}}} \mathrm{P}
$$

$$
\begin{gathered}
\mathrm{P} \xrightarrow{k_{-1}} \mathrm{~A} \xrightarrow{k_{1}} \mathrm{P} \\
\frac{\mathrm{~d}[\mathrm{~A}]}{\mathrm{d} t}=k_{-1}[\mathrm{P}]-k_{1}[\mathrm{~A}]
\end{gathered}
$$

$$
\mathrm{A} \xrightarrow{k_{1}} \mathrm{P} \xrightarrow{k_{-1}} \mathrm{~A}
$$

$$
\frac{\mathrm{d}[P]}{\mathrm{d} t}=k_{1}[\mathrm{~A}]-k_{-1}[\mathrm{P}]=-\frac{\mathrm{d}[A]}{\mathrm{d} t}
$$

## Opposing reactions have a simple analytical solution

$$
\begin{gathered}
\frac{\mathrm{d}[\mathrm{P}]}{\mathrm{d} t}+\frac{\mathrm{d}[A]}{\mathrm{d} t}=\mathrm{o} \Longrightarrow[\mathrm{~A}]+[\mathrm{P}]=[\mathrm{A}]_{0} \\
\frac{\mathrm{~d}[A]}{\mathrm{d} t}=k_{-1}[\mathrm{~A}]_{0}-\left(k_{1}+k_{-1}\right)[\mathrm{A}] \\
\ln \left(\frac{k_{-1}[\mathrm{~A}]_{0}-\left(k_{1}+k_{-1}\right)[\mathrm{A}]}{k_{1}[\mathrm{~A}]_{0}}\right)=-\left(k_{1}+k_{-1}\right) t \\
{[\mathrm{~A}]=\frac{[\mathrm{A}]_{0}}{k_{1}+k_{-1}}\left(k_{-1}+k_{1} \mathrm{e}^{-\left(k_{1}+k_{-1}\right) t}\right)}
\end{gathered}
$$

to be studied by conventional means
$k_{1}=1.4 \times 10^{11} \mathrm{dm}^{3} \mathrm{~mol}^{-1} \mathrm{~s}^{-1}$ and $k_{-1}=$
$2.5 \times 10^{-5} \mathrm{~s}^{-1} \Longrightarrow K \approx 10^{15} \mathrm{dm}^{3} \mathrm{~mol}^{-1}$.
In pure water, $\left[\mathrm{H}^{+}\right]_{0}=\left[\mathrm{OH}^{-}\right]_{0}=$
$1 \times 10^{-7} \mathrm{dm}^{-3} \mathrm{~mol}, t_{1 / 2}=7.1 \times 10^{-5} \mathrm{~s}$ is
less than mixing time.

Approach to equilibrium after a perturbation to study fast reactions


Time

$$
\begin{gathered}
\mathrm{A} \stackrel{k_{1}}{\underset{k_{-1}}{ }} \mathrm{P} \\
\frac{\mathrm{~d}[\mathrm{~A}]}{\mathrm{d} t}=k_{-1}(T)[\mathrm{P}]-k_{1}(T)[\mathrm{A}]
\end{gathered}
$$

At equilibrium,

$$
\begin{gathered}
\mathrm{o}=k_{-1}(T)[\mathrm{P}]_{\mathrm{eq}}-k_{\mathrm{I}}(T)[\mathrm{A}]_{\mathrm{eq}} \\
K(T)=\frac{k_{1}}{k_{-1}}=\frac{[\mathrm{P}]_{\mathrm{eq}}}{[\mathrm{~A}]_{\mathrm{eq}}}
\end{gathered}
$$

$$
\begin{gathered}
\frac{\mathrm{d}[\mathrm{~A}]}{\mathrm{d} t}=k_{-1}[\mathrm{P}]-k_{1}[\mathrm{~A}] \\
{[\mathrm{A}]=[\mathrm{A}]_{\mathrm{eq}}+\Delta \mathrm{A} \quad[\mathrm{P}]=[\mathrm{P}]_{\mathrm{eq}}+\Delta \mathrm{P}} \\
\frac{\mathrm{~d}\left([\mathrm{~A}]_{\mathrm{eq}}+\Delta \mathrm{A}\right)}{\mathrm{d} t}
\end{gathered}
$$

Stoichiometry of the reaction demands that $\Delta \mathrm{A}=-\Delta \mathrm{P}$.

$$
\frac{\mathrm{d} \Delta \mathrm{~A}}{\mathrm{~d} t}=-\left(k_{-1}+k_{1}\right) \Delta \mathrm{A}
$$

Exponential approach of the displacement from equilibrium to its equilibrium value.

$$
\Delta \mathrm{A}=\Delta \mathrm{A}_{0} \mathrm{e}^{-t / \tau}
$$

with a time-constant (relaxation time)

$$
\tau=\frac{1}{\left(k_{-1}+k_{1}\right)}
$$

## Experimental results from a pressure jump relaxation study



Relaxation time for $\mathrm{H}^{+}+\mathrm{OH}^{-} \rightleftharpoons \mathrm{H}_{2} \mathrm{O}$

$$
\frac{\mathrm{d}\left[\mathrm{H}_{2} \mathrm{O}\right]}{\mathrm{d} t}=k_{1}\left[\mathrm{H}^{+}\right]\left[\mathrm{OH}^{-}\right]-k_{-1}\left[\mathrm{H}_{2} \mathrm{O}\right]
$$

Steps:

- $\left[\mathrm{H}^{+}\right]=\left[\mathrm{H}^{+}\right]_{\text {eq }}+\Delta \mathrm{H}^{+}$and $\ldots$.
(2) Use $\Delta \mathrm{H}^{+}=\Delta \mathrm{OH}^{-}=-\Delta \mathrm{H}_{2} \mathrm{O}$
relaxation time, $\tau, \frac{1}{\left(k_{-1}+k_{1}\left(\left[\mathrm{H}^{+}\right]_{\mathrm{eq}}+\left[\mathrm{OH}^{-}\right]_{\mathrm{eq}}\right)\right)}$


## Quiz on relaxation kinetics

Calculate the relaxation time when 0.015 M
benzoic acid is subjected to a temperature-jump experiment to a final temperature of 298 K. For the reaction

$$
\mathrm{H}^{+}+\mathrm{C}_{6} \mathrm{H}_{5} \mathrm{COO}^{-} \stackrel{k_{1}}{\underset{k_{-1}}{ }} \mathrm{C}_{6} \mathrm{H}_{5} \mathrm{COOH}
$$

the rate constants are $k_{1}=$
$3.5 \times 10^{10} \mathrm{dm}^{3} \mathrm{~mol}^{-1} \mathrm{~s}^{-1}$ and $k_{-1}=$
$2.2 \times 10^{6} \mathrm{~s}^{-1}$

