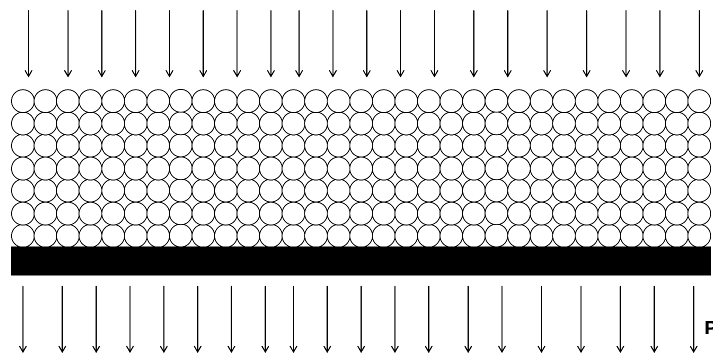
Tutorial Sheet 3 11.09.07

1. The process of filtration is shown schematically in figure below. In constant pressure filtration, the pressure difference p_s-p_l is kept constant. Suppose that a slurry contains a volume fraction ϕ of essentially spherical particles of diameter D_p which pack into a filter cake with void fraction ε. Obtain a design equation for the height of the filter cake as a function of time. You should assume that the filter cloth is a porous medium whose pressure drop-velocity equation is

$$\Delta p = -K_f \; \mu v_\infty$$

and that the low Reynolds number asymptote applies at all times in the filter cake.



2. A bed of spheres of identical diameter Dp and density ρ_p is fluidized by a Newtonian liquid of density ρ and viscosity μ. The fluidization is uniform; i.e., the void fraction ε is the same everywhere. Estimate the void fraction of the fluidized bed. The empirical Richardson-Zaki equation for the velocity V_{sw} of a uniform swarm of particles with respect to the surrounding fluid is expressed in terms of the Stokes settling velocity of single sphere, V_{st}, as

$$V_{sw} = V_{st}\, \varepsilon^n$$

n = 4.65 for Re < 0.1. Compare your result to the rule of thumb $\varepsilon^3/(1 - \varepsilon) = 0.091$.

3. The following data were obtained for a plate-and-frame filter of total area A = 500 sq cm. operating under a constant pressure drop of Δp = 0.1 atm:

Time t (min):	50	75	100
Volume of filtrate V (liter):	9.7	12.3	14.0

The filtrate is essentially water. The volume of the cake is one-tenth the volume of the filtrate passed. The resistance of the filter medium may be neglected.

(a) Make an appropriate plot and estimate the permeability k (darcies) of the cake.

(b) At a certain time t after start-up, the filter is shut down. There follows a cleaning time t_c, in which the accumulated cake is removed, the cloth is cleaned and the filter is reassembled. This cyclical pattern of productive operation, followed by cleaning, etc., is continued indefinitely. If t_c = 30 min, what value of t will maximize the average volume of filtrate produced per unit time?

(c) What is the value (liter/min) of this maximum average volumetric flow rate of filtrate?

4.The production of a filtrate from a compressible sludge is maintained by using both a plate-and-frame filter and a continuously operating rotary vacuum filter. The plate-and-frame filter produces 50 gpm of filtrate averaged over a day, using a slurry pump with a capacity of 100 gpm fitted with a relief valve to insure that the pressure does not exceed 60 psig. The time needed to clean the filter is 25 min.

The rotary vacuum filter has a diameter of 5 ft and a width of 5 ft, and the internal segments are arranged so that 20% of the total filtering surface is effective at any time. The filter drum rotates at half a revolution each minute, and the filtrate is produced at 25 gpm when the pressure inside the drum equals 8.2 psia, and at 30 gpm when the drum pressure is reduced to 3.2 psia. The slurry trough is at atmospheric pressure.

What is the minimum surface area required for the plate-and-frame filter? Assume that the rate of filtration is given by:

$$\frac{dV}{dt} = \frac{c\,(\Delta p)^n}{V},$$

where V is the amount of filtrate produced per unit area of filter in time t, Δp is the pressure difference across the filter, c and n are constants.

5. Consider the centrifugal filter shown in the lecture class with the same notations and equations given there, prove that the pressure in the cake and slurry are given in dimensionless form by:

Slurry: $P = R^2 - R_1^2$; Cake: $P = R^2 - 1 + \frac{(1-R_1^2)\ln R}{\ln R_a}$.

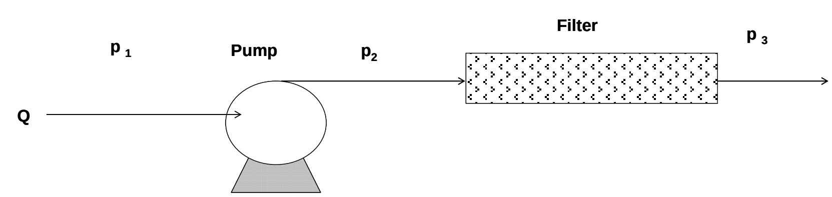
Here, $P = 2p/\left(\rho\omega^2 r_2^2\right)$, $R = r/r_2$, and subscripts 1 and a correspond to radii of r_1 and a, respectively.

6. . Consider the centrifugal filter shown in the lecture class with the notations and equations given there. If the flow rate of filtrate is steady and there is initially no cake, prove that after a time t the inner radius of the slurry is given by:

$$r_1 = \sqrt{r_2^2 - \frac{\mu Q}{\pi \kappa H \rho \omega^2} \ln \frac{r2}{\sqrt{r_2^2 - (Q \varepsilon t / \pi H)}}}$$

A centrifugal filter operates with the following values: r_2 = 0.5 m, ρ = 1,000 kg/ m^3, ω = 40π rad / s, κ = 3.2 x 10^{-13} m^2, H = 0.5 m, Q = 0.005 m^3 / s, ε = 0.1, and μ = 0.001 kg / m s. After five minutes, what are the values of r_1 and a?

7. Pumping into a filter – E Figure given below shows a centrifugal pump with pressure increase $p_2 - p_1 = a - bQ^2$ atm, where a and b are known constants, pumping a slurry into a plate-and-frame filter. The filter has a cross-sectional area A cm^2, cake permeability κ darcies [(cm/s) cP / (atm/cm)], filtrate viscosity μ cP, and cake-to-filtrate volumetric ratio ε. A total volume V cm^3 of filtrate has been passed. The pump inlet and filter exit pressures are equal.



Pump feeding into a filter.

If a = 0.2, b = 10^{-5}, A = 100, κ = 100, μ = 1, ε = 0.1, and V = 10^4, all in units consistent with the above definitions, calculate the current volumetric flow rate Q cm^3 /s. Ignore the slight difference between slurry and filtrate volumes.

8. Derive the general filtration equation for a constant-rate period followed by a constant-pressure period:

$$\frac{t - t_r}{V - V_r} = \frac{r}{A \Delta p}\left[\frac{V_c}{2A}(V - V_r) + \frac{V_c V_r}{A} + \frac{c}{r}\right],$$

Where t is the total filtration time, t_r is the time of filtration at constant rate, V is the total filtrate volume, V_r is the filtrate volume collected in the constant-rate period, A is the total cross-sectional area of the filtration path, r is the specific resistance of the filter cake, c is the resistance coefficient of the filter cloth, V_c is the volume of filter cake formed per unit volume of filtrate collected, and Δ_p is the pressure drop across the filter.

A plate-and-frame filter is to be designed to filter 300 m^3 of slurry in each cycle of operation. A test on a small filter of area 0.1 m^2 at a pressure drop of 1 bar gave the following results:

Time (s):	250	500	750	1,000
Volume of filtrate collected (m^3):	0.0906	0.1285	0.1570	0.1815

Assuming negligible filter-cloth resistance, estimate the filtration area required in the full-scale filter when the cycle of operation consists of half an hour at a constant rate of 2 x 10^{-3} m^3 / s m^2 , followed by one hour at the pressure attained at the end of the constant-rate period. Also evaluate this pressure.