Multi-Comp. Distillation Design Short-Cut Method
Example Problem

A liquid feed of 100 mol/h at the boiling point is fed to distillation column at 405.3 kPa and is to be fractionated so that 95 % of the n-pentane (B) is recovered in the distillate and 95 % of the n-hexane (C) in the bottoms. The feed composition is as follows: n-butane (XA = 0.40), n-pentane (XB = 0.25), n-hexane (XC = 0.20), n-heptane (XD = 0.15).

Calculate the following:
1. Moles per hour and composition of distillate and bottoms
2. Top and bottom temperature of tower
3. Minimum no. of stages for total reflux and distribution of other components in the distillate and bottoms
4. Minimum reflux ratio and number of theoretical stages at R = 1.3 Rm
5. Location of feed tray

[K values are supplied]
K Value

Figure 11.3-2. Equilibrium K values for light hydrocarbon systems at 403.3 kPa (46 atm) absolute.
A Material Balance

\[
\text{Comp B } x_{BF} F = 0.125 (100) \\
= y_{BD} D + x_{BW} W \\
= 25 \text{ moles/h.}
\]

\[
x_{BD} D = 0.95 \times 25 \\
= 23.75 \text{ moles/h.}
\]

\[
x_{BF} F \\
x_{BW} W = 0.05 (25) \\
= 1.25 \text{ moles/h}
\]
\[ \text{COMP C} \]

\[ X_{CF} F = 0.20 (100) = 20 \]
\[ = Y_{CD} D + Y_{CW} W \]
\[ \therefore X_{CW} W = 0.95 (20) \]
\[ = 19.0 \text{ MOLES/h} \]

\[ X_{CF} F \]

\[ Y_{CD} D = 0.05 (20) \]
\[ = 1.0 \text{ MOLES/h} \]
D
\[
\frac{0.15}{1.0} \quad 15 \quad 0.0 \quad 6.0 \quad 6.4 \quad 15 \quad 0.1 \quad 0.4 \quad 25
\]

HEAVY KEY
\[
0.20 \quad 20 \quad 1.00 \quad 0.0154 \quad 19 \quad 0.054
\]

C
\[
0.15 \quad 1.0 \quad 1.0 \quad 0.0 \quad 6.4 \quad 15 \quad 0.1 \quad 0.4 \quad 25
\]

LIGHT KEY
\[
0.25 \quad 25 \quad 23.75 \quad 6.468 \quad 1.25 \quad 0.035
\]

B
\[
0.25 \quad 20 \quad 1.00 \quad 0.0154 \quad 19 \quad 0.054
\]

A
\[
0.15 \quad 1.0 \quad 1.0 \quad 0.0 \quad 6.4 \quad 15 \quad 0.1 \quad 0.4 \quad 25
\]

COMP.

FEED

DI SST.

Bottom

YD 7= YD

\[
X = \frac{120}{8+178} \quad 0.08
\]

\[
X = \frac{120}{8+178} \quad 0.08
\]

\[
X = \frac{120}{8+178} \quad 0.08
\]
\[
\sum_{i=1}^{m} \frac{X_i}{K_i} = \frac{1}{K_C} \sum \frac{X_i}{x_i} = 1
\]

\[
K_C = \sum (Y_i/x_i) \quad x_i = \frac{X_i \times x_i}{\sum Y_i x_i}
\]

<table>
<thead>
<tr>
<th>Comp</th>
<th>X_i D</th>
<th>K_i x_i</th>
<th>\frac{X_i}{x_i}</th>
<th>X_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.6178</td>
<td>1.72</td>
<td>6.745</td>
<td>0.0916</td>
</tr>
<tr>
<td>B</td>
<td>0.3668</td>
<td>0.645</td>
<td>2.529</td>
<td>0.1450</td>
</tr>
<tr>
<td>C</td>
<td>0.0154</td>
<td>0.26</td>
<td>0.8</td>
<td>0.0154</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0.095</td>
<td>0.373</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
K_C (\text{cal.}) = 0.252
\]

66 °C

SAME AS REPORTED
**Bottom Temp. of Toner (Bubble Point)**

\[ \sum Y_i = \sum k_i x_i = k_c \sum d_i x_i + 10 \]

\[ = k_c \sum d_i x_i = 1.0 \]

\[ k_c = \frac{1}{\sum d_i x_i} \]

**Trial 134 °C**

<table>
<thead>
<tr>
<th>Comp</th>
<th>X_i</th>
<th>W</th>
<th>K_i</th>
<th>d_i</th>
<th>( k_i \cdot X_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>5.20</td>
<td>437</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0.0355</td>
<td>2.45</td>
<td>2.059</td>
<td>0.073</td>
<td>0.028</td>
</tr>
<tr>
<td>C</td>
<td>0.539</td>
<td>1.19</td>
<td>1.00</td>
<td>0.539</td>
<td>0.641</td>
</tr>
<tr>
<td>D</td>
<td>0.4255</td>
<td>0.64</td>
<td>0.538</td>
<td>0.2288</td>
<td>0.244</td>
</tr>
</tbody>
</table>

\[ k_c (\text{calc}) = 1.189 \]

Same as (\( \approx \)) 1.193 (134 °C)
MINIMUM NO STAGES

\[ d_{LD} = 2.529 \text{ (66°C) } \] \[ x_{LN} = 2.059 \text{ (134°C) } \]

\[ d_{L,an} = \sqrt{2.529 \times 2.059} = 2.282 \]

Fenske's Eq:

\[ N_{m} = \frac{\log \left( \frac{d_{LD}}{d_{RD}} \right) \left( \frac{x_{HN}}{x_{LN}} \right)}{\log (d_{L, an})} \]

\[ N_{m} = \log \left( \frac{0.36 \times 64.75}{0.04 \times 64.75} \right) \left( \frac{0.539 \times 35.25}{0.0355 \times 35.25} \right) \]

\[ = 7.139 = 7.14 \text{ (theoretical stages) } \]
Calculation of Trace Element

<table>
<thead>
<tr>
<th></th>
<th>$x_D$</th>
<th>$x_N$</th>
<th>$\sqrt{x_D x_N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.745</td>
<td>4.37</td>
<td>5.429</td>
</tr>
<tr>
<td>B</td>
<td>2.529</td>
<td>2.059</td>
<td>2.282</td>
</tr>
<tr>
<td>C</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>D</td>
<td>0.373</td>
<td>0.538</td>
<td>0.448</td>
</tr>
</tbody>
</table>

Modified Penske's Eqtn.

$$\frac{x_{AD} D}{x_{AN} N} = \left(\frac{x_{A,AN}}{x_{A,D}}\right)^N \frac{X_{HD} D}{X_{HW} W}$$

$$= \left(5.429\right)^{0.015 \times 6.175} \left(0.539 \times 3.525\right)$$

$$= 9229$$

Overall balance on A

$$x_{AF} = 40 = x_{AD} D + x_{AN} N$$

$$= 9229 x_{AN} N + x_{AN} N$$

$$x_{AN} N = 0.0043 \text{ mol}$$

$$x_{AD} D = 39.96 \text{ mol}$$
\[
\frac{\text{Comp on}}{D} = \frac{X_{DD}D}{X_{DN}^N} = (0.488) \left(\frac{0.0154 \times 64.75}{0.539 \times 35.25}\right)
\]

\[= 1.710 \times 10^{-4}\]

**Overall balance on D**

\[X_{DF} F = 15'000 = X_{DD}D + X_{DN}^N\]

\[= 1.710 \times 10^{-4} X_{DN}^N + X_{DN}^N\]

\[X_{DN}^N = 14.9974\]

\[X_{DD}D = 0.0026\]
New Compositions

<table>
<thead>
<tr>
<th>Comp.</th>
<th>Distillate D</th>
<th>Bottom W</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_D = x_D$</td>
<td>$x_{DW}$</td>
</tr>
<tr>
<td>A</td>
<td>0.61776</td>
<td>39.986</td>
</tr>
<tr>
<td>B</td>
<td>0.3668</td>
<td>23.75</td>
</tr>
<tr>
<td>C</td>
<td>0.0154</td>
<td>1.00</td>
</tr>
<tr>
<td>D</td>
<td>0.00004</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

No change in top and bottom temp. (traces are too small)
\[
\text{Min. ReFLUX RATIO} \\
1 - q = \frac{\sum x_i u_i}{x_i - \theta}
\]

\[q = 1.0 \quad (\text{feed at boiling point})\]

\[T_{av} = \frac{(T_{top} + T_{bottom})}{2} = \frac{(66 + 134)}{2} = 100^\circ C\]

\[x_{IF} \quad x_{i(000^\circ C)} \quad x_i \quad x_D\]

\[
\begin{array}{cccccc}
A & 0.40 & 3.15 & 5.25 & 0.6177 \\
B & 0.25 & 1.38 & 2.30 & 0.3668 \\
C & 0.20 & 0.60 & 1.00 & 0.0154 \\
D & 0.15 & 0.285 & 0.475 & 0.0004 \\
\end{array}
\]

\[1.0 \quad \frac{\sum x_i w_i}{x_i - \theta} = \frac{1.0}{\text{Rmod } \theta}\]
Reflux Ratio Control:

\[ 1 - q = 1 - 1 = 0 = \frac{5.25 \times (0.4)}{5.25 - 0} + \frac{2.3 \times (0.25)}{2.3 - 0} + \frac{1.0 \times (0.20)}{1 - 0} + \frac{0.475 \times (0.15)}{0.475 - 0} \]

By trial and error method
\[ \theta = 1.2104 \quad \text{(In between } \theta_B \text{ and } \theta_C \text{)} \]

Use computer program → Algorithm

- Newton's Method
- Bisection Method
- Secant Method
Min. Reflux Ratio

Underwood Eqn.

\[ R_m + 1 = \frac{\sum di x_i D}{dx_i - D} \]

\[ = \frac{5.25 (0.6177)}{5.25 - 1.2104} + \frac{2.3 (0.3668)}{2.3 - 1.2104} + \]

\[ = \frac{1.0 (0.0154)}{1 - 1.2104} + \frac{0.475 (0.00004)}{0.475 - 1.2104} \]

\[ R_m + 1 = 1.5039; \quad R_m = 0.504 \]

\[ R = 1.3 \quad R_m = 0.6552 \]
Eqbm. No. Stages

GILLILAND E&N,

$$\frac{N - N_m}{N + 1} = 0.75 \left[ 1 - \left( \frac{R - R_m}{R + 1} \right)^{0.85} \right]$$

$$N = 17 \text{ (Theoretical)}$$

Feed Tray Location

$$\log \frac{M}{P} = 0.206 \log \left[ \frac{x_{HF}}{x_{LF}} \frac{N}{D} \left( \frac{x_{LN}}{x_{AD}} \right)^2 \right]$$

$$= 0.075$$

$$\frac{M}{P} = 1.18 \text{ if } M + P = 17$$

Feed on q.m. base from Top.