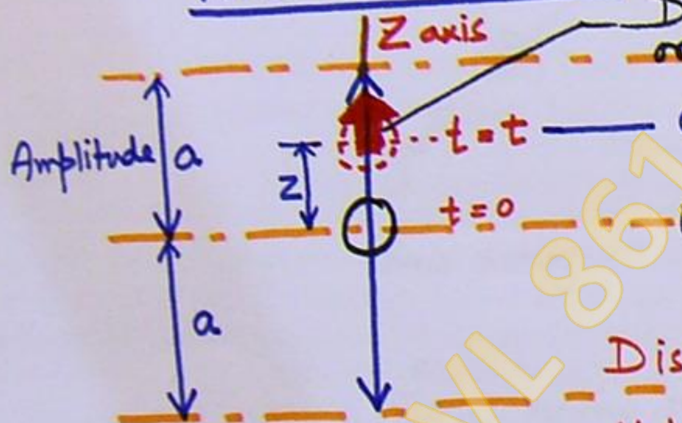
FORCE TRANSMISSION MECHANISMS IN MACHINES

There are two mechanisms → Reciprocating, Rotating. → Generate dynamic forces.

RECIPROCATING



Direction of force acting on fdn.

Current position

Mean position → x-axis

ω_m = Angular frequency (rad/s)

$= 2\pi f_m$ ← Frequency in Hz (no. of oscillations per second)

Disp. $z = a \sin \omega_m t$

Velocity $\dot{z} = a\omega_m \cos \omega_m t$

Accel. $\ddot{z} = -a\omega_m^2 \sin \omega_m t$

$\Rightarrow \ddot{z} = -\omega_m^2 z \Rightarrow$ Acceleration acts opposite to the direction of displacement

$\therefore$ Force acting on particle

$F = m\ddot{z}$

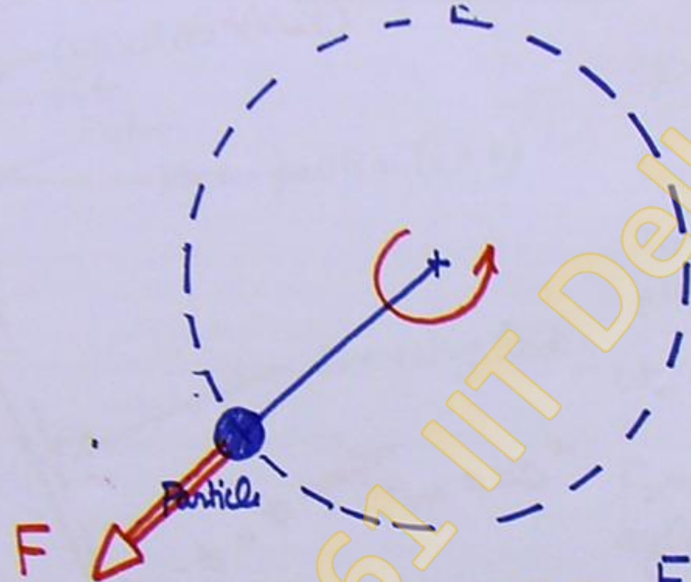
$= -m\omega_m^2 z$

$F = -ma\omega_m^2 \sin \omega_m t$ $\therefore$ Force acting on the particle is always acting opp. to direction of disp.

$\Rightarrow$ ALWAYS directed towards mean position.

Force acting on foundation : Always acting in the direction of disp.

ROTATING



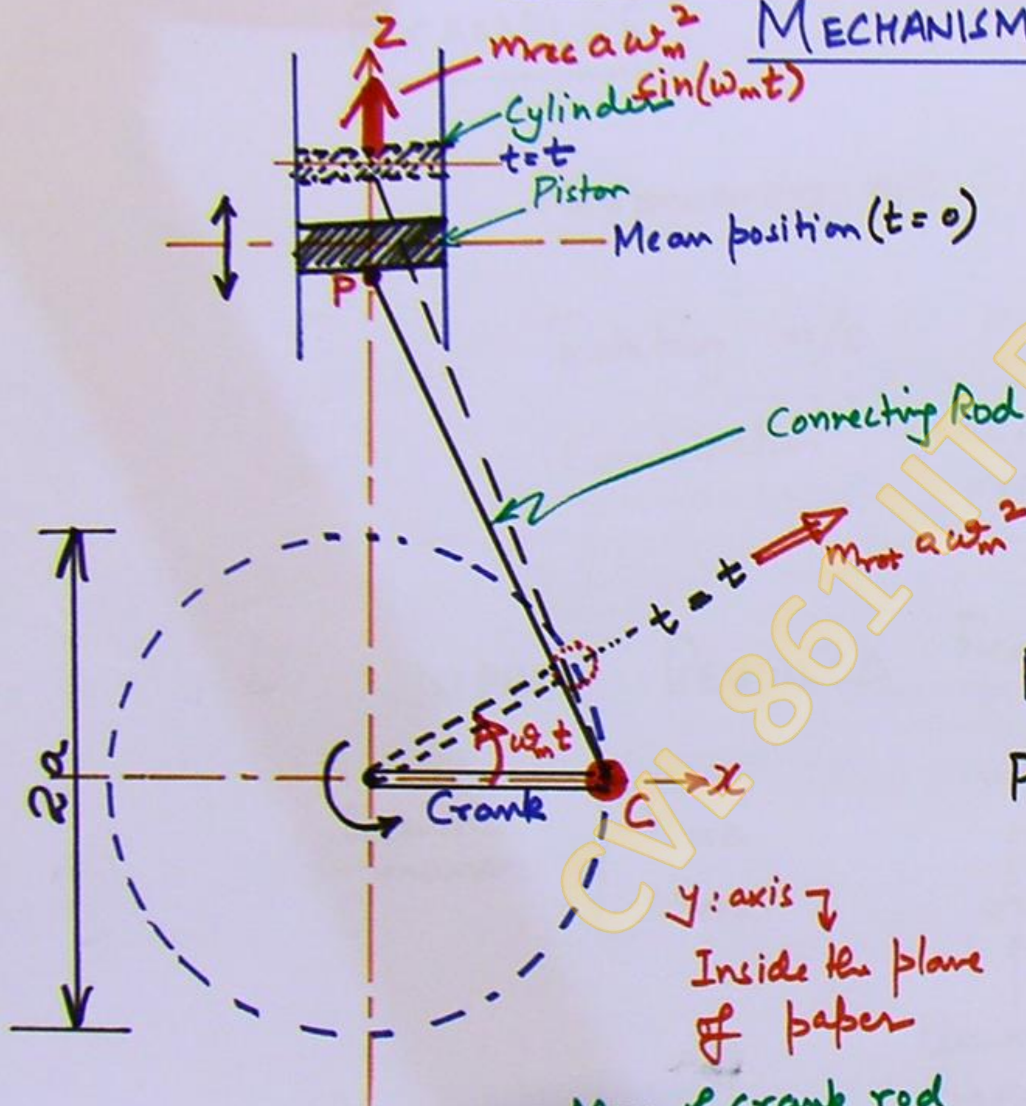
ω_m = Angular frequency.

Force acting on particle = $F = m a \omega_m^2$

Directed towards centre.

Force acting on foundation : Directed from centre to the particle.

OPERATION OF PISTON - CRANK MECHANISM



P: Piston Pin – Linear motion

C: Crank Pin – Circular motion.

Point in between 'P' & 'C' follow elliptical motion.

ω_m = Angular frequency.

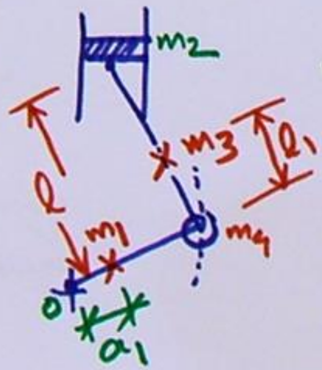
Forces acting on the foundation system.

$$P_z = (m_{rec} + m_{rot})\, a\omega_m^2 \sin(\omega_m t)$$

$$P_x = m_{rot}\, a\omega_m^2 \cos(\omega_m t)$$

m_1 = Mass of crank rod

m_2 = Mass of piston



m_3 = Mass of connecting rod

m_4 = Mass of crank pin.

$$m_{rec} = m_2 + \left(\frac{\ell_1}{\ell}\right) m_3$$

$$m_{rot} = m_4 + \left(\frac{a_1}{a}\right) m_1 + \left(1 - \frac{\ell_1}{\ell}\right) m_3$$

EXAMPLES :

Reciprocating m/c : Internal combustion (I/c) engines.

Rotating m/c : Motors, Turbines.

Combination : I/c engine coupled to crank mechanism.

INPUTS REQUIRED From Manufacturer -

Eccentric masses $\begin{cases} m_{rec} \\ m_{rot} \end{cases}$

ω_m or f_m ← Operating frequency.

a ← Eccentricity.

Piston : 5 kg

Uniform
- Connecting rod : 1 kg → 2.5 m.
- Crank rod : 1 kg → 1 m

Crank pin : 50 kg

100 Hz → f_m

Determine ~~Ers~~ → Amplitude < P_z, P_x

TYPES OF MACHINES

A) Based on operating frequency.

B) Based on mechanism of operation.

C) Based on nature of forces generated by m/c.

D) Based on geometry of fdn.

A) BASED ON FREQUENCY

IS 2974 (III)
- Low frequency $f_m \leq 25\,Hz$ (1500 RPM)
- Medium frequency $25 < f_m \leq 50\,Hz$ ← 3000 RPM
- High frequency $> 50\,Hz$

Low : Eg : Compressors, pumps. $f_m < f_N$. Block type fdn. resting on soil directly.

Medium : Eg : Motors $f_m \approx f_N$ :: Often problematic due to resonance
springs :

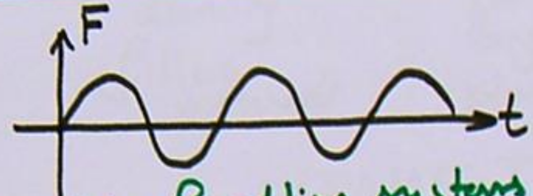
High : Eg. Turbo generators ⟶ Frame type fdn. $\langle f_m > f_N \rangle$

B) BASED ON OPERATING MECHANISM

1. Reciprocating type → eg. I/C engine
2. Rotating type → eg. motors, turbines.
3. Combined type → Reciprocating + Rotating mechanism
   eg. piston coupled to crank system.

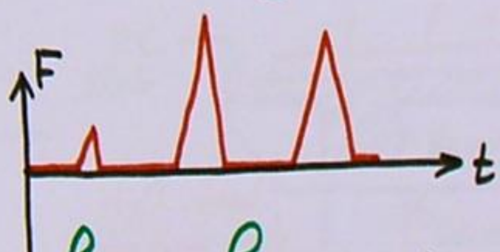
C) BASED ON NATURE OF FORCES GENERATED

1. Periodic forces



eg. Compressors, Air handling systems in bldgs.

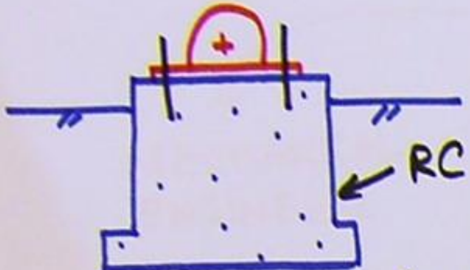
2. Impact type forces



eg. UTM, forge hammers.

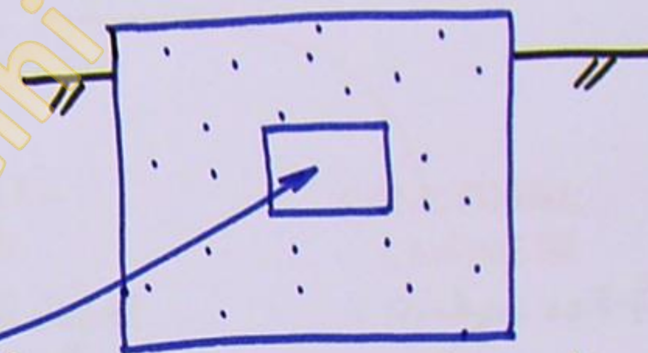
D) BASED ON GEOMETRY OF FOUNDATION

BLOCK TYPE



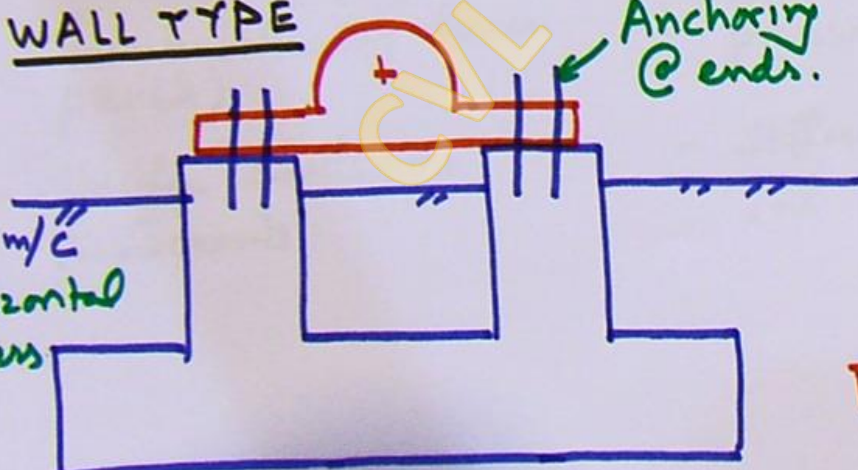
- Low frequency m/c generating periodic/impact forces
- Low to medium frequency/capacity pumps/compressors.

BOX TYPE



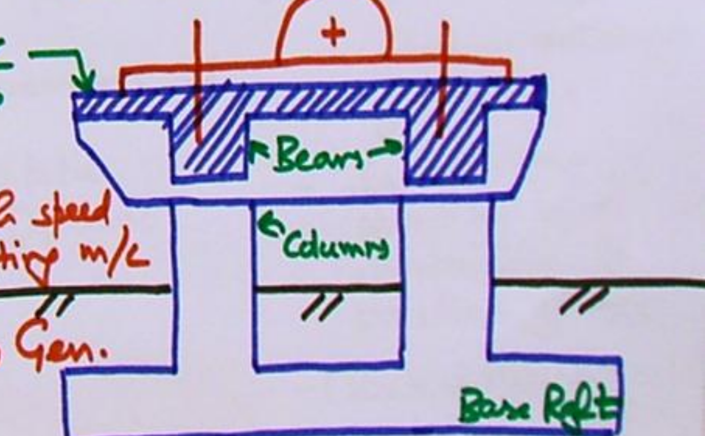
For m/c needing large fdns. in plan.

WALL TYPE



- Large m/c
- eg. Horizontal boilers

FRAME TYPE



High speed rotating m/c

Turbo Gen.

DESIGN PROCESS FOR MACHINE FOUNDATIONS

Multidisciplinary activity

MECHANICAL ENGINEER

- Magnitude, location & direction of machine induced forces.
- Ensure minimal forces generated (apply counterbalancing wherever possible)
- Specify limits on displacements

GEOTECHNICAL ENGINEER

- Location & depth criteria for foundations
- Type of fdn. < Shallow / Deep
- Allowable net bearing pressure (or pile capacities)
- Stiffness parameters of soil
  a) Static
  b) Dynamic

STRUCTURAL ENGINEER

- Analyse soil-fdn system
- Ensure $\sigma < \sigma_{all,net}$
- Vibrations under limit
- Proper anchorage of machine
- Satisfactory operation after construction & erection of m/c
- Check → Measurement

GENERAL DESIGN GUIDELINES

Bldg. fdns. → Predominantly static loads.

Machine fdns. → * Very small static forces.
* High dynamic forces – continuous action.

1. Fdn. should be able to carry static & dynamic forces with adequate FOS against shear failure. $\sigma_{safe-net}$

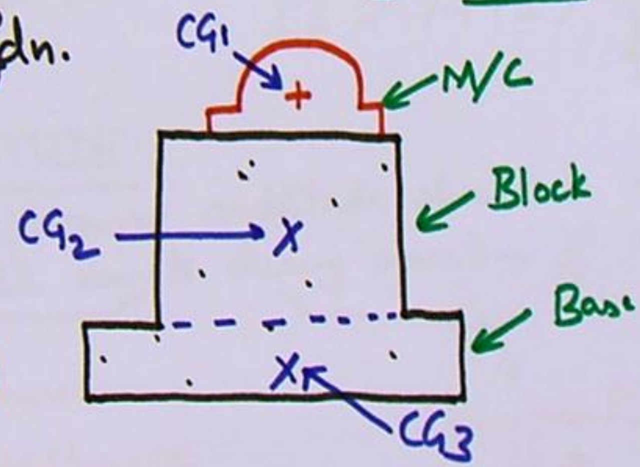
2. Settlement should be within limits $\sigma_{safe-pr.}$

$$\sigma_{all,net} = \text{Lower of } \sigma_{safe-net} \ \& \ \sigma_{safe-pr.}$$

IS 1904

3. Ensure stability of fdn.

CG1, CG2, CG3 should preferably lie on same vertical line.
(max 5% eccentricity)

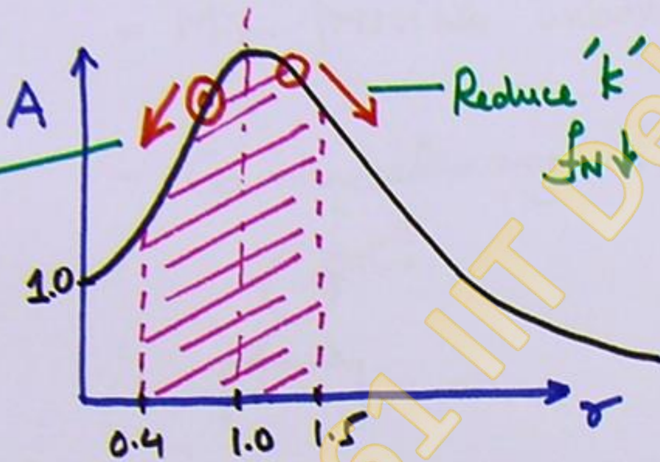


4. RESONANCE

Must be avoided.

$$r = \text{Frequency ratio} = \frac{f_m}{f_N}$$

f_N ← Natural frequency of m/c + fdn. + soil system



Increase 'k' $\Rightarrow f_N \uparrow$

Prohibited Zone.

A = Amplification factor.

$r \leq 0.4$ or $r \geq 1.5$

IS 2974 (I) Block type fdns.

5. VIBRATION AMPLITUDE

Dyn. displacement amplitude should be under permissible limits during operation of m/c.

Specified by manufacturer
eg. $\not> 50\mu m$ @ anchorage points

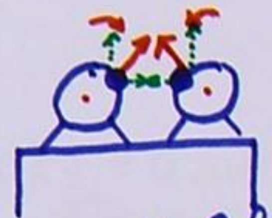
User point of view
- Structural integrity
- User comfort

6. CONTROLLING MACHINE GENERATED FORCES

Mech. engrs. → Manufacturers

- Min. possible unbalanced forces/vibrating/rotating masses.

- Counterbalancing wherever possible.



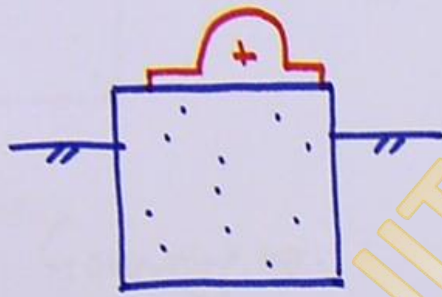
- Net vertical forces only
- Stability due to cancellation of horizontal forces.

7. POSSIBILITY OF FUTURE EXPANSIION

Onus on – Client, manufacturers, structural engineers

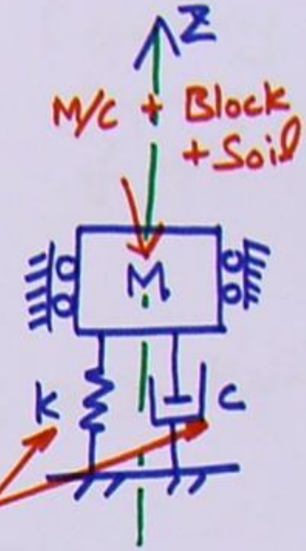
ANALYSIS OF MACHINE-FOUNDATION - SOIL SYSTEM

FREE VIBRATIONS -



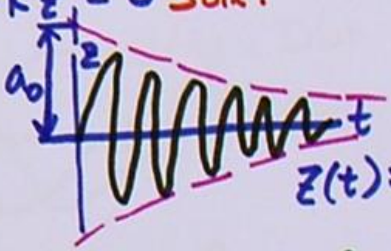
No external forces.

$\equiv$



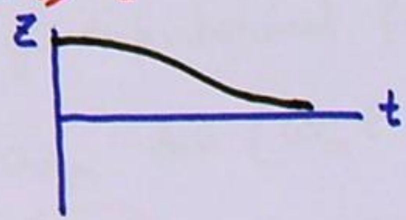
Governing eqn. $M\ddot{z} + c\dot{z} + kz = 0$ Soil.

Solution → Underdamped $c < c_c$
Solution → Overdamped $c \geq c_c$



$z(t) = a_0 e^{-\xi \omega_N t} \sin(\omega_d t)$

ω_d = Damped natural frequency



c_c = Critical damping

$= 2M\omega_N = 2\sqrt{kM}$

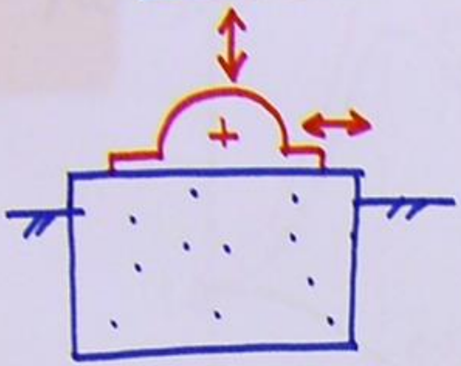
$c = \xi c_c = 2M\omega_N \xi$

$= 2\sqrt{kM}\,\xi$

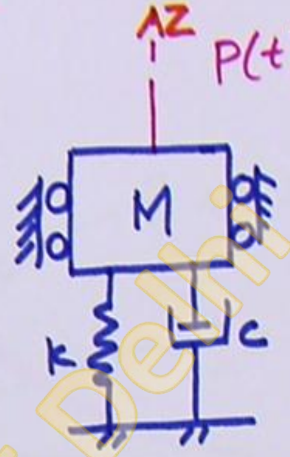
$\omega_d = \omega_N\sqrt{1-\xi^2}$

$\omega_N = \sqrt{\frac{k}{M}}$ $\quad \xi = \frac{c}{c_c}$

FORCED VIBRATIONS



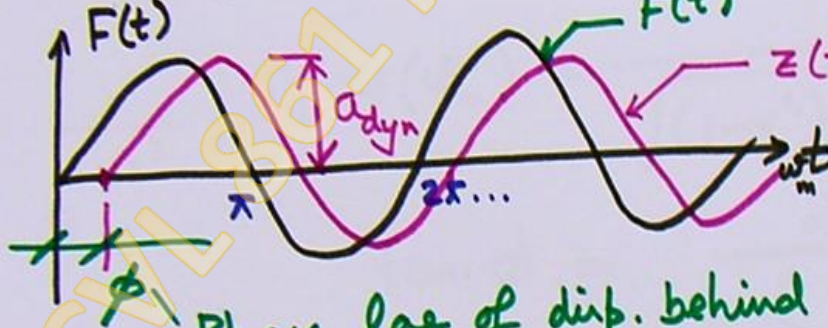
$\equiv$



$P(t) = P_o \sin \omega_m t$

↑ operating frequency of m/c.

Governing eq. : $M\ddot{z} + c\dot{z} + kz = P(t) = P_o \sin(\omega_m t)$



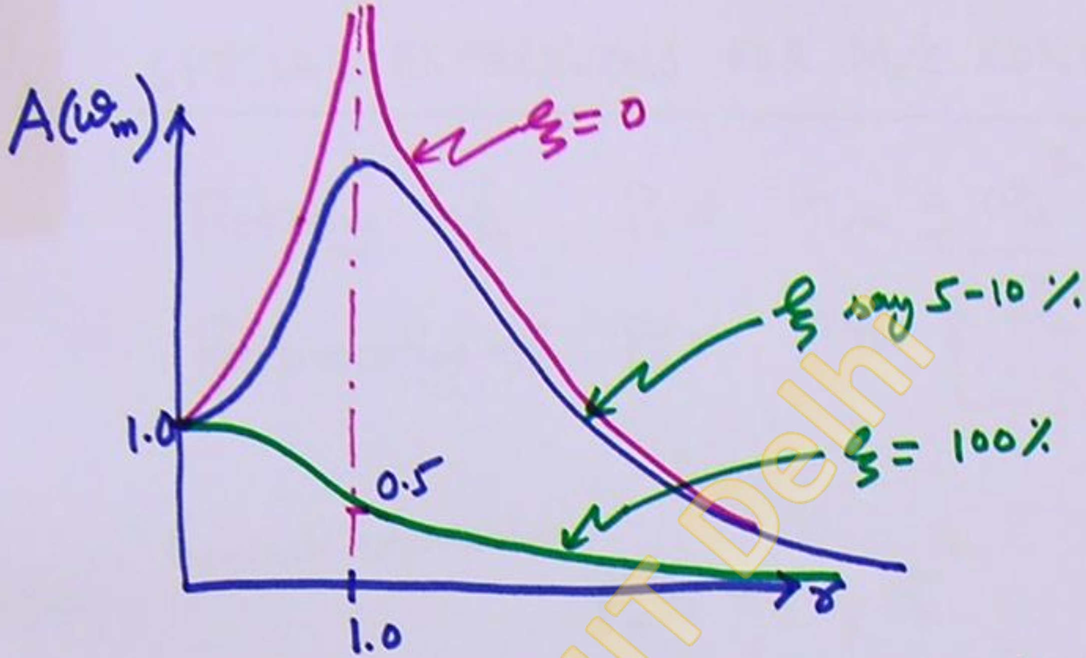
Phase lag of disp. behind force.

Response $z(t) = a_{dyn} \sin(\omega_m t - \phi)$

$$a_{dyn} = a_{static} \times A(\omega_m)$$

$a_{static} = \left(\frac{P_o}{k}\right)$

↑ Amplification factor



<u>If $\xi = 0$</u>

$$A(\omega_m) = \frac{1}{1-r^2}$$

$$\phi = 0$$

$$A(\omega_m) = \frac{1}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$$

$$\tan\phi = \frac{2r\xi}{(1-r^2)}$$

At resonance, $r = 1 \Rightarrow \phi = 90^\circ$

SPECIAL EXPRESSIONS FOR M/C FDNS.

Rotating m/c $\quad P_0 = m_{rot}\, \underset{\text{Radius.}}{a}\, \omega_m^2$

Reciprocating m/c $\quad P_0 = m_{rec}\, \underset{\text{Disp. amplitude.}}{a}\, \omega_m^2$

∴ General expression

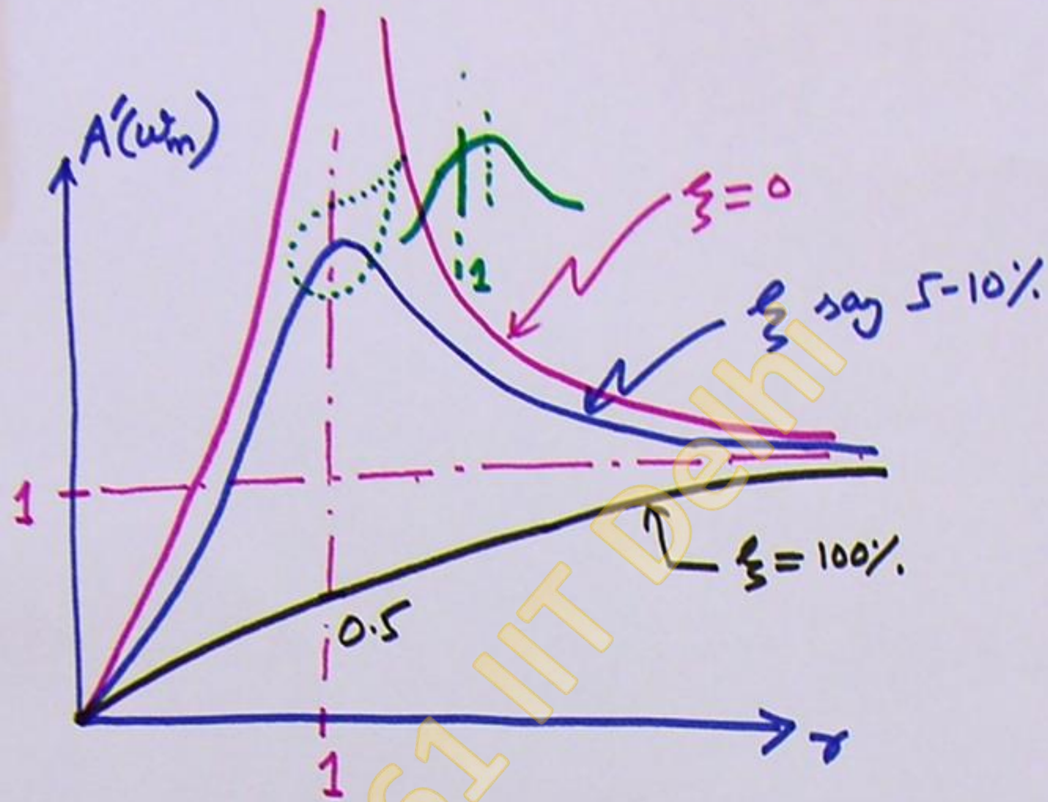
$$P_0 = \underset{\text{Eccentric mass.}}{m_e}\ \underset{\text{Eccentricity.}}{a}\ \omega_m^2$$

$$a_{dyn} = \left(\frac{\overset{m_e a \omega_m^2}{P_0}}{\underset{\omega_N^2 M}{K}}\right) \frac{1}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$$

$$= a\left(\frac{m_e}{M}\right) r^2 \frac{1}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$$

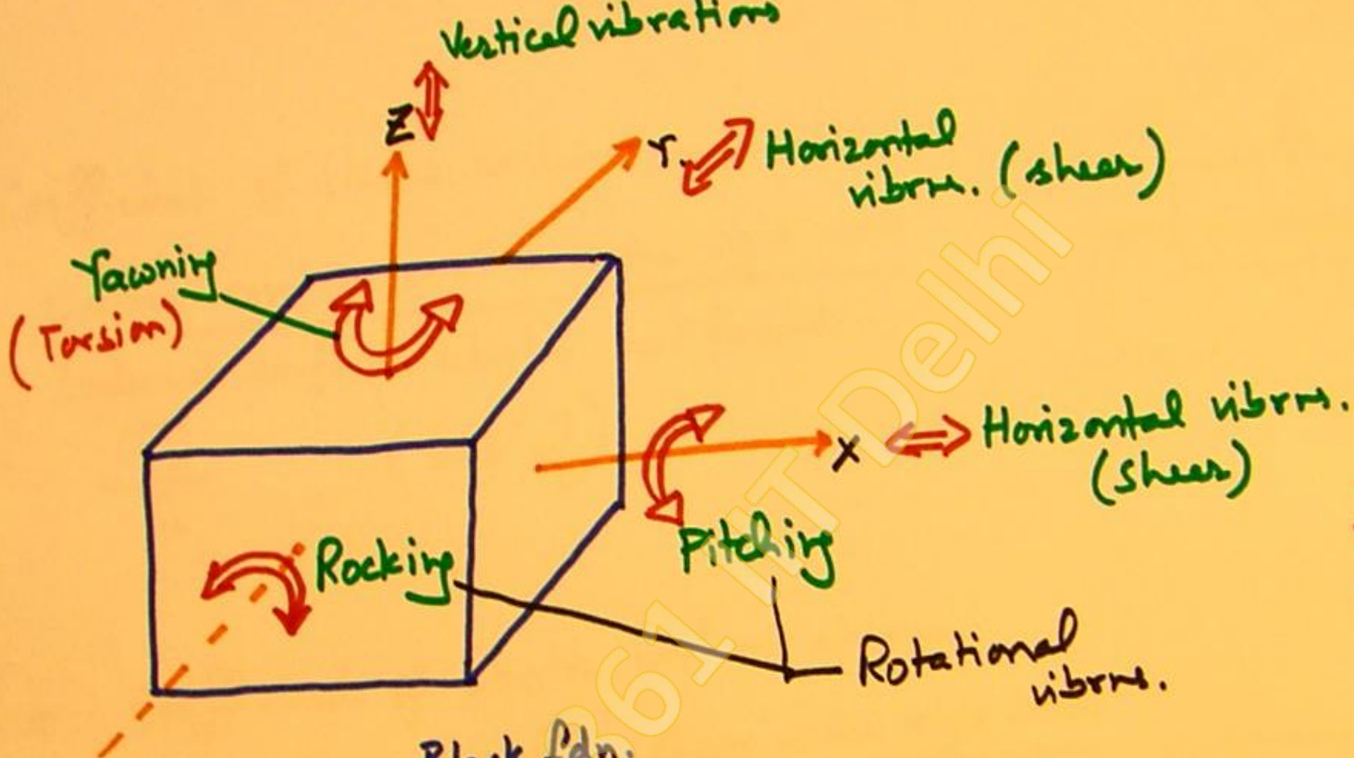
Modified amplification factor

$$\therefore \quad a_{dyn} = a\left(\frac{m_e}{M}\right) A'(\omega_m)$$



$$A'(\omega_m) = r^2 A(\omega_m)$$

PARAMETERS ESSENTIAL FOR ANALYSIS



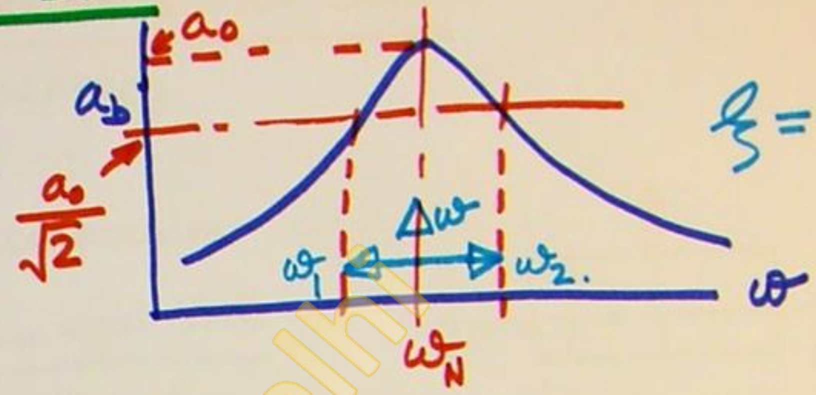
Block fdn.

SIX Degrees of freedom.

- Parameters
  - Soil → Stiffnes, damping
  - Block → Geometrical parameters
    - Area of base
    - Mass moment of inertia
    - Area moment of inertia (base)

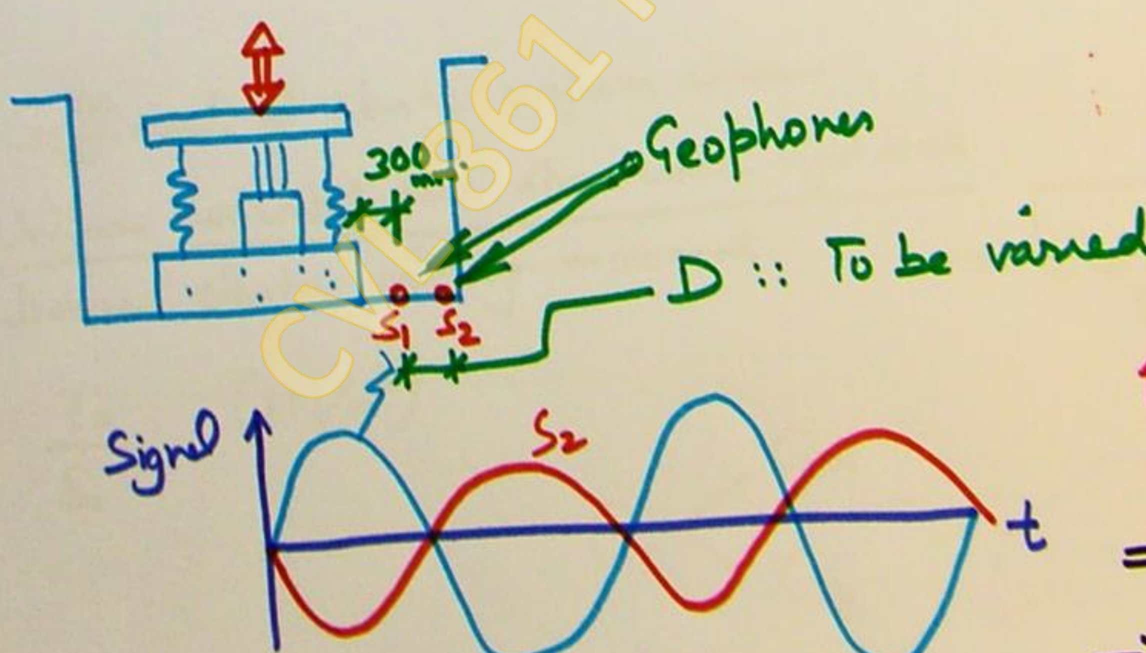
DAMPING OF SOIL

Half-power band method.

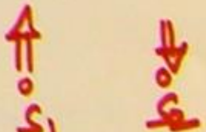


$$\xi = \frac{\Delta\omega}{2\omega_N}$$

INDIRECT METHOD OF MEASURING C_2



Phase diff. = π



$$\Rightarrow \frac{1}{2}\lambda = D$$

$$\Rightarrow \lambda = 2D$$

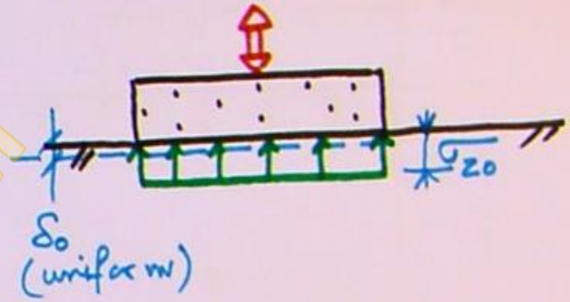
ELASTIC CONSTANTS OF SOIL

(DYNAMIC)

C_z = Coefficient of elastic uniform compression

$$= \frac{\text{Uniform compressive stress under rigid block}}{\text{Induced uniform elastic settlement.}}$$

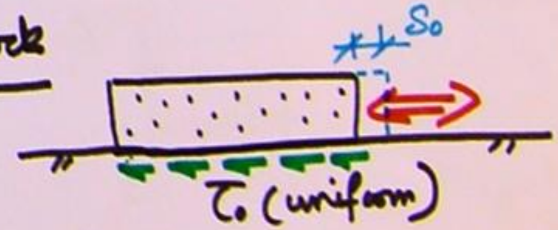
$$= \frac{\sigma_{z0}}{\delta_0} \qquad (kN/m^3)$$



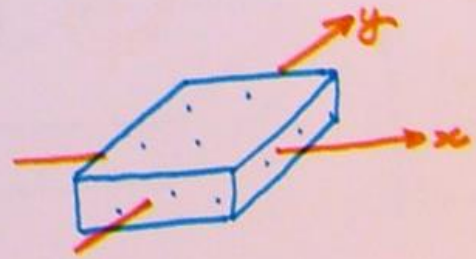
C_τ = Coefficient of elastic uniform shear

$$= \frac{\text{Uniform horizontal shear stress under rigid block}}{\text{Induced elastic sliding movement}}$$

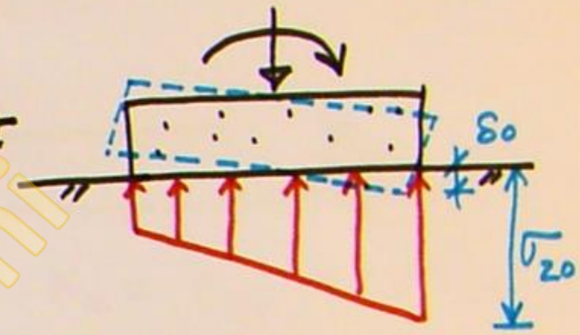
$$= \frac{\tau_0}{\delta_0} \qquad (kN/m^3)$$



C_τ < $C_{\tau x}$, $C_{\tau y}$



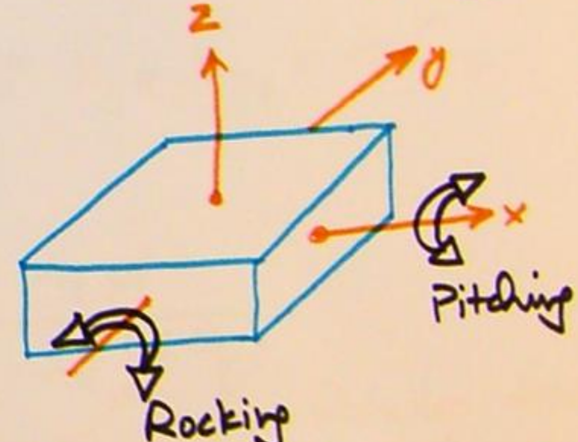
C_θ = Coefficient of elastic non-uniform compression



$$= \frac{\text{Local compressive stress under rigid block}}{\text{Induced elastic settlement at same point}}$$

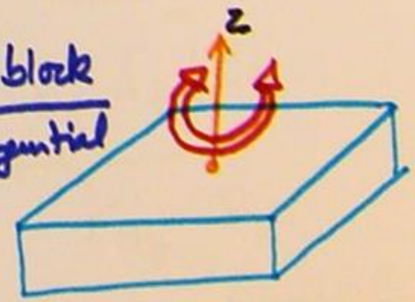
$$= \frac{\sigma_{z0}}{S_0}$$

$$C_\theta \begin{cases} C_{\theta x} \\ C_{\theta y} \end{cases}$$



C_ϕ or C_ψ or $C_{\theta z}$

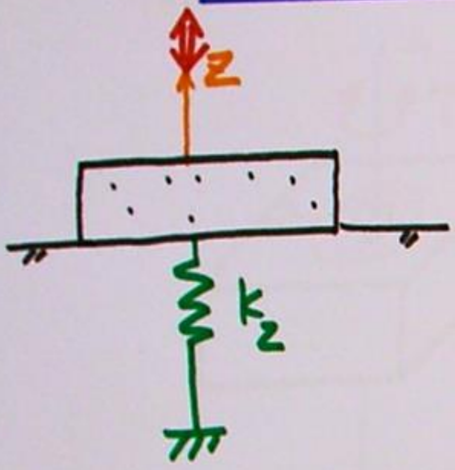
= Coefficient of non-uniform elastic shear



$$= \frac{\text{Local (non-uniform) shear stress under rigid block}}{\text{Induced elastic movement (sliding) in tangential direction at same point}}$$

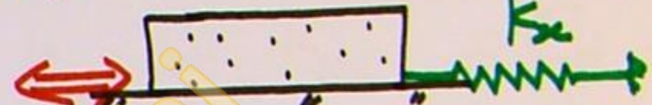
$$= \frac{\tau_0}{S_0}$$

OVERALL STIFFNESS OF SOIL



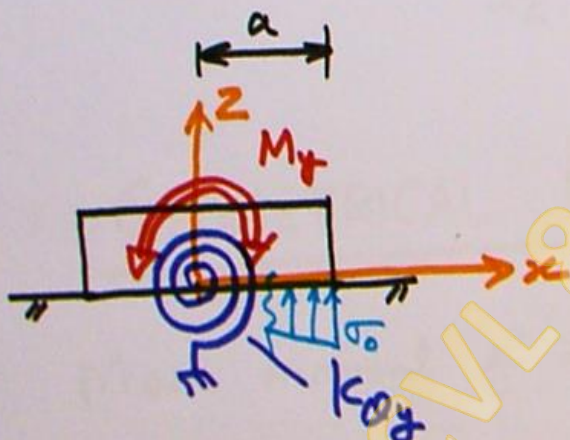
$$k_z = A_f C_z \text{ (kN/m)}$$

Area of base of fdn.



$$k_x = k_{\tau x} = A_f C_{\tau x}$$

Similarly $k_y = k_{\tau y} = A_f C_{\tau y}$



$$C_{\theta y} = \frac{\sigma_0}{\delta_0} = \frac{M_y \cancel{\alpha}}{I_y(\cancel{\alpha}\,\theta_y)}$$

Rotation about y-axis.

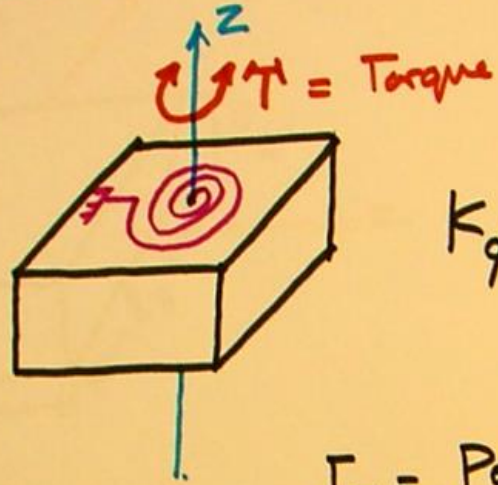
$$\therefore\ M_y = \underbrace{\left[C_{\theta y} I_y\right]}_{k_{\theta y}} \theta_y = k_{\theta y}\,\theta_y$$

$\therefore\ k_{\theta y} = C_{\theta y} I_y$ — Rocking

Unit kN-m/rad

Similarly Pitching $k_{\theta x} = C_{\theta x} I_x$

Torsional vibrations.



K_ϕ or K_ψ or $K_{\theta z}$ $= C_\phi \, I_z$

C_ϕ or C_ψ or $C_{\theta z}$

I_z = Polar moment of inertia

$= I_x + I_y$

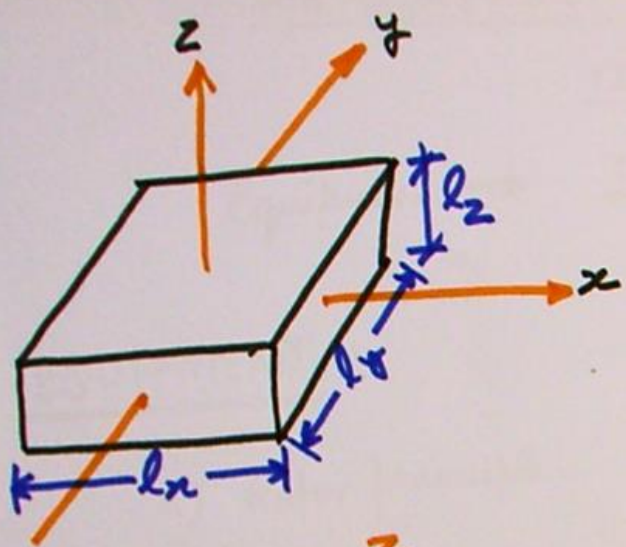
GEOMETRICAL PARAMETERS

Mass moment of inertia

$$\phi = \int_V r^2 \, dm$$

x or y or z

$$= \int_V r^2 \rho \, dV$$

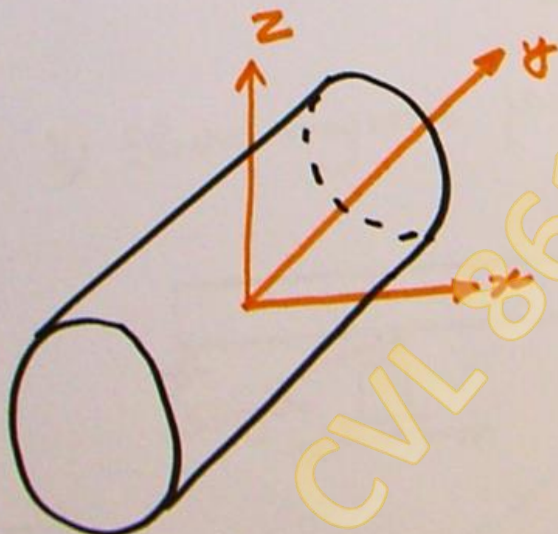


Mass of block

$$\phi_x = \frac{m}{12}\left(l_y^2 + l_z^2\right)$$

$$\phi_y = \frac{m}{12}\left(l_x^2 + l_z^2\right)$$

$$\phi_z = \frac{m}{12}\left(l_x^2 + l_y^2\right)$$

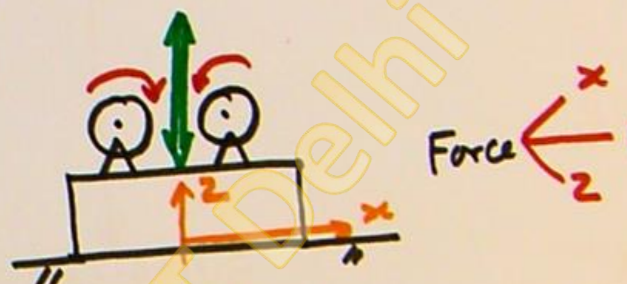


$$\phi_y = \frac{m}{8} D^2$$

$$\phi_x = \phi_z = \frac{m}{12}\left(L^2 + \frac{3}{4} D^2\right)$$

MEASUREMENT OF ELASTIC PARAMETERS OF SOIL.

Equipment → Dynamic forces Sensors → Dynamic displacements/accelerations

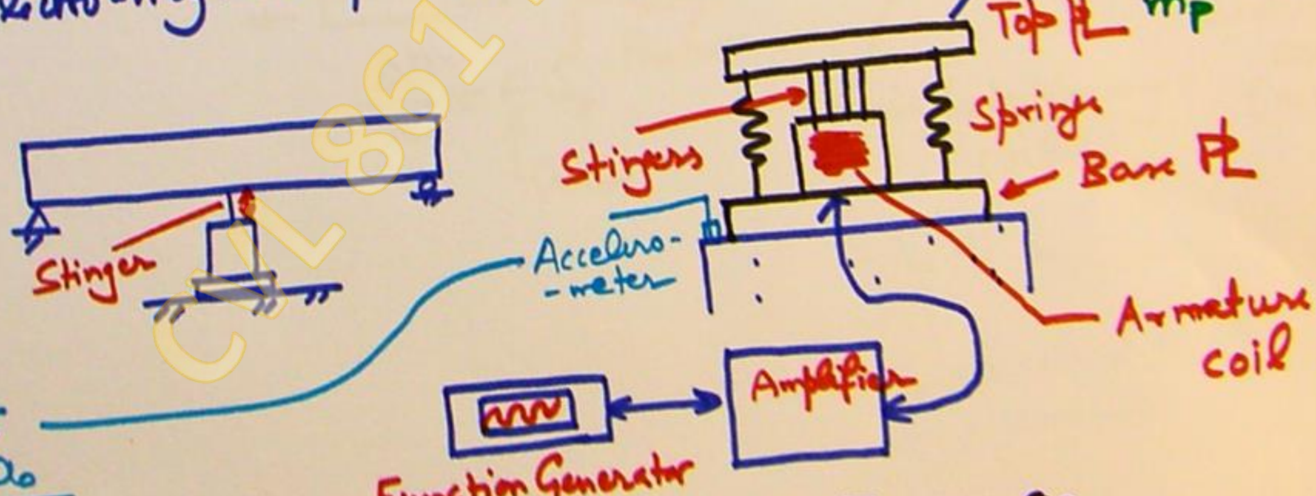


EQUIPMENT :

A) Rotor principle.

B) Electro-magnetic principle. ⟷ Shaker

Acceleration = a_p.



Response measurement

$$d_o = \frac{a_o}{\omega_m^2}$$

(Indirect dynamic disp. measurement)

Force generated = $F = m_p a_p$

Direct Displacement Measurement

ANALOG

Direct dial gauge

RESISTIVE

$$\frac{\Delta R}{R} = \epsilon\, S_g$$

Gauge Factor

Strain

ELECTRO-MAGNETIC

Induction coil (movable)

Main/stationary coil

$V_o \propto L$

Calibrated with displacement

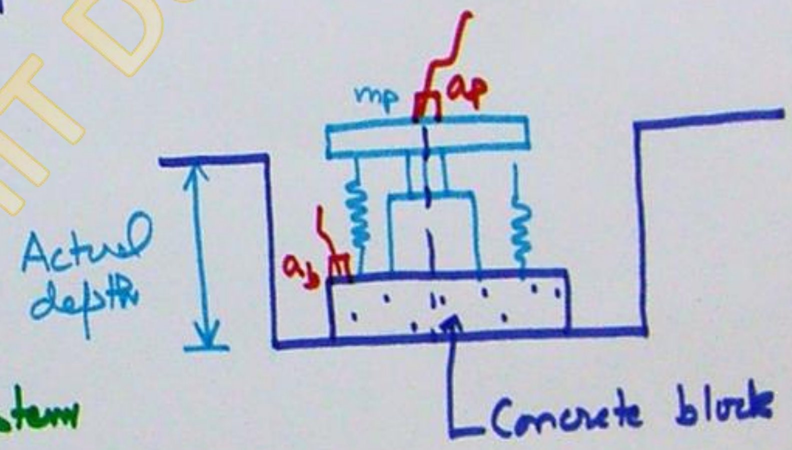
DETERMINATION OF C_2

Direct   Indirect

DIRECT METHOD

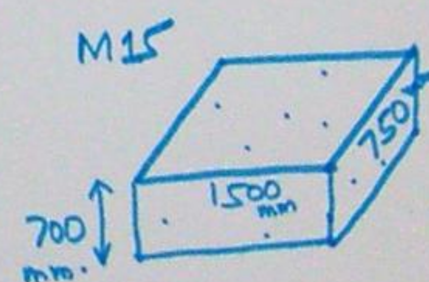
- Dig excavation of same depth as actual fdn.

Line of force should pass through the base of block



$$\left(\frac{\text{Dynamic force}}{\text{Static mass}}\right) = \text{Same as in actual system}$$

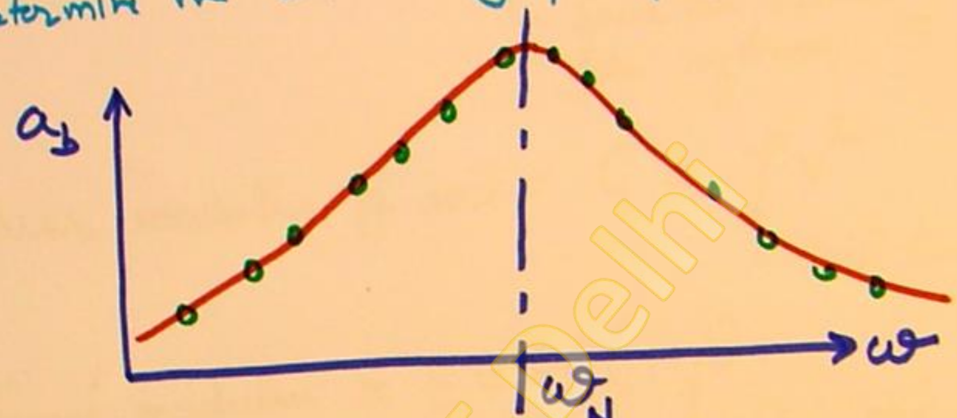
$\omega_m \Longleftrightarrow$ same as m/c frequency.



M = Mass of block + Shaker

Excite the shaker over a wide range of frequencies –

→ To determine the resonance frequency of (m/c + fdn) system



$K_z = M\omega_N^2$

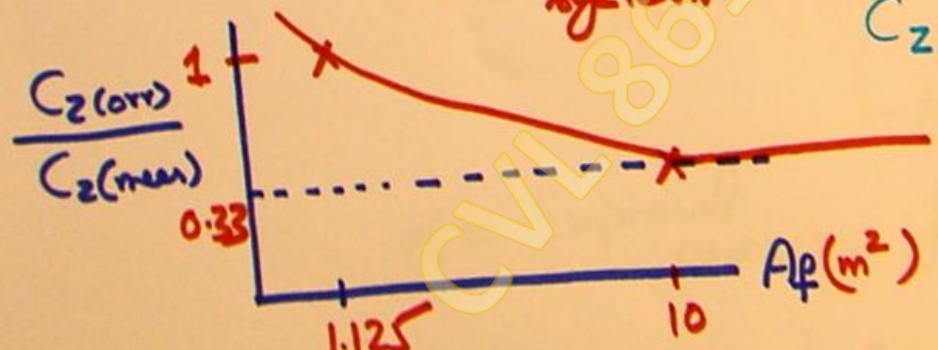
$C_z = \dfrac{K_z}{A_b}$

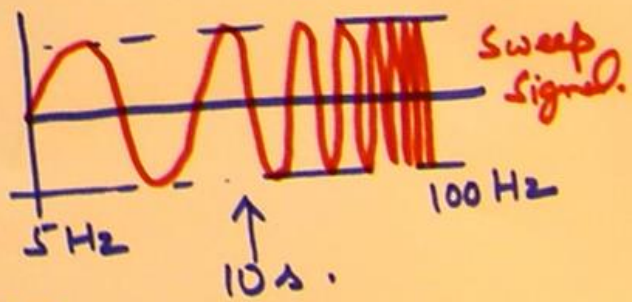
↑ $1.5 \times 0.75\,m^2$ of the experimental system.

ω_N = Natural frequency. $= 2\pi f_N$

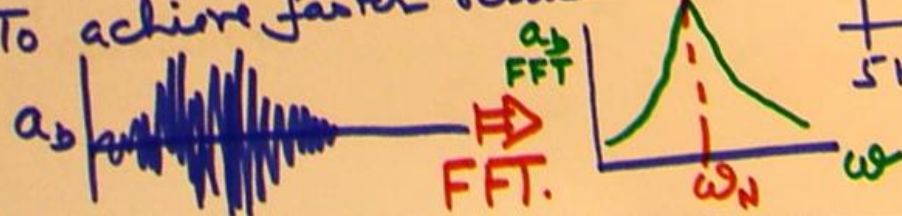
$C_{z(corr)} = C_{z(meas)} \sqrt{\dfrac{A_b}{A_f}}$

↑ Actual fdn. (max. $10\,m^2$)





To achieve faster result →



Wave velocity $v = f_m \lambda$

f_m: Same as actual fdn. system $= \dfrac{\omega_m}{2\pi}$.

Shear modulus of soil $= G = \rho v^2$

ρ: soil density

Young's modulus $= E = 2G(1+\nu)$

ν: Poisson's ratio

Clay	0.5
Sand	0.3 – 0.35
Rock	0.15 – 0.25

$$C_z = \frac{\alpha E}{(1-\nu^2)} \frac{1}{\sqrt{BL}}.$$

B, L: Actual fdn.

α = Empirical correction factor

Empirical corelations

$$C_\tau = \frac{1}{2} C_z \qquad C_\theta = 2 C_z$$

$$C_\phi = 1.5\, C_\tau = 0.75\, C_z$$

METHODS OF ANALYSIS

EMPIRICAL

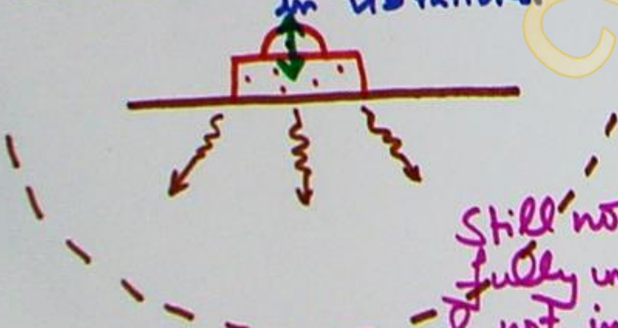
- No theoretical basis
- Based on cumulative experience
- Preferred for less important m/c or preliminary design.

THEORETICAL

I

Soil as semi-infinite elastic medium

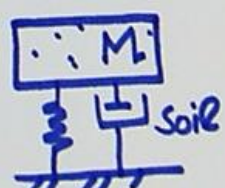
Assumption: Soil actively participates in vibrations.



Still not fully understood & not in practice

II

Soil as finite spring.



M = m/c + Block + Part of soil.

Still not fully understood & not in practice.

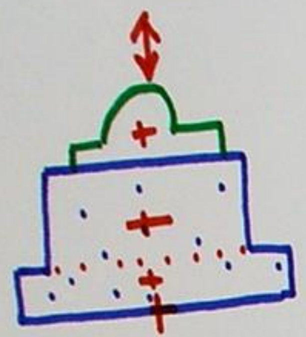
III

Barkan's method.

Soil as lumped spring.

ZERO → Mass, damping.

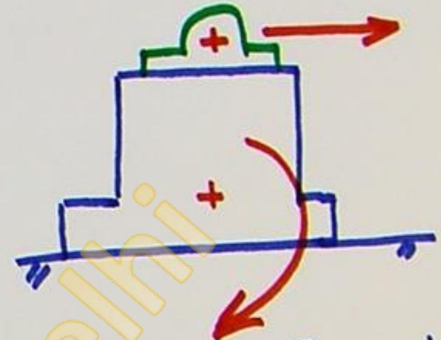
- Most popular
- Simple
- Results have good correlation with experimental observations (conservative side)



Pure vertical vibrations.

Vertical vibrations independent of other types of vibrations

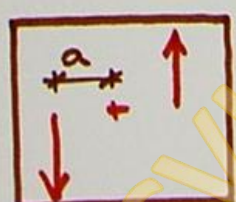
Rocking, pitching, sliding.



Sliding & rocking/pitching occur in combination.

COUPLED

$\overset{\leftrightarrow}{T_o} = 2F_o a$



$F_o \sin \omega_m t$

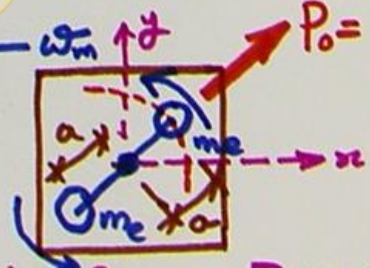
$F_o \sin \omega_m t$

Plan

Pair of horizontally operating reciprocating m/c OUT OF PHASE

Induce pure YAWNING

Const. — ω_m



$P_o = m_e a \omega_m^2$

Shall induce horizontal force continuously changing direction

$\Rightarrow$ Rocking, pitching, sliding

No yawning.

$\dot{\omega}_m$ = Angular acceleration $= \ddot{\theta}_m \neq 0$

$\overset{\leftrightarrow}{T_o} = 2 m_e \dot{\omega}_m a^2$ — Pure yawning

Vertical & Yawning vibrations are analysed independently

VERTICAL

$$M\ddot{z} + kz = P_z(t) = P_{z0}\sin(\omega_m t)$$

Natural frequency $\omega_{zN} = \sqrt{\dfrac{k_z}{M}}$ — $C_z A_f$

Amplitude $a_{z0} = \left(\dfrac{P_{z0}}{k_z}\right) A(\omega_m)$ — $\dfrac{1}{1-r^2}$

$= \left(\dfrac{m_e}{M}\right) A'(\omega_m)\, a$ — $\dfrac{r^2}{1-r^2}$

$r = \dfrac{f_m}{f_N}$

≤ 0.4 or ≥ 1.5

Amplitude to be within limits as IS 2974 (I)

YAWNING

Angular acceleration

$$\phi_z \ddot{\theta}_z + k_\phi \theta_z = T_z(t)$$
$$= T_0 \sin(\omega_m t)$$

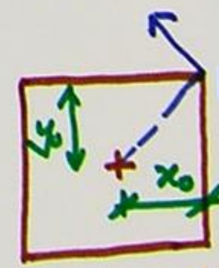
Mass Moment of inertia.

$C_\phi I_p$

Natural frequency $\omega_{\theta zN} = \sqrt{\dfrac{k_\phi}{\phi_z}}$

Amplitude of angular displacement

$$a_{\theta z0} = \frac{T_0}{k_\phi} A(\omega_m)$$



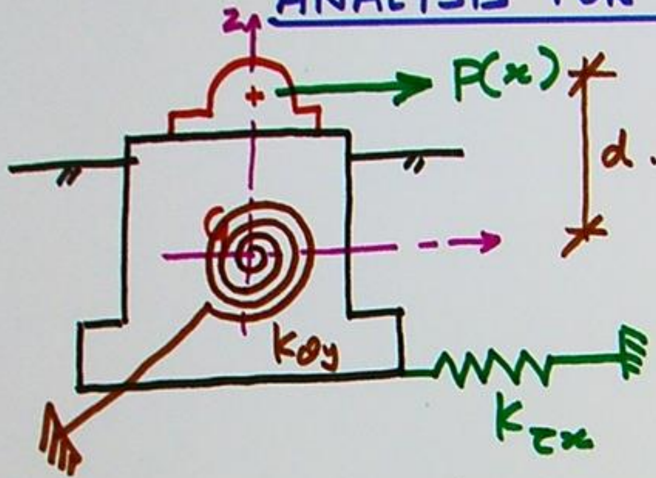
Farthest point (x_0, y_0).

$a_{y0} = a_{\theta z0}\, x_0$

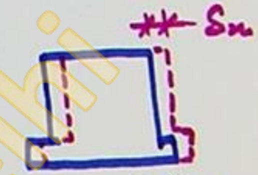
$a_{x0} = a_{\theta z0}\, y_0$

Net amplitude $\sqrt{a_{x0}^2 + a_{y0}^2}$ should be within limits

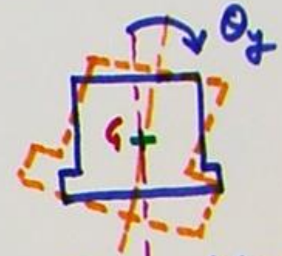
ANALYSIS FOR ROCKING + SLIDING.



$M(y) = P(x) \times d.$

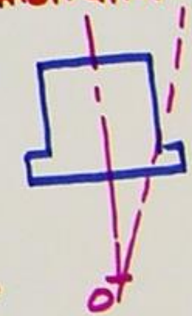


Pure sliding



Pure rocking

Combination



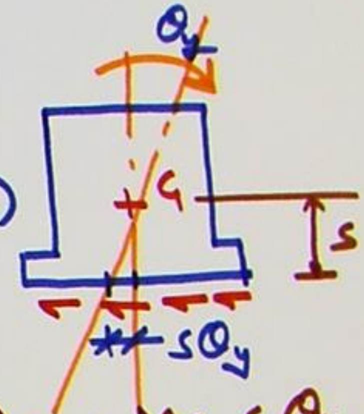
Alternate school of thought

Bhatia

Positioned @ C.G of Base.

IS 2974, Vaidyanathan DOF positioned at combined CG "G" of block + Base + M/c.

EQUATIONS OF MOTION

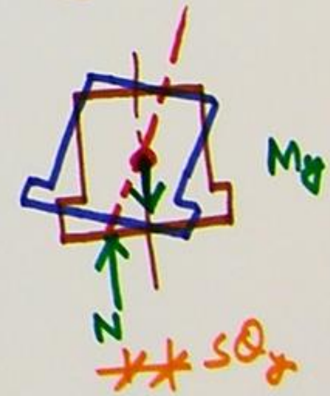


Horizontal → $M\ddot{x} + K_{\tau x}(x - s\theta_y) = P_x(t)$

$= P_{x0}\sin(\omega_m t)$

Rotation → $\phi_y \ddot{\theta}_y + K_{\theta y}\theta_y = M(t) + \underbrace{K_{\tau x}(x - s\theta_y)s}_{\text{Moment of shear stress}} + Mg\,s\,\theta_y$

This is TWO-DOF problem



SOLUTION PROCEDURE

⟨1⟩ Limiting frequencies

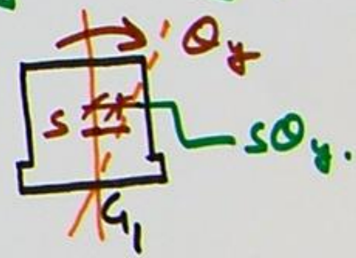
Assuming soil has ∞ rotational stiffness $\Rightarrow$ Pure sliding.

$$M\ddot{x} + K_{\tau x}\,x = P_x(t) \qquad \omega_{xN} = \sqrt{\frac{K_{\tau x}}{M}}$$

Assume soil has ∞ sliding stiffness

$\Rightarrow$ Pure rotation. No sliding at all.

Rotation shall occur w.r.t CG of base of fdn.



$$\phi_{yo}\,\ddot{\theta}_y + K_{\theta y}\,\theta_y = M_y(t) + Mgs\,\theta_y$$

$$\Rightarrow \phi_{yo}\,\theta_y + \left[K_{\theta y} - Mgs\right]\theta_y = M_y(t)$$

ϕ_{yo} w.r.t G_1

$$\Rightarrow \omega_{\theta_y N} = \sqrt{\frac{K_{\theta y} - Mgs}{\phi_{yo}}}.$$

$$\alpha_y = \frac{\phi_y}{\phi_{yo}}$$

ϕ_y w.r.t block + Base + M/c

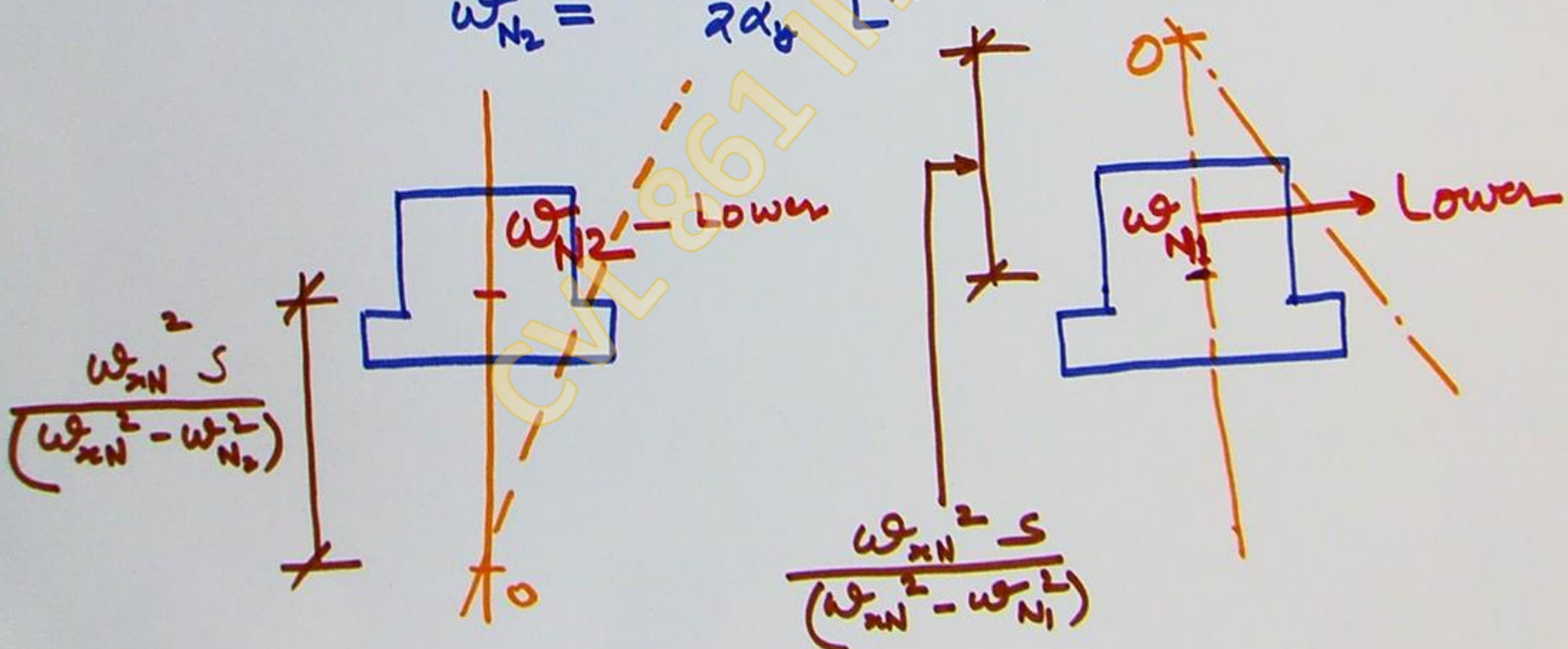
⟨2⟩ Determination of Coupled Frequencies

$$A = \omega_{xN}^2 + \omega_{\theta yN}^2$$

$$B = 2\sqrt{\alpha_y}\, \omega_{xN}\, \omega_{\theta yN}.$$

$$\omega_{N_1}^2 = \frac{1}{2\alpha_y}\left[A + \sqrt{A^2 - B^2}\right]$$ Higher

$$\omega_{N_2}^2 = \frac{1}{2\alpha_y}\left[A - \sqrt{A^2 - B^2}\right]$$ Lower



⟨3⟩ Amplitude of vibration.

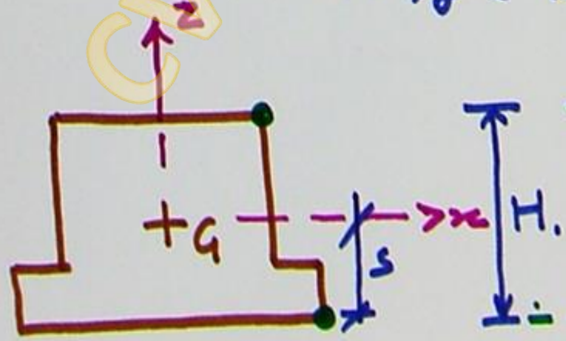
Values @ CG of block + Base + M/C.

$$a_{x0} = \frac{\left(K_{\theta y} - M g s + K_x s^2 - \phi_y \omega_m^2\right) P_{x0} + \left(K_x s\right) M_{y0}}{M \phi_y \left(\omega_{N_1}^2 - \omega_m^2\right)\left(\omega_{N_2}^2 - \omega_m^2\right)}$$

Disp. amplitude @ G

Rotational amplitude @ G

$$a_{\theta y0} = \frac{\left(K_x s\right) P_{x0} + \left(K_x - M \omega_m^2\right) M_{y0}}{M \phi_y \left(\omega_{N_1}^2 - \omega_m^2\right)\left(\omega_{N_2}^2 - \omega_m^2\right)}$$



-- $a_{x0} + (H-s)\, a_{\theta y0}$

To be checked

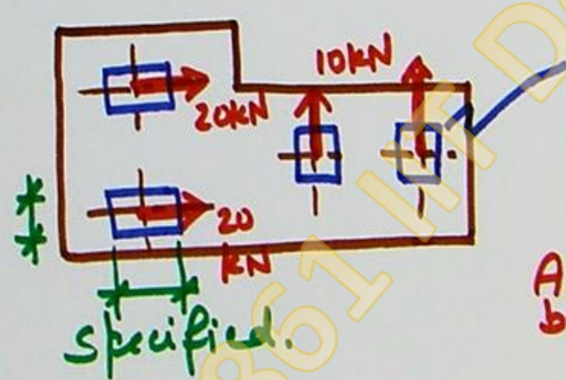
...... $a_{x0} + - s\, a_{\theta y0}$.

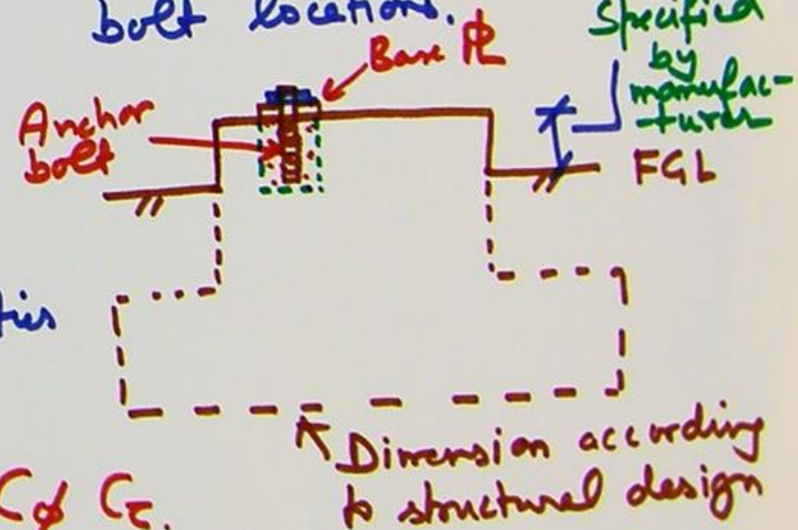
DESIGN INPUTS REQUIRED

1) Operating frequency ω_m or f_m.

2) Layout of top of fdn., Loading diagram.

↑ Anchorage positions.





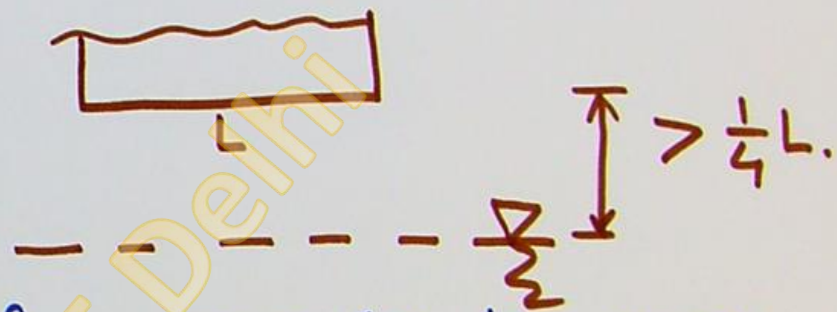
3) Static & dynamic properties of soil

$\sigma_{all, net}$ C_z C_Θ C_ϕ C_τ.

E ν

IMPORTANT POINTS

1) Ground water table to be as low as possible below the base



2) Fdn. to be isolated from surroundings by means of expansion joints

3) Hot pipes to be isolated.

4) Concrete to be protected by acid-resisting coating.

5) M/c fdn. preferably at a level lower than bottom of surrounding structure fdn.

ANALYSIS/DESIGN STEPS

1. Assume layout in plan & elevation as per inputs from manufacturer & Geotech. report.

   A, I, ϕ, M etc.

2. CG of m/c + Block + Base.

3. Stiffness of fdn. system K_{zx} K_{zy} $K_{\theta y}$ $K_{\theta x}$ K_{ϕ}

4. Natural frequencies
   - (a) Limiting ω_{zN} $\omega_{\theta xN}$ $\omega_{\theta yN}$ $\omega_{\theta zN}$

     Actual

   - (b) Coupled ω_{N_1} ω_{N_2} sliding + rocking

Repeatition may be necessary

5. Displacements — vertical, angular (horizontal/vertical)

6. Check for pressure in soil under fdn.

7. Inertial forces.

FORCES ON MACHINE - FDN. SYSTEM

$$P(t) = M\ddot{x} + \underline{C\dot{x}} + kx$$

Induced force.

Generated by the m/c.

Inertial force

Damping force ≈ 0

Elastic force.

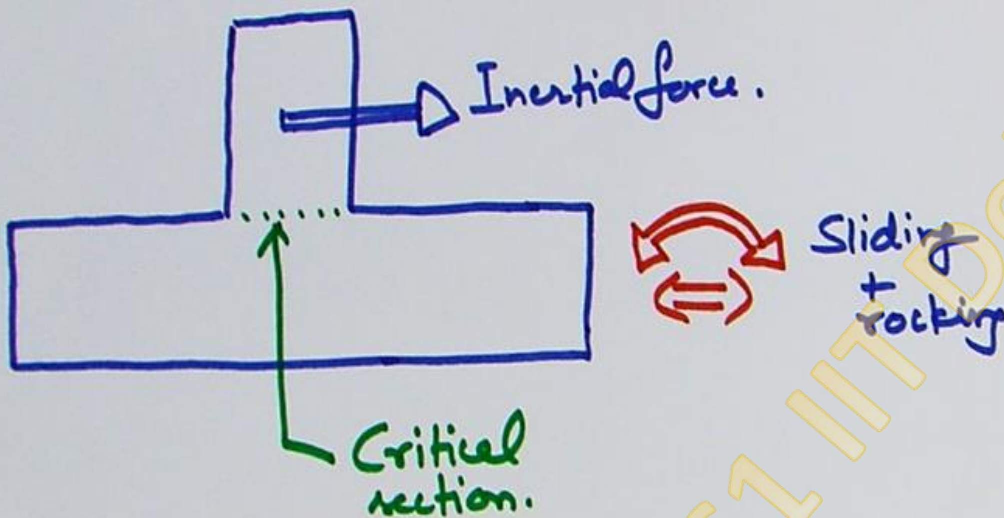
Magnitude to be determined.
Multiplied by Fatigue Factor
$FF = 3.0$

Low frequency m/c : $f_m \ll f_N \Rightarrow$ Low inertial forces $\rightarrow$ Elastic forces dominate

High frequency m/c : $f_m \gg f_N \Rightarrow$ High inertial forces $\rightarrow$ Inertial forces dominate.

Medium frequency m/c : $f_m \approx f_N$
Both elastic & inertial forces of comparable magnitude

INERTIAL FORCES



Concept similarly extendable to vertical, yawning motions.

Acceleration amplitude

Total inertial force $F_{ix} = M \ddot{a}_{x0} FF$

$\ddot{a}_{x0} = \omega_m^2 a_{x0}$

Total mass block + Base + M/c.

Disp. amplitude

Total inertial moment

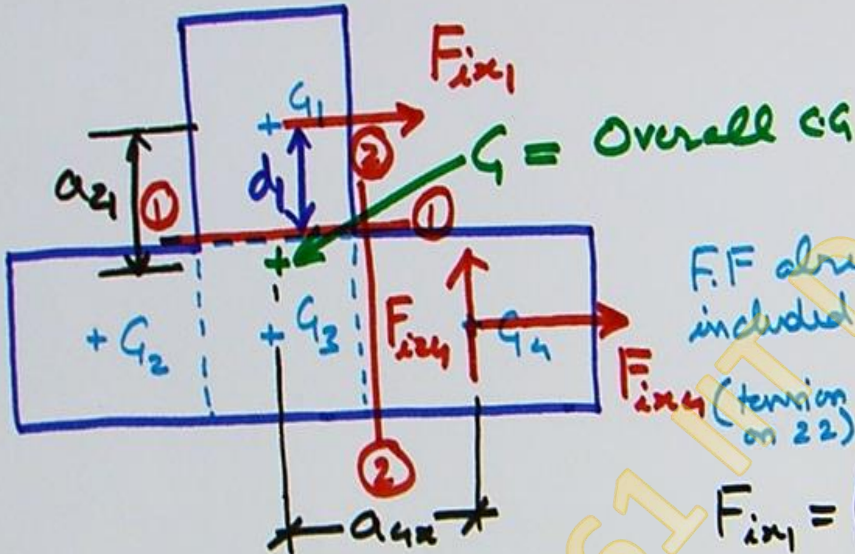
$M_{iy} = \phi_y \ddot{a}_{\theta y0} FF$

Angular acc. amplitude

$= \phi_y (\omega_m^2 a_{\theta y0}) FF$

Angular displacement amplitude

INERTIAL FORCE ON COMPONENT



mass of component $\langle 1 \rangle$

$$F_{ix_1} = \left(\frac{m_1}{M}\right) F_{ix} + m_1 \left(a_{z_1} \ddot{a}_{\theta y o}\right)$$

?

Tangential acceleration of $\langle 1 \rangle$ w.r.t CG

Dist. (vertical) between G_1 & G

F.F already included F_{ix} & M_{yi}

$$F_{ix_1} = \left(\frac{m_1}{M}\right) F_{ix} + m_1 a_{z_1} \left(\frac{M_{yi}}{\phi_y}\right)$$

$a_{\theta y o}$

Bending moment acting on plane 1 1 $= F_{ix_1} \times d_1$

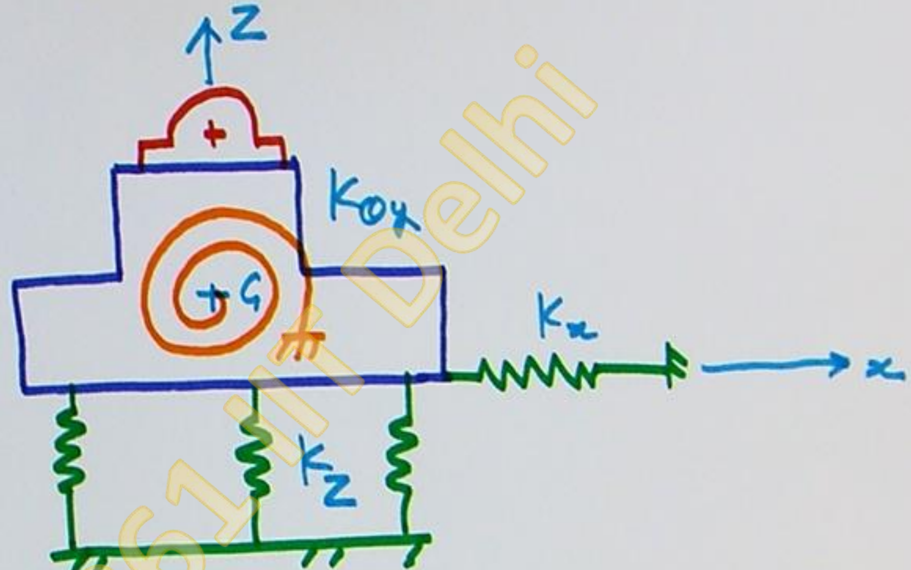
Shear force $= F_{ix_1}$

This section to be adequate for <u>shear force</u> & <u>Bending moment</u>

$$F_{iz_4} = \left(\frac{m_4}{M}\right) F_{iz} + m_4 \left(\frac{M_{yi}}{\phi_i}\right) a_{4x}$$

ELASTIC / DYNAMIC FORCES

Soil acting as elastic medium is subjected to elastic forces.



Vertical vibrations : Elastic force in z-direction

$$= K_{zx} [a_{x0} - s\, a_{\theta y}$$

Vertical vibrations: Elastic force in z-direction

$$F_{ez} = K_z \, a_{z0} \, FF$$

Sliding + Rocking: Elastic force in x-direction

$$F_{ex} = K_{\tau x} \underbrace{\left(a_{x0} - s\, a_{\theta y0}\right)}_{\text{Net disp. @ base of fdn.}} FF$$

Elastic moment

$$M_{ey} = K_{\theta y} \, a_{\theta y0} \, FF$$

Check for soil pressure:

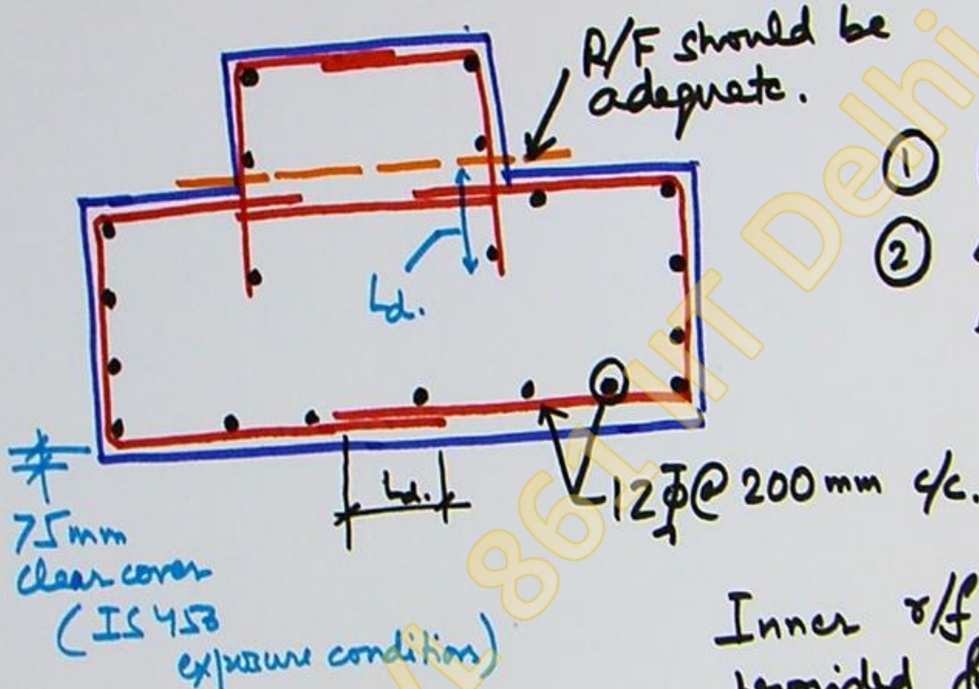
$$\sigma_{\text{total (gross)}} = \overset{*}{\sigma_{\text{static}}} + \overset{\#}{\sigma_{\text{dyn.}}}$$

$$= \left[\frac{W_{\text{static}}}{A_f} + \frac{(M_{\text{static}})_y}{Z_f}\right] \pm \overbrace{\frac{F_{ez}}{A_f} \pm \frac{M_{ey}}{Z_f}}^{\#}$$

> 0, $< 80\%$ of $\sigma_{\text{all, gross}}$.

REINFORCEMENT DETAILING

Mainly subjected to rigid body vibrations.



① Reinf. ≮ 25 kg/m^3

② 40 kg/m^3 for m/c. pumping explosive gases.

Inner r/f should also be provided for large blocks.

Min. concrete grade M25

Refer IS: 456, IS 2974 (I) 5.4.5

SUMMARY

1) Frequency ratio check
$r \leq 0.4$ ≥ 1.5

2) Vibration amplitude :
- Value specified by manufacturer
- IS 2974(I) for desired comfort level.
- $\not>$ 200 μm (0.2 mm) under any situation.

3) Check for soil pressure.

4) Critical sections for inertial forces.

Perform for the example m/c fdn. problem

VIBRATION ISOLATION

- Often problems encountered after construction.
- Disturbing vibrations.
- Vibration isolation means to reduce/suppress unacceptable or disturbing vibrations so as to bring them to acceptable level.
- Remedial measures post operation

CAUSES :

1) Insufficient/erroneous inputs during design.
2) Wear & tear in machine during operation
3) Any other unanticipated reason.

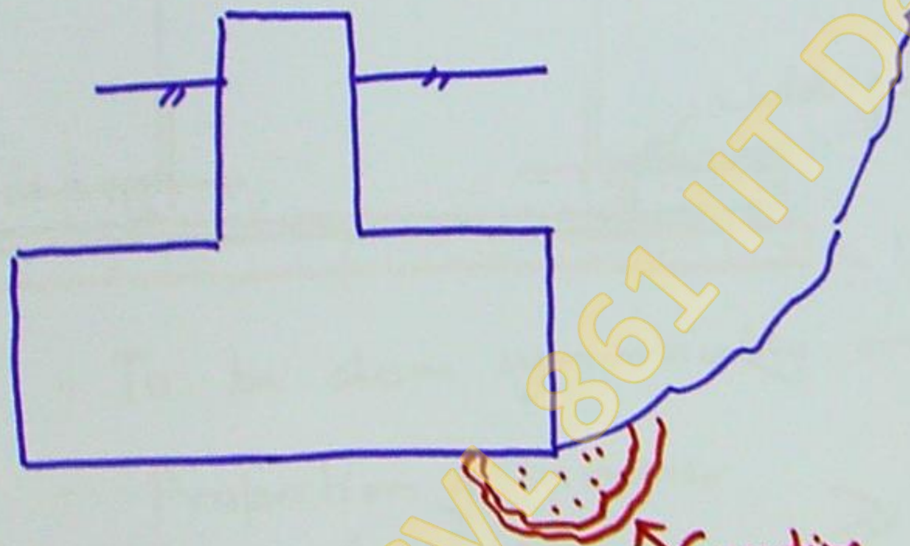
1) COUNTERING VIBRATIONS AT SOURCE

Sometimes sudden/abrupt vibrations reported.

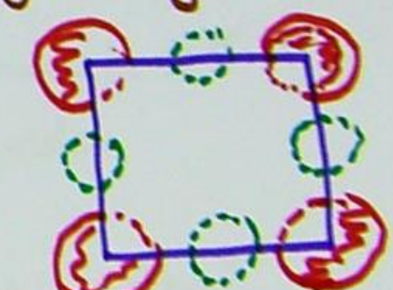
- Machine expert/ manufacturer.
- Wear & tear of components (eg. bearings)
- Loosening of tension in belts.
  [older type of compressors AC of buildings]
- Clogged filters.
- Regular servicing of m/c necessary
- counterbalancing ..... ??

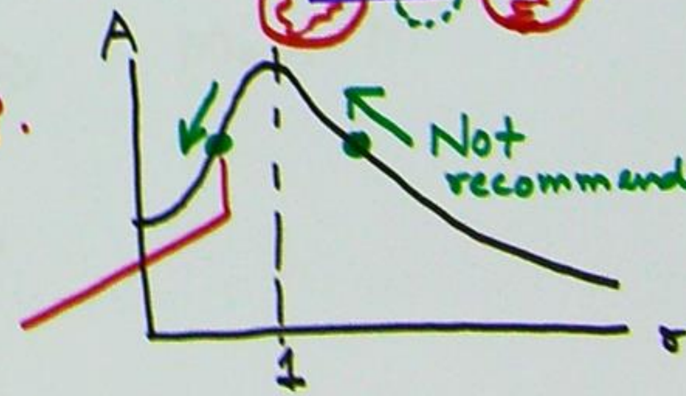
2) REMEDIAL MEASURES IN FOUNDATION AFTER CONSTRUCTION

: SOIL STABILIZATION



- Excavate
- Pocket-by-pocket grouting





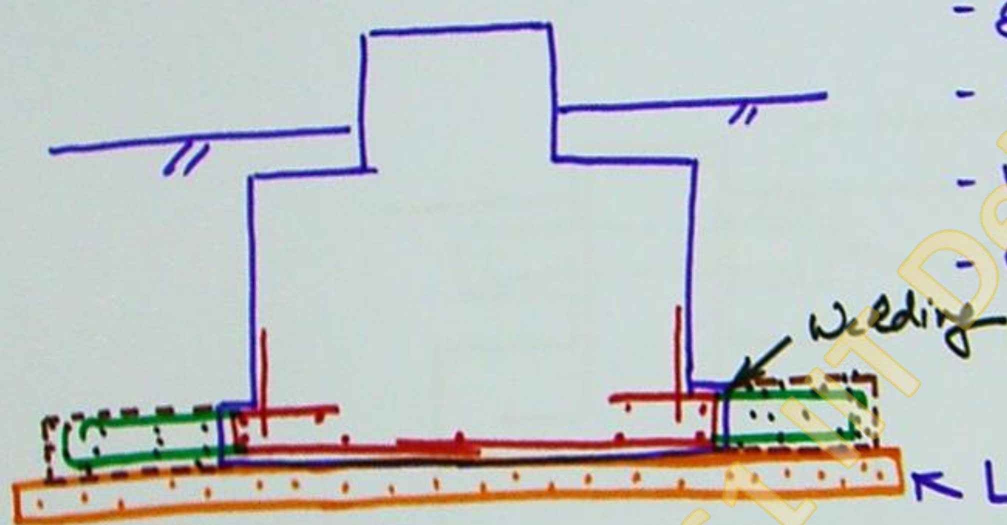
When helpful? $f_N \uparrow$ $r \downarrow$

Counterproductive $r > 1$

Recommended $r < 1$

2) INCREASE AREA OF BASE OF FDN.

- Excavate around fdn.
- Chipping of concrete
- Weld new reinforcement
- Cast extension of base slab.

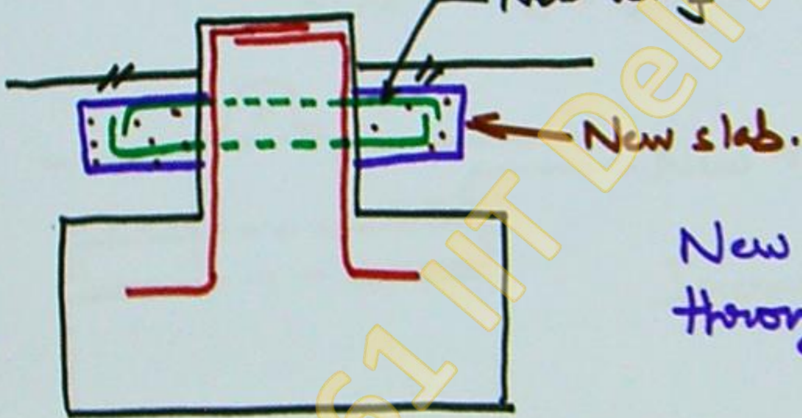


- To be done symmetrically on all sides
- Proportion of increase in "K" $\gg$ Mass "M"

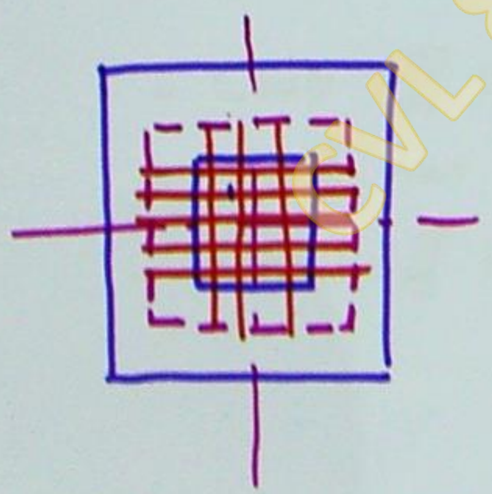
Recommended for $r < 1$

4) ATTACHMENT OF SLAB NEAR TOP

$\gamma > 1$: Not recommended.



New r/f should pass through the pedestals

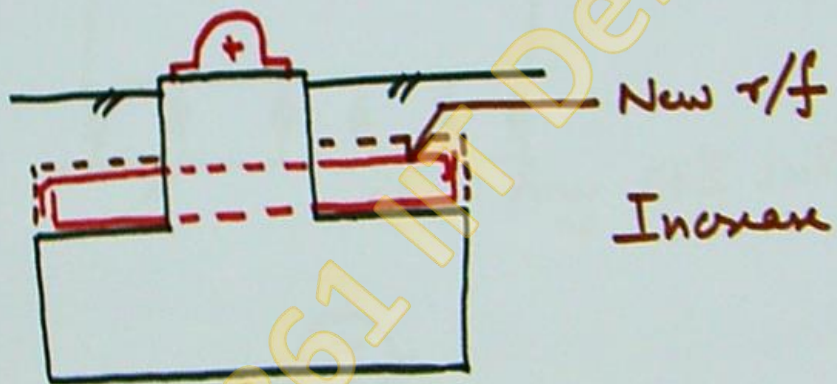


3)

5) INCREASE MASS OF FOUNDATION

Effective $r > 1$

Since $f_N \downarrow$ $r \uparrow$

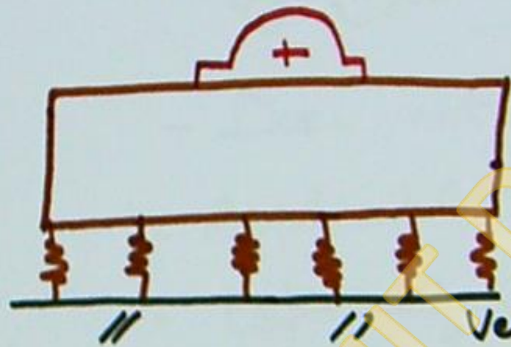


Increase 'M' but not 'K'

- Chip off concrete cover
- Cast new concrete after laying r/f
- New r/f to pass through pedestal.

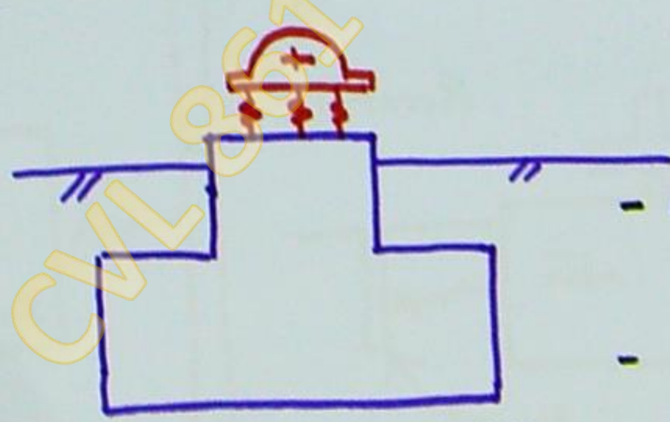
6) INTRODUCTION OF SPRINGS

$r > 1 \quad f_N \downarrow \quad r \uparrow .$



Very stiff soil / rock

or



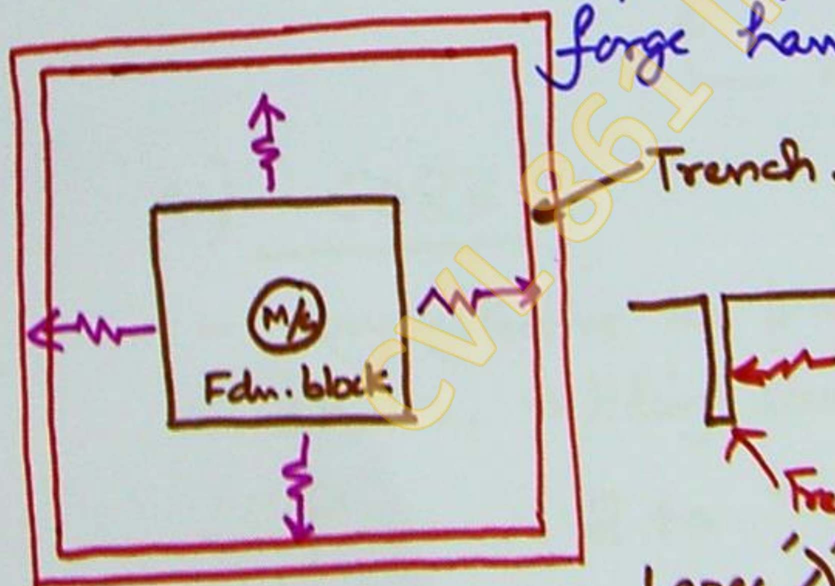
- Machine vibration might increase
- Higher wear & tear

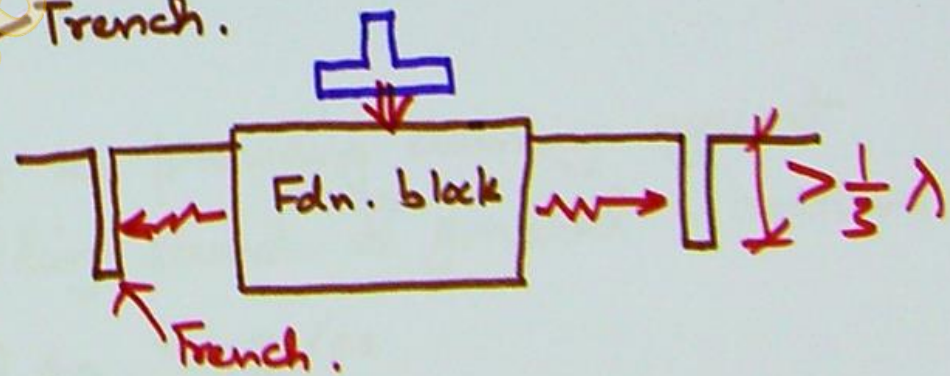
$\therefore$ Check with manufacturer.

7) TRENCH BARRIER

Principle – To cut off waves generated by (m/c + fdn.) system.

- Does not involve modifying "K" or "M"
- Applicable to large variety of m/c.
- Especially expedient for impact/ forge hammers.





Large 'λ' waves may cross over less deep trench.

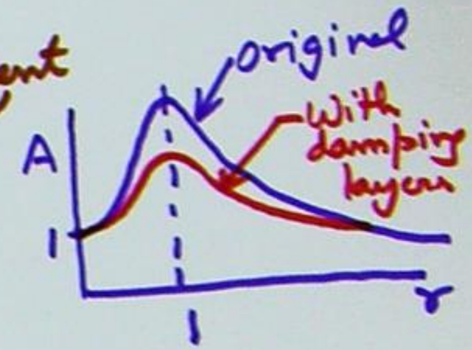
8) USE OF HIGHLY DAMPED/ELASTIC MATERIALS

A) Cork
B) Felt

} Large ξ → Independent of 'r'

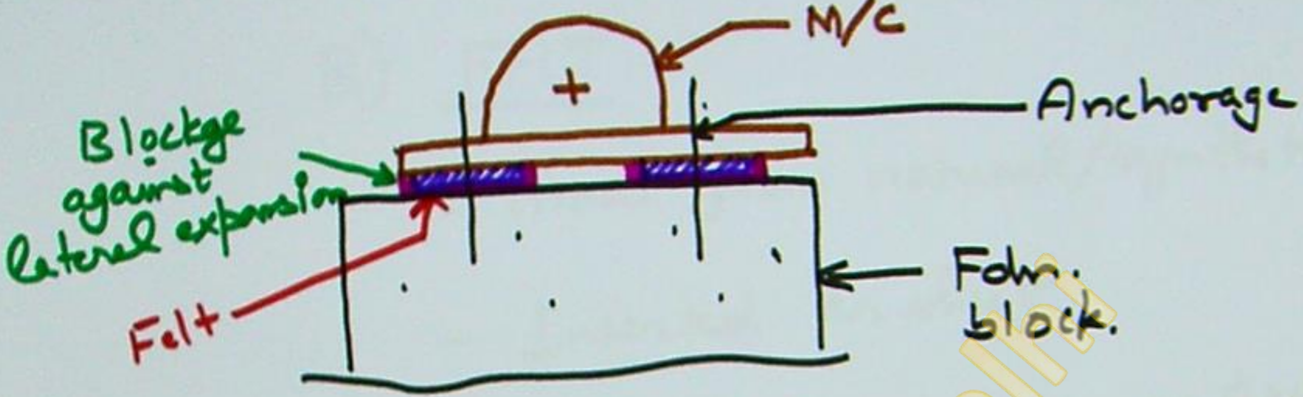
C) Rubber
- Alters 'K'
- Has inherent damping also

↑ $r > 1$



A) CORK

- Very effective in providing damping against shocks, rattling sounds & general vibrations.
- ρ → 2 to 4 g/cc
- Effective in sheet form.



Precautions:

1) Should not come in contact with water/oil

2) Effective only if prevented from lateral expansion. (confining action)

Else it will disintegrate

B) FELT

- Made from natural/synthetic fibres
- Inserted as sheet
- Compressive strength ~ 8 MPa.
  Young's modulus ~ 80 MPa.

Precaution : Must not be wet by water/oil.

c) RUBBER

- More effective as spring/elastic material than damper.
- Effective for $r > 1$

Prerequisites –

a) Allow lateral expansion.

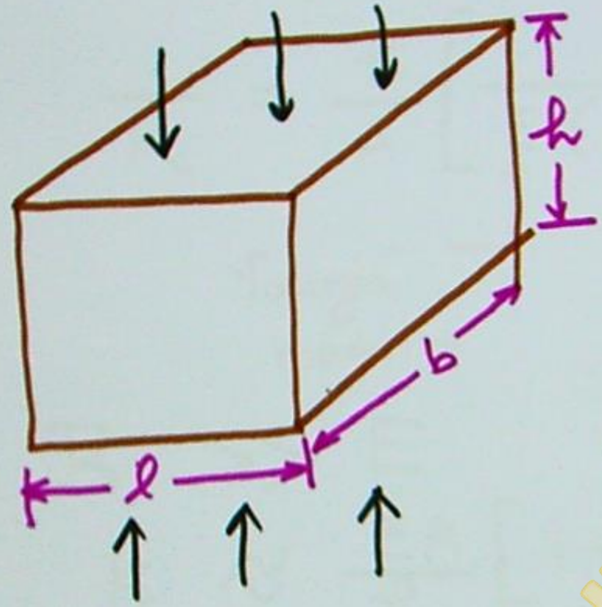
b) Avoid contact with water or oil or chemicals.

[water/oil resistant variants available in market]

Grades of rubber: SHORE HARDNESS

40° 45° 50° etc.

All of varying properties



Both "E" & "B" play role

$$A_r = \text{Area ratio} = \frac{\text{Area}}{\text{Perimeter} \times h}$$

$$A_r = \frac{lb}{2(l+b)h}$$

$$\frac{1}{K_c} = \frac{h}{A}\left[\frac{1}{E(1+2\alpha A_r^2)} + \frac{1}{B}\right]$$

Youngs' modulus

Constant [Grade of rubber]

Bulk modulus

$$B \gg E$$

$$K_c \approx \frac{EA}{h}\left[1 + 2\alpha A_r^2\right] \approx \frac{EA}{h}$$

$$\frac{1}{k_c} = \frac{h}{A}\left[\frac{1}{E(1+2\alpha A_r^2)} + \frac{1}{B}\right]$$

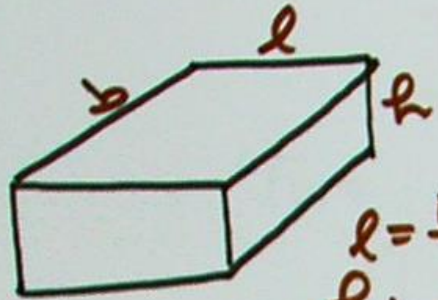
Young's modulus

Constant (grade of rubber)

Bulk modulus

$$k_c \approx \frac{EA}{h}\left[1+2\alpha A_r^2\right] \approx \frac{EA}{h}$$

Large $h >> l, b$



$l = b = 100$ mm
h: variable

55°

G 0.652 MPa
E 2.243 MPa
B 1050 MPa
α 0.73

h (mm)	K_c (Exact)	K_c (exclude 'B')		K_c ($\sim EA/h$)	
		Value	% Error	Value	% Error
50 (plate)					
200					
500					

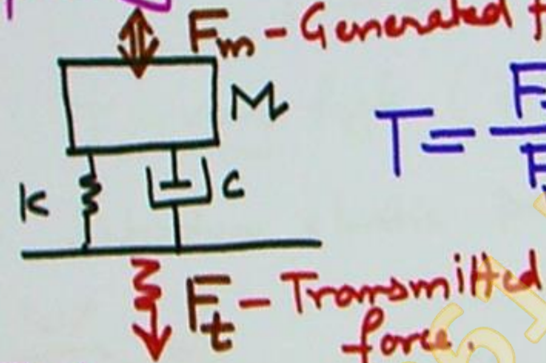
N/mm

TRANSMISSIBILITY

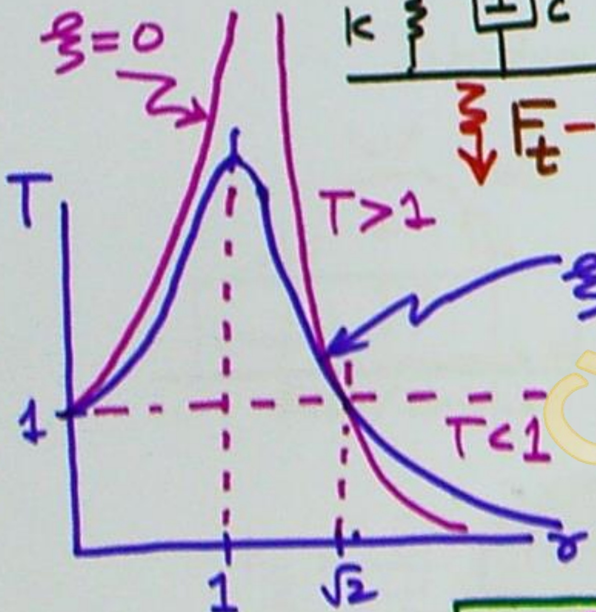
Two scenarios

⟨1⟩

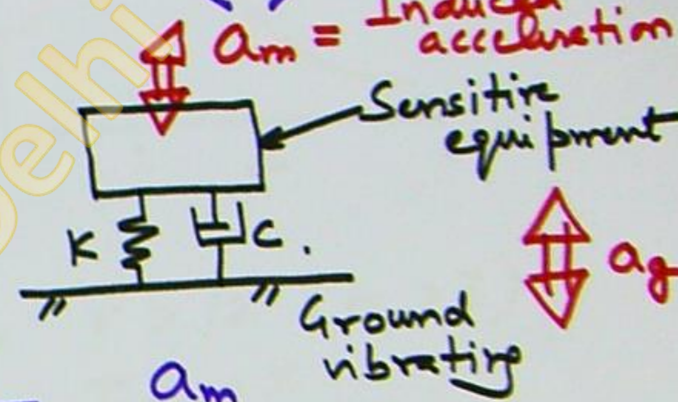
M/C operating on a fdn.



$$T = \frac{F_t}{F_m}$$



⟨2⟩



$$T = \frac{a_m}{a_g}$$

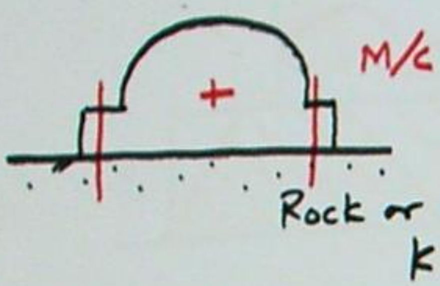
$$T = \sqrt{\frac{1 + (2r\xi)^2}{(1-r^2)^2 + (2r\xi)^2}}$$

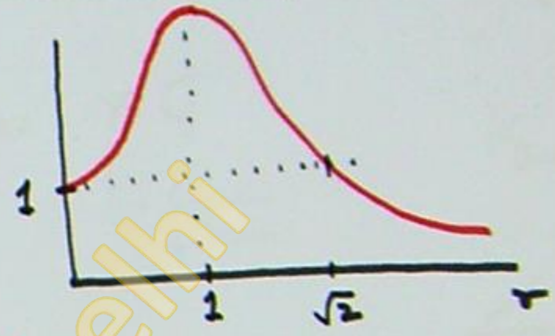
Valid for ⟨1⟩ & ⟨2⟩

High frequency m/c : $\boxed{r > \sqrt{2}}$ $\approx \left| \frac{1}{1-r^2} \right|$ ignore sign $r > 1$

$r < 1 \Rightarrow T \geqslant 100\%$ (Unavoidable) Rigid ground. $k \to \infty$ $T = 100\%$

FOUNDATIONS ON ELASTIC PADS

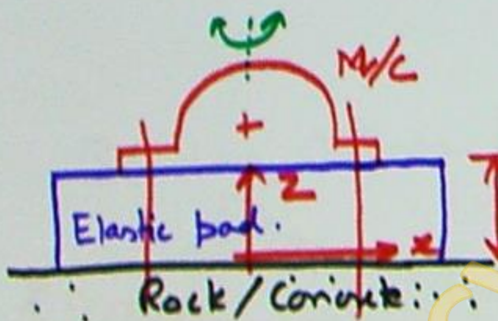




T – very high ($\geq 100\%$)

$\therefore$ Introduce elastic pads $k \downarrow$ $\quad r > \sqrt{2}$ ($r > 3$)

$T < 10\%$



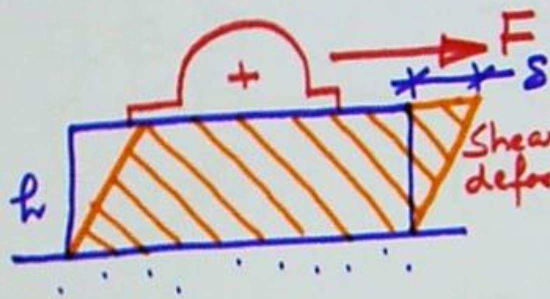
$k_x \rightarrow$ $\quad k_z \uparrow$ $\quad k_{\theta y}$ $\quad k_{\theta x}$

$k_{\theta z}$

$$k_z = \frac{EA}{h}\left[1 + 2\alpha A_r^2\right] \qquad B \gg r. \qquad t < \text{Lateral dimensions.}$$

$$\approx \frac{EA}{h} \qquad h \geq \text{Lateral dimensions}$$

Horizontal stiffness K_x or K_y



$$\tau = \gamma G$$

Shear stress — Shear strain

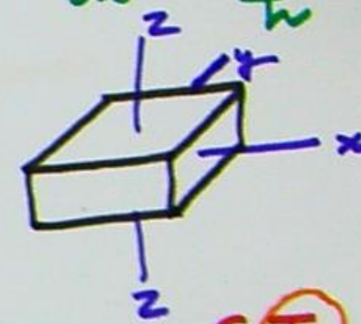
$$\therefore K_x = \frac{GA}{h} = K_y$$

$$\frac{F}{A} = \left(\frac{\delta}{h}\right) G$$

$$F = \left(\frac{GA}{h}\right)\delta \qquad K_x$$

$$K_{\theta y} = \frac{EI_y}{h}$$

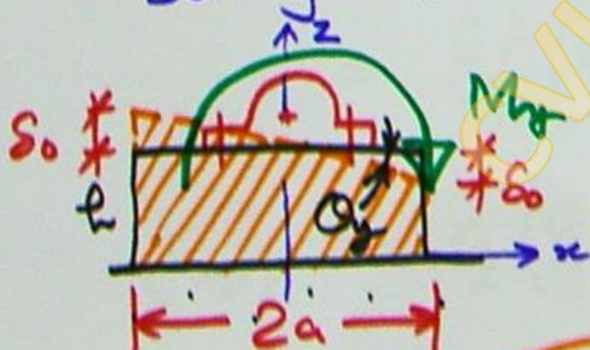
$$K_{\theta x} = \frac{EI_x}{h}$$



$$K_\psi = \frac{GJ}{h}$$

Torsional constant

Bending stiffness $K_{\theta y}$ or $K_{\theta x}$



$$\delta_0 = a\theta_y$$

Tensile/compressive strain in the extreme fibre $\epsilon_0 = \frac{\delta_0}{h}$

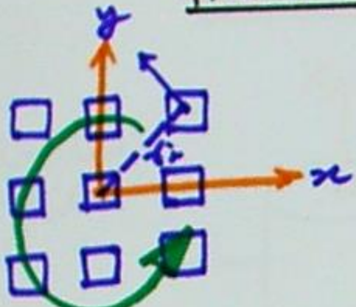
$$\frac{\sigma_0}{E} = \frac{\delta_0}{h}$$

$$\frac{M_y a}{I_y E} = \frac{\delta_0}{h}$$

$$M_y = \left(\frac{EI_y}{h}\right)\frac{\delta_0}{h a}$$

$$K_{\theta y} \qquad M_y = \left(\frac{EI_y}{h}\right)\theta_y$$

GROUPS OF ELASTIC PADS



- Identical pads
- Symmetrical placement

$$(K_z)_g = \left(\frac{EA}{h}\right) N$$

N — No. of pads

$$(K_x)_g = \left(\frac{GA}{h}\right) N$$

or

$$(K_y)_g$$

$$I_{group} = \int_A x^2\, dA = \sum_{i=1}^{N} \left(\cancel{I_{yai}}^{\,0} + x_i^2 A_i^2 \right) \approx A \sum_{i=1}^{N} x_i^2$$

A

$$(K_{\theta x})_g = \frac{E\, I_{group}}{h} = \frac{EA}{h}\left(\sum_{i=1}^{N} x_i^2\right)$$

$$(K_{\theta y})_g = k_z \left(\sum_{i=1}^{N} x_i^2\right)$$

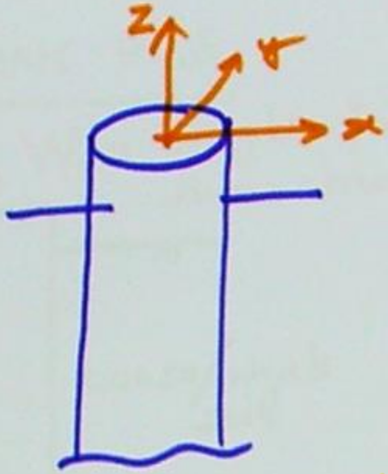
$$(K_{\theta x})_g = k_z \left(\sum_{i} y_i^2\right)$$

$$(K_{\phi})_g = \frac{G\, J_{group}}{h} = \frac{GA}{h} \sum_{i=1}^{N} x_i^2$$

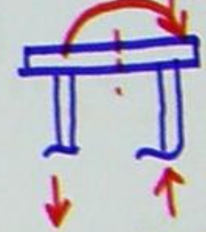
$$= \frac{GA}{t} = \frac{GA}{h} \sum_{i=1}^{N} (x_i^2 + y_i^2)$$

MACHINE FDNS. ON PILES.

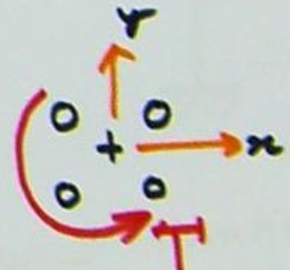
Individual pile



K_z ✓ $\quad K_x$ ✓ $\quad \underbrace{K_{\theta y} \times \quad K_{\theta x} \times \quad K_{\theta z} \times}_{\text{Min. 2 piles}}$



$K_z \rightarrow K_{\theta y}$ or $K_{\theta x}$



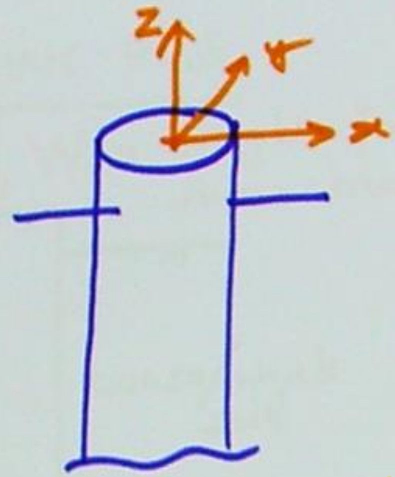
K_x or $K_y \rightarrow K_{\theta z}$

PILE GROUP

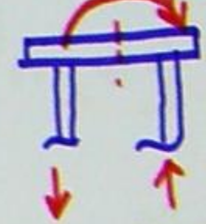
- Vertical K_z
  - End bearing action
  - Friction action
- Horizontal K_x
  - Modulus of subgrade reaction
- Bending $K_{\theta x}$ or $K_{\theta y}$
  - Dependent on K_z
- Torsion or yawning
  - Dependent on K_x or K_y

MACHINE FDNS. ON PILES.

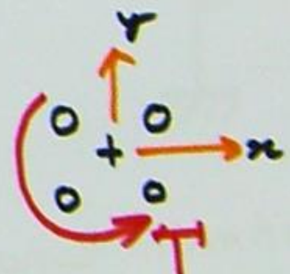
Individual pile



K_z ✓ K_x ✓ $\underbrace{K_{\theta y} \times \quad K_{\theta x} \times \quad K_{\theta z} \times}_{\text{Min. 2 piles}}$



$K_z \rightarrow K_{\theta y}$ or $K_{\theta x}$



K_x or $K_y \rightarrow K_{\theta z}$

PILE GROUP

- Vertical K_z
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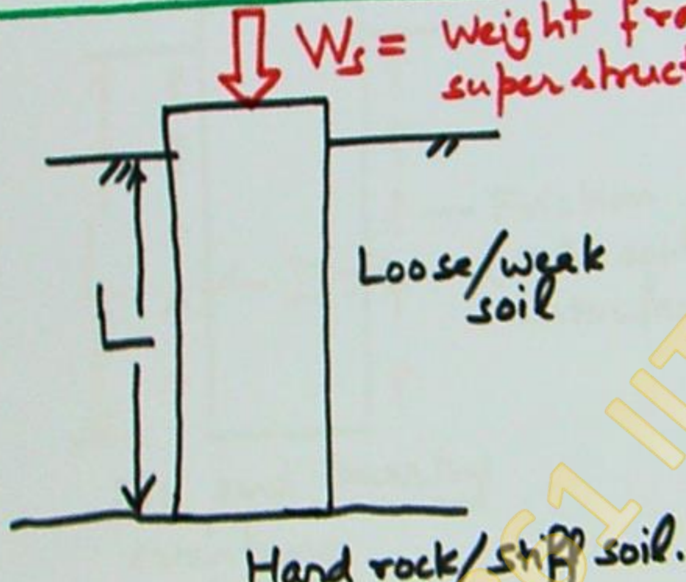
  - Dependent on K_x or K_y

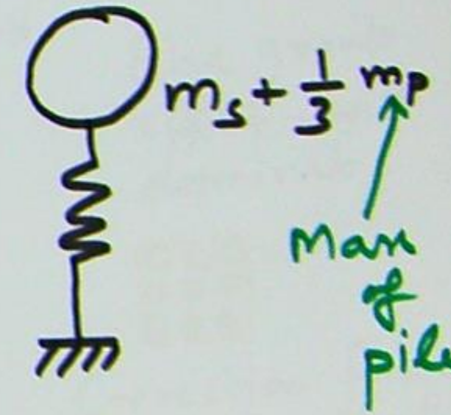
VERTICAL STIFFNESS

A) END BEARING PILE



Lumped mass = $\frac{W_s}{g} = m_s$ (N / Kg)

$\equiv$



Idealize pile as a rigid bar undergoing axial vibrations

$$\omega_N = \frac{\beta}{L}\sqrt{\frac{E}{\rho}}$$

E — Young's modulus

ρ — Density

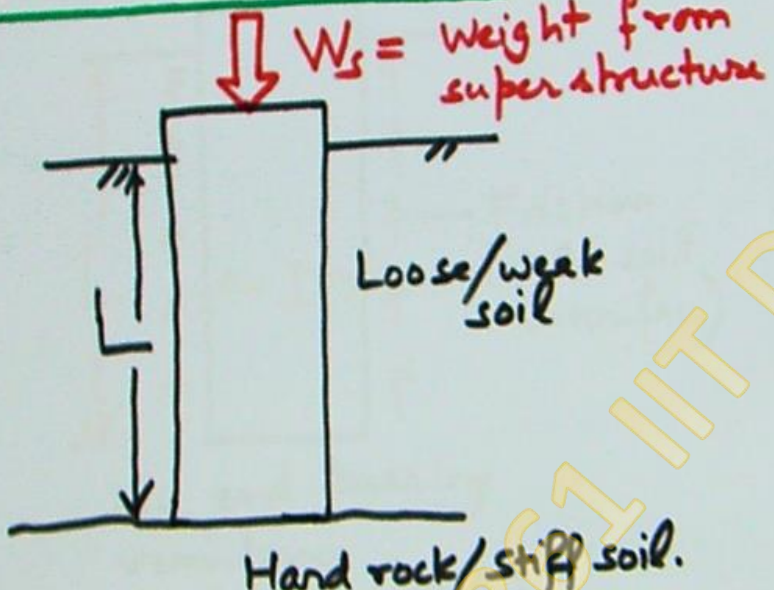
$\beta \tan\beta = \alpha = \frac{m_p}{m_s}$

Table 3.5 Text book

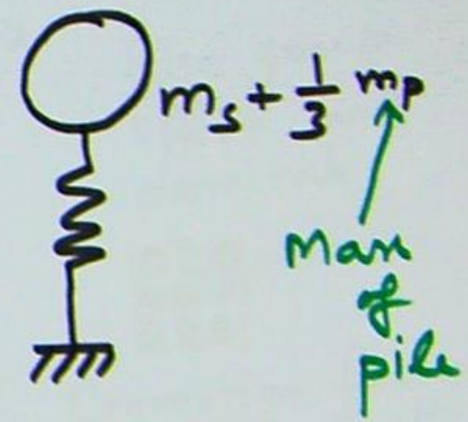
$$k_z = \left(1 + \frac{\alpha}{3}\right) m_s \, \omega_N^2$$

VERTICAL STIFFNESS

A) END BEARING PILE



$\equiv$



Lumped mass = $\frac{W_s}{g} = m_s$

N

kg

Idealize pile as a rigid bar undergoing axial vibrations

$$\omega_N = \frac{\beta}{L}\sqrt{\frac{E}{\rho}}$$

E — Young's modulus

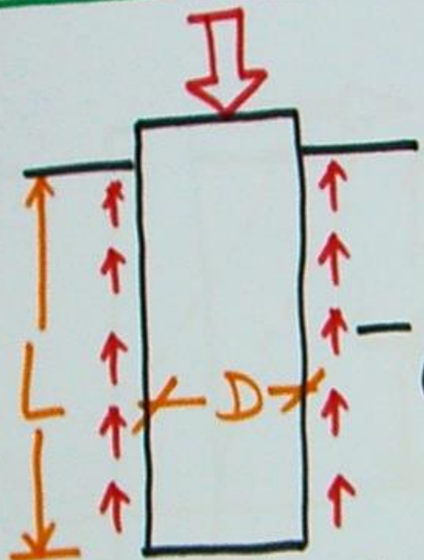
ρ — Density

$$\beta \tan\beta = \alpha = \frac{m_p}{m_s}$$

Table 3.5 Text book

$$k_z = \left(1+\frac{\alpha}{3}\right) m_s \omega_N^2$$

B) FRICTION PILE



— Friction (pile-soil interface)

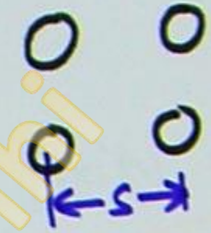
No end bearing resistance (loose soil)

$$K_z = C_p \, A_{pile}$$

C_p (kN/m³)

$A_{pile} = \pi D L$

Soviet Code CH-18-58



$s > 6 D_p$

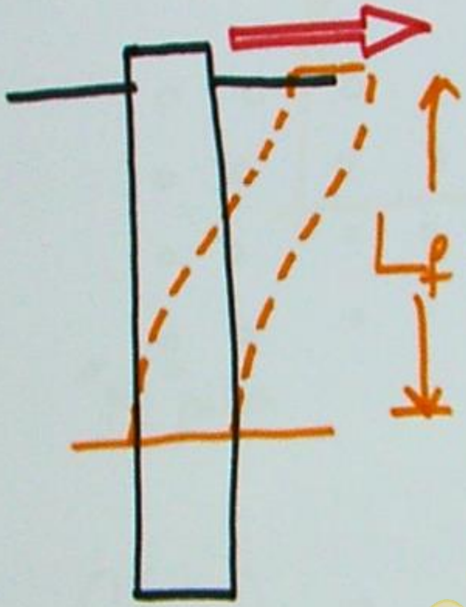
Else group effect will dominate.

Correction factor "α"

$$3 < \frac{s}{D_p} < 6$$

$$\frac{s}{D_p} \nless 3$$

HORIZONTAL CAPACITY



L_f = Length of fixity $\sim\left(\frac{1}{4} \text{ to } \frac{1}{2}\right)L$

I_x

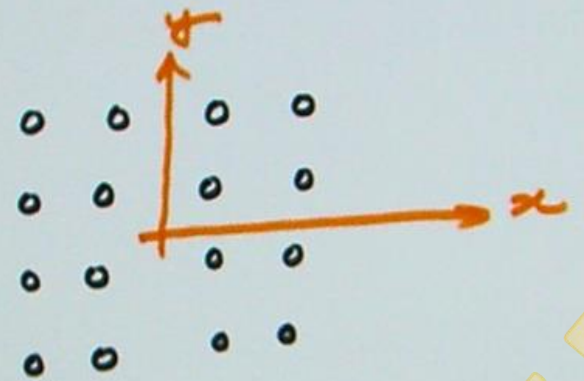
$$K_x \text{ or } K_y = \frac{12EI}{L_f^3}$$

I_y

Not dependent on type of pile

FRICTION or **END BEARING**

GROUP ACTION



"N" piles

$$(K_z)_g = N K_z$$

$$(K_x)_g = N K_x$$

or $(K_y)_g$

$$(K_{\theta y})_g = K_z \left(\sum_N x_i^2\right)$$

$$(K_{\theta x})_g = K_z \left(\sum_N y_i^2\right)$$

$$(K_{\theta z})_g \text{ or } (K_\phi)_g = K_x \left(\sum_N r_i^2\right) = K_x \left[\sum_N (x_i^2 + y_i^2)\right]$$

$$= K_x \left[\sum_N (x_i^2 + y_i^2)\right]$$

FRAME TYPE FOUNDATIONS

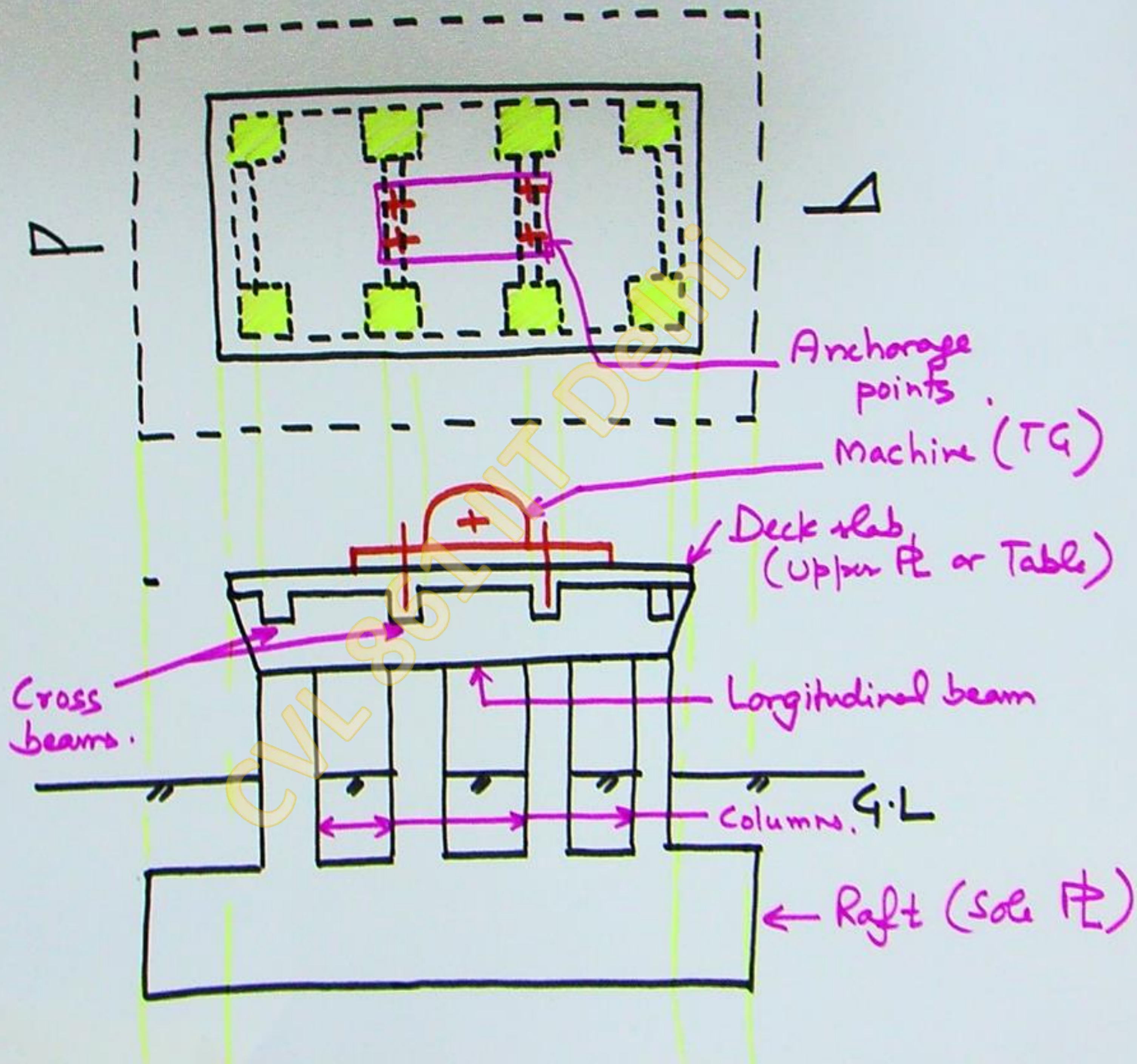
Recommended for medium to high frequency machines generating exceptionally high forces.

Eg. TURBOGENERATORS (power plants)

Earlier, wall type foundations were provided for these.

ADVANTAGES -

1. Savings (space, materials)
2. Greater accessibility to m/c parts.
3. Structurally better systems.
   * Less cracking
   * Less settlement
   * Less thermal effects


Anchorage points.
Machine (TG)
Deck slab (Upper PL or Table)
Cross beams.
Longitudinal beam
Columns.
G.L
Raft (Sole PL)
CVL 801 IIT Delhi

PRINCIPAL DESIGN CRITERIA

1. FREQUENCY

f_N should be at least 20% (preferable 50%) away from f_m.

$f_N < 0.8 f_m$ OR $f_N > 1.2 f_m$

$r > 1.25$ $\quad$ $r < 0.833$

Preference →

$f_N < 0.5 f_m$ OR $f_N > 1.5 f_m$

$r > 0.2$ or $r < 0.67$

2. AMPLITUDE :

	RPM < 3000 (50 Hz)	RPM ≥ 3000 (50 Hz)
Vertical	40 μm	20 μm
Horizontal	70 μm	40 μm

Horizontal/vertical amplitudes need not be combined since these do not occur simultaneously.

3. ALLOWABLE NET BEARING PRESSURE :

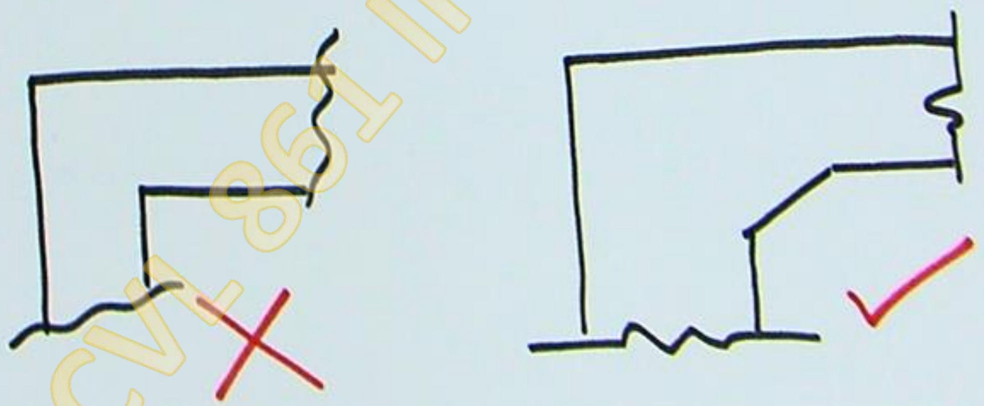
Under worst load combination

$$\sigma_{all,\cancel{net}} \leq 80\% \text{ of } \sigma_{all,net}$$

SPECIAL CONSIDERATIONS IN ANALYSIS/DESIGN

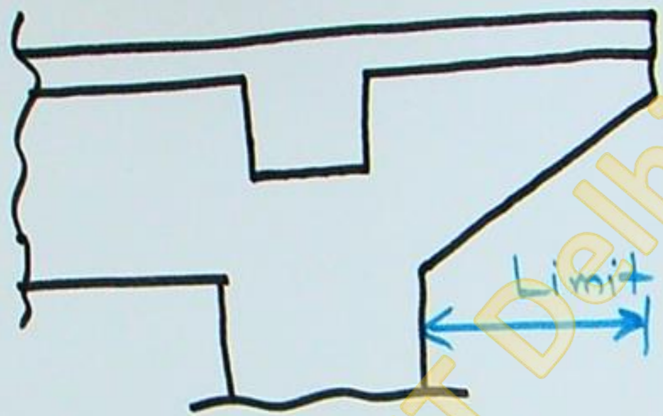
1. Should be isolated/seperated from main building. Gaps, expansion joints, trenches etc.

2. Stress concentration to be avoided



Less cracking, higher joint stiffness

3. Avoid overhangs



Large moments due to inertial effects.

4. Base slab should be very rigid.

$\Longrightarrow$ Ensure high degree of fixity @ column base

T_b

$T_b \not< \begin{cases} 1.\ \text{Width of col.} \\ 2.\ 1/10\ L_{slab} \\ 3.\ 2m \\ 4.\ 0.07\ L^{4/3} \\ L = \text{Avg. of 2 adjacent clear spans} \end{cases}$

$$\left(\frac{EI}{L}\right)_{slab} > 2\left(\frac{EI}{L}\right)_{cols}$$

5. Weight of base slab > Weight of superstructure + Weight of m/c

6. Beams/frames to be located directly under bearing/loading points.

Eccentric loading ✗

7. Vertical forces due to m/c (including DL) should pass through CG of base PL.

8. Fatigue factor FF = 2.0

METHODS OF ANALYSIS

Static Analysis

Dynamic Analysis

ANALYSIS

MANUAL

Single or Double DOF idealization.

- Resonance method. SDOF
- Amplitude method. DDOF
- Combined method.

Tedious, less accurate

NUMERICAL

Space frame analysis

- 2D
- 3D

Braces

Rigid diaphragm

Solid 3D FEM.

- ANSYS
- ABACUS

Soil - structure interaction

DYNAMIC ANALYSIS

1. Free Vibrational Analysis

$$[M]\{\ddot{x}\} + \cancelto{0}{[C]}\{\dot{x}\} + [K]\{x\} = 0$$

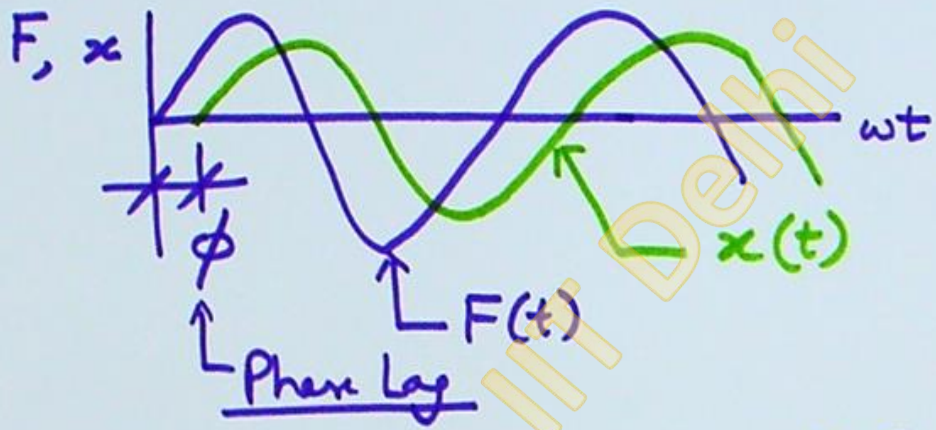
Standard EIGEN VALUE problem

Natural frequencies, mode shapes.

In operating range

3D frame, solid FEM

2. Harmonic (Forced Vibration) Analysis



$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F_0\} e^{j\omega_m t}$$

$$\{\dot{x}\} = j\omega\{x\} \qquad \{\ddot{x}\} = -\omega_m^2\{x\}$$

$$\left\{-\omega_m^2[M] + [K] + j\omega[C]\right\}\{x_0\}\cancel{e^{j\omega_m t}}\, e^{-j\phi} = \{F_0\}\cancel{e^{j\omega_m t}}$$

Close-form soln. like static analysis

3. Transient Dynamic Analysis