DEPARTMENT OF CIVIL ENGINEERING

# IIT DELHI

# IMPEDANCE BASED IDENTIFICATION AND SHM USING EMI TECHNIQUE

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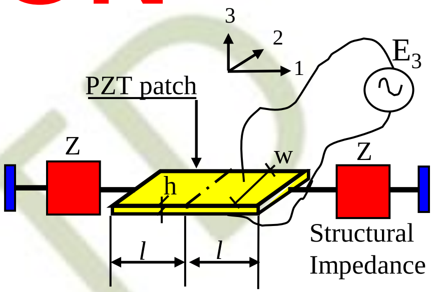
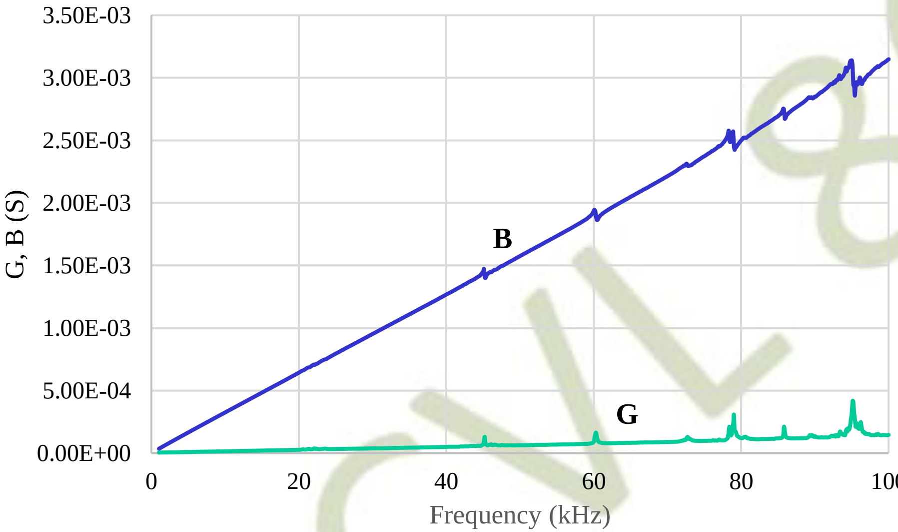
# OBSERVATIONS ON SIGNATURES

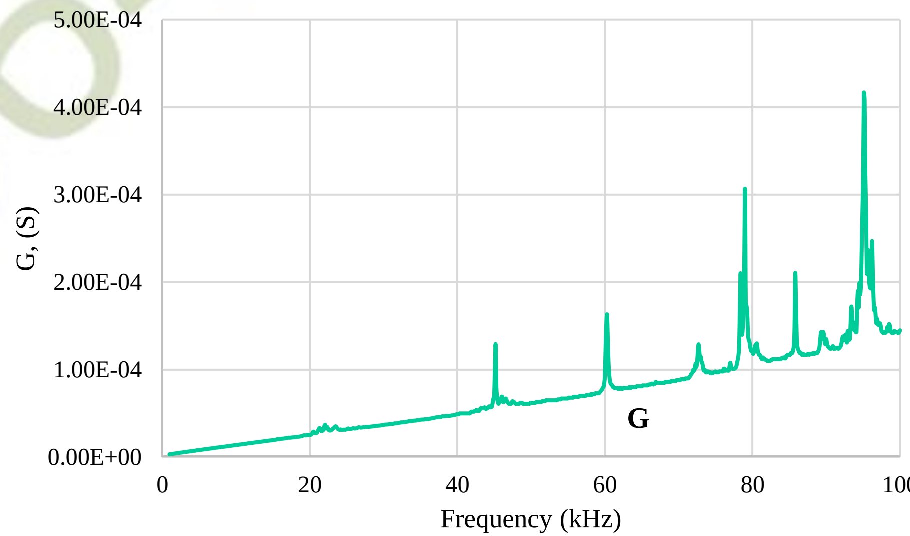



$$\overline{Y} = 2\omega j \frac{w\,l}{h}\left[\overline{\varepsilon_{33}^{T}} + \left(\frac{Z_a}{Z+Z_a}\right) d_{31}^{2}\,\overline{Y^{E}}\left(\frac{\tan kl}{kl}\right) - d_{31}^{2}\,\overline{Y^{E}}\right] = G + Bj$$

**1 D Impedance Approach (Liang, Sun and Rogers, 1993, 1994)**

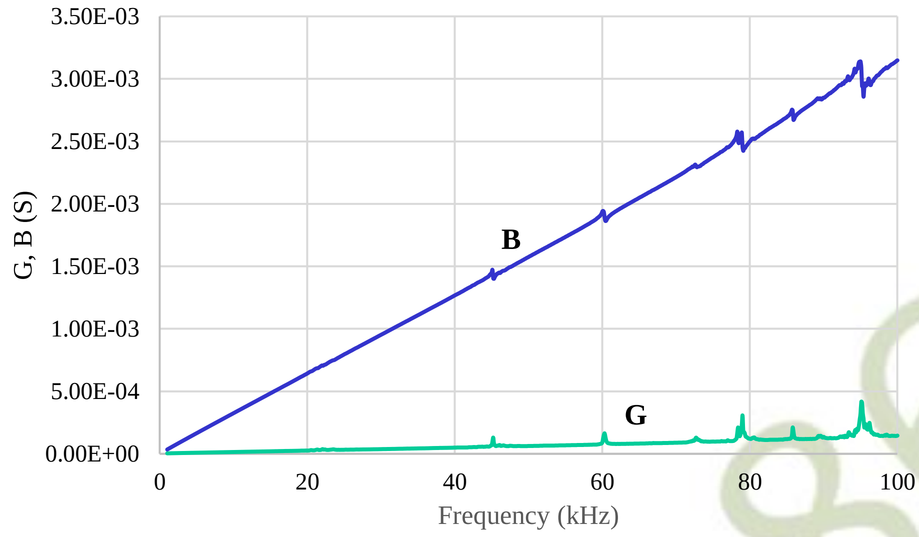
## Typical characteristics of two parts G and B

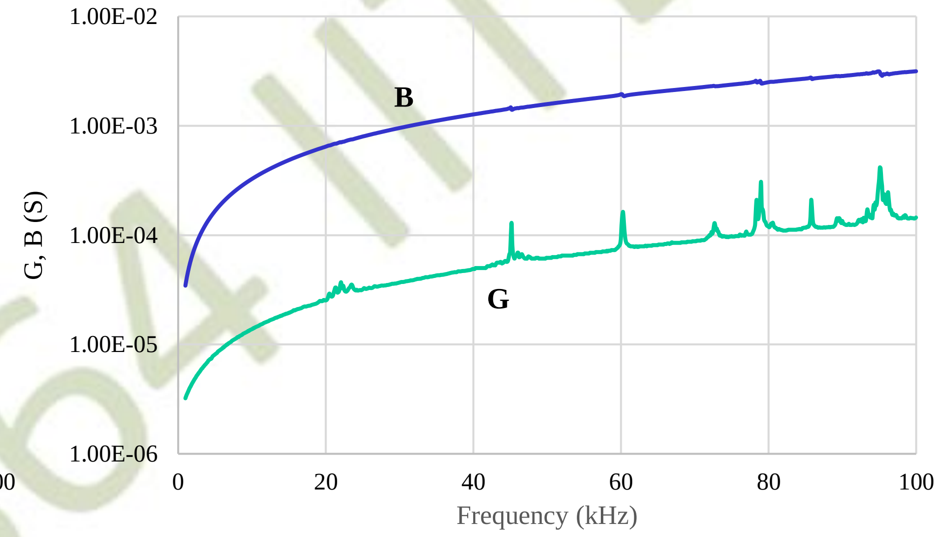



**It can be shown that if |Z| >> |$Z_a$| peaks shall correspond to structural resonance only**

# OBSERVATIONS ON SIGNATURES




## MAIN OBSERVATIONS:

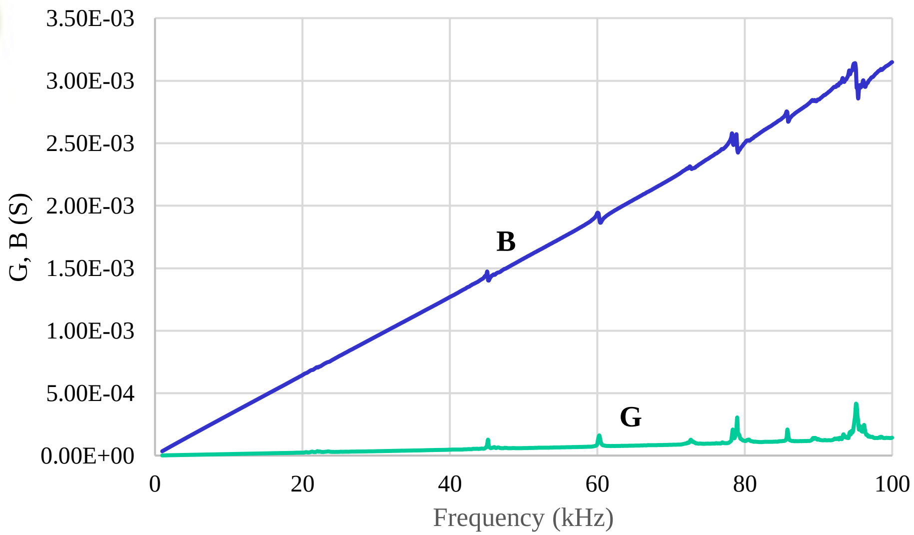
1. Magnitude of B several times that of G.
2. B largely straight line, peaks less dominant.
3. Both G and B exhibit linear variation with frequency, especially exhibited in B

# EXPLAINATION OF OBSERVATIONS

$$\overline{Y} = 2\omega j \frac{w\,l}{h}\left[\overline{\varepsilon_{33}^{T}} + \left(\frac{Z_a}{Z+Z_a}\right) d_{31}^{2}\overline{Y^{E}}\left(\frac{\tan kl}{kl}\right) - d_{31}^{2}\overline{Y^{E}}\right] = G + Bj$$

**CAPACITIVE PART (Large value compared to other components)**

**Expand the real and imaginary parts and show that observations can be explained using 1D impedance model**



# EXPLAINATION OF OBSERVATIONS

$$\bar{Y} = 2\omega j \frac{w\,l}{h}\left[\overline{\varepsilon_{33}^{T}} + \left(\frac{Z_a}{Z+Z_a}\right) d_{31}^{2}\overline{Y^{E}}\left(\frac{\tan kl}{kl}\right) - d_{31}^{2}\overline{Y^{E}}\right] = G + Bj$$

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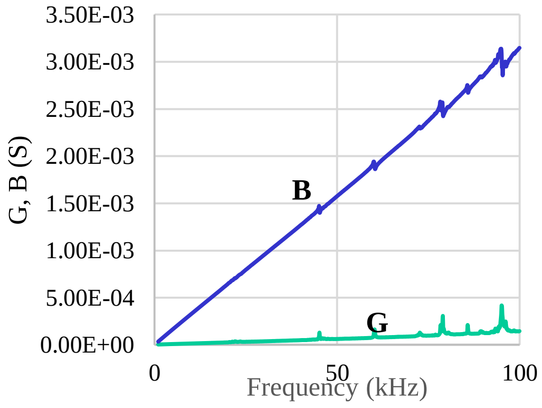


# QUASITATIC APPROXIMATION

$$\overline{Y} = 2\omega j \frac{w\,l}{h}\left[\overline{\varepsilon_{33}^{T}} + \left(\frac{Z_a}{Z+Z_a}\right) d_{31}^2 \overline{Y^E}\left(\frac{\tan kl}{kl}\right) - d_{31}^2 \overline{Y^E}\right] = G + Bj$$

Under small frequency of excitation $\omega < \frac{1}{5}\omega_N$

$$\frac{\tan kl}{kl} \rightarrow 1$$

$$\overline{Y_{QS}} = 2\omega j \frac{w\,l}{h}\left[\overline{\varepsilon_{33}^{T}} - \left(\frac{Z}{Z+Z_a}\right) d_{31}^2 \overline{Y^E}\right]$$

Much more convenient to explain the previous observations using this equation

....... Derive. final expression in form of A+Bj

# RESONANCE FREQUENCY OF PZT PATCH

**Mechanical impedance of PZT patch**

$$Z_a = \frac{kwh\overline{Y^E}}{(j\omega)\tan(kl)}$$

How to determine the natural frequency of PZT patch? (a) Theoretically (b) Experimentally

At resonance frequency of the PZT patch (under axial vibrations), $Z_a$ would be ???

$$kl = (2n-1)\frac{\pi}{2}$$

$$k = \omega\sqrt{\frac{\rho}{\overline{Y^E}}}$$

Find out first three natural frequencies for 10x10x0.3 mm PZT patch

= 143 (2n-1) (kHz)

# Material Data

SPECIFIC PARAMETERS OF THE STANDARD MATERIALS

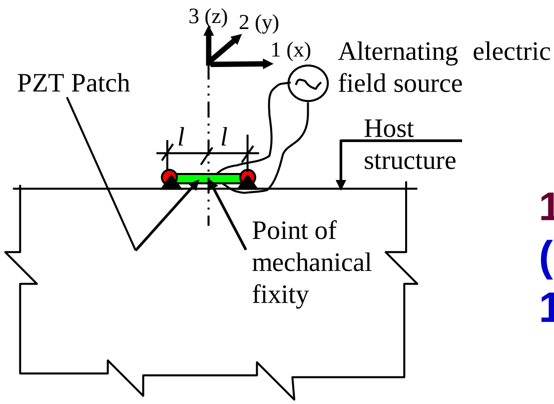
Soft PZT materials

| | | | | Soft PZT materials | | | |
|---|---|---|---|---|---|---|---|
| | | | Unit | PIC151 | PIC255/ PIC252[1)] | PIC155 | PIC153 |
| **Physical and dielectric properties** | | | | | | | |
| Density | | $\rho$ | $g/cm^3$ | 7.80 | 7.80 | 7.80 | 7.60 |
| Curie temperature | | $T_c$ | °C | 250 | 350 | 345 | 185 |
| Relative permittivity | in the polarization direction | $\varepsilon_{33}^T/\varepsilon_0$ | | 2400 | 1750 | 1450 | 4200 |
| | ⊥ to polarity | $\varepsilon_{11}^T/\varepsilon_0$ | | 1980 | 1650 | 1400 | |
| Dielectric loss factor | | $\tan\delta$ | $10^{-3}$ | 20 | 20 | 20 | 30 |
| **Electromechanical properties** | | | | | | | |
| Coupling factor | | $k_p$ | | 0.62 | 0.62 | 0.62 | 0.62 |
| | | $k_t$ | | 0.53 | 0.47 | 0.48 | |
| | | $k_{31}$ | | 0.38 | 0.35 | 0.35 | |
| | | $k_{33}$ | | 0.69 | 0.69 | 0.69 | |
| | | $k_{15}$ | | | 0.66 | | |
| Piezoelectric charge coefficient | | $d_{31}$ | | -210 | -180 | -165 | |
| | | $d_{33}$ | $10^{-12}$ C/N | 500 | 400 | 360 | 600 |
| | | $d_{15}$ | | | 550 | | |
| Elastic compliance coefficient | | $S_{11}^E$ | $10^{-12}$ $m^2/N$ | 15.0 | 16.1 | 15.6 | |
| | | $S_{33}^E$ | | 19.0 | 20.7 | 19.7 | |



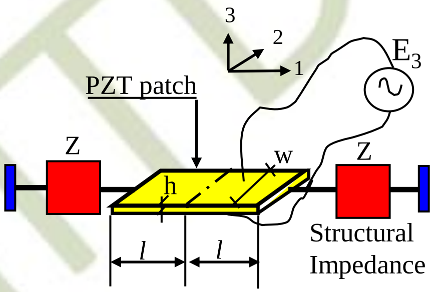
**Permittivity of free space = 8.85e-12 F/m**

# EXISTING ELECTRO-MECHANICAL IMPEDANCE MODELS




## 1D Vibrations

**1 D Impedance Approach (Liang, Sun and Rogers, 1993, 1994)**




$$\overline{Y} = G + Bj = \omega j \frac{wl}{h}\left[\overline{\varepsilon_{33}^{T}} + \left(\frac{Z_a}{Z + Z_a}\right) d_{31}^{2}\overline{Y^{E}}\left(\frac{\tan \kappa l}{\kappa l}\right) - d_{31}^{2}\overline{Y^{E}}\right]$$

$$Z = x + yj \qquad\qquad F_{PZT} = -Z\dot{u}$$

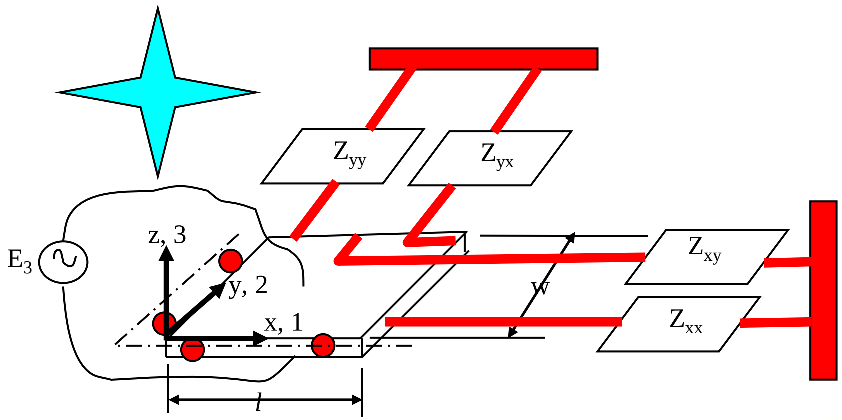
"Z" can be extracted from measured "G" and "B" (computational procedure).

But……………..**(1) How about structures which interact in 2D manner**

**(2) Is it possible to accurately predict PZT parameters?**

**(3) End conditions**

# 2D ELECTRO-MECHANICAL IMPEDANCE MODELs




2 D Impedance Approach
(Zhou, Liang, Rogers, 1995)

## 2D Vibrations

$$F_{PZT} = -Z\dot{u}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = -\begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} \quad \overline{Y} = G + Bj = j\omega\frac{wl}{h}\left[\overline{\varepsilon_{33}^{T}} - \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)} + \frac{d_{31}^2\overline{Y^E}}{(1-\nu)}\begin{Bmatrix}\frac{\sin\kappa l}{l} & \frac{\sin\kappa w}{w}\end{Bmatrix}N^{-1}\begin{Bmatrix}1\\1\end{Bmatrix}\right]$$

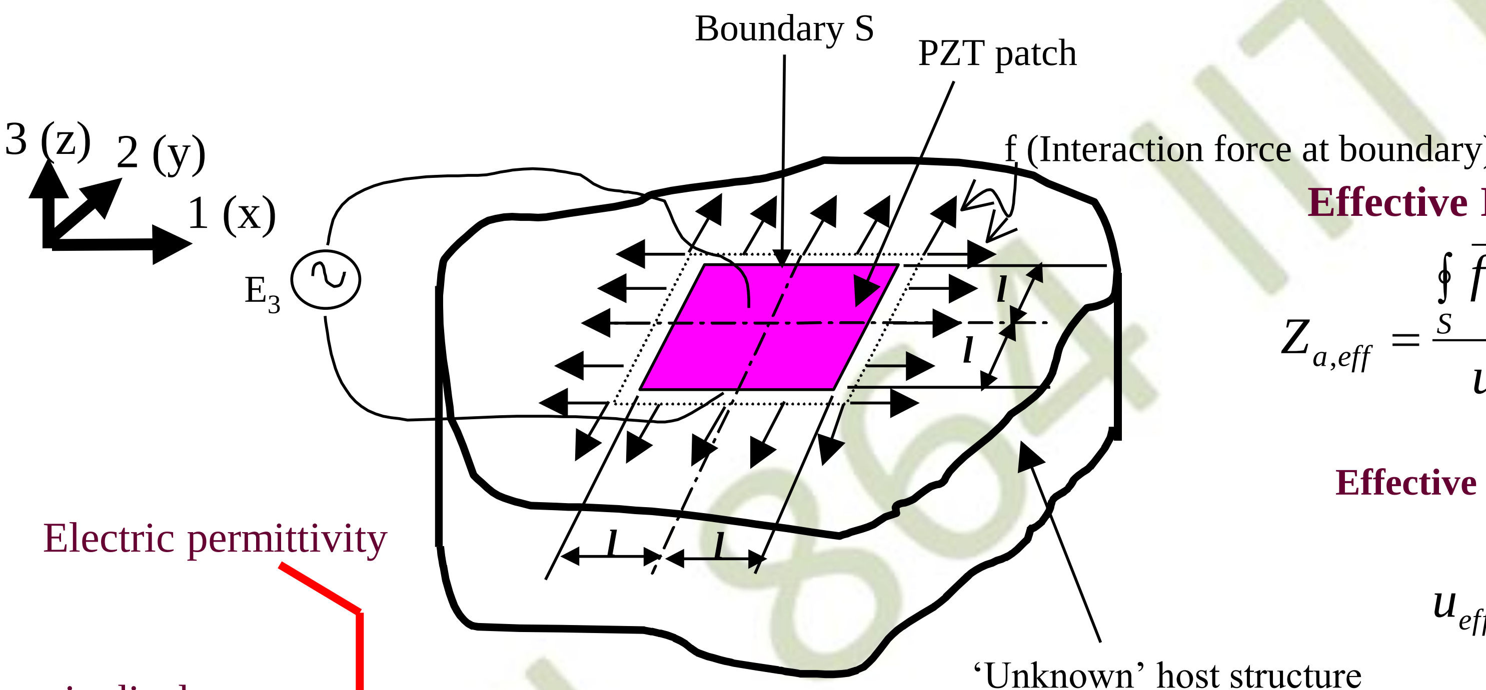
$$N = \begin{bmatrix} \kappa\cos(\kappa l)\left\{1-\nu\frac{w}{l}\frac{Z_{xy}}{Z_{axx}}+\frac{Z_{xx}}{Z_{axx}}\right\} & \kappa\cos(\kappa w)\left\{\frac{l}{w}\frac{Z_{yx}}{Z_{ayy}}-\nu\frac{Z_{yy}}{Z_{ayy}}\right\} \\ \kappa\cos(\kappa l)\left\{\frac{w}{l}\frac{Z_{xy}}{Z_{axx}}-\nu\frac{Z_{xx}}{Z_{axx}}\right\} & \kappa\cos(\kappa w)\left\{1-\nu\frac{l}{w}\frac{Z_{yx}}{Z_{pyy}}+\frac{Z_{yy}}{Z_{ayy}}\right\} \end{bmatrix}$$

**LIMITATIONS:**

(1) 2 equations and 8 unknowns- cannot extract structural impedance.

(2) Force transmission is assumed to occur at end points of PZT patch only.

# SIMPLIFIED 2D IMPEDANCE MODEL




**Effective Impedance**

$$Z_{a,eff} = \frac{\oint_S \vec{f}.\hat{n}ds}{\dot{u}_{eff}} = \frac{F}{\dot{u}_{eff}}$$

**Effective Displacement**

$$u_{eff} = \frac{\delta A}{p_o}$$

$$\begin{bmatrix} D \\ S \end{bmatrix} = \begin{bmatrix} \varepsilon^T & d^d \\ d^c & s^E \end{bmatrix} \begin{bmatrix} E \\ T \end{bmatrix}$$

Electric displacement ($D$), Strain ($S$), Electric permittivity ($\varepsilon^T$), Strain coefficient ($d^c$), Elastic compliance ($s^E$), Electric field ($E$), Stress ($T$)

$$D_3 = \overline{\varepsilon_{33}^T} E_3 + d_{31}(T_1 + T_2)$$

$$S_1 = \frac{T_1 - \nu T_2}{\overline{Y^E}} + d_{31} E_3$$

$$S_2 = \frac{T_2 - \nu T_1}{\overline{Y^E}} + d_{31} E_3$$

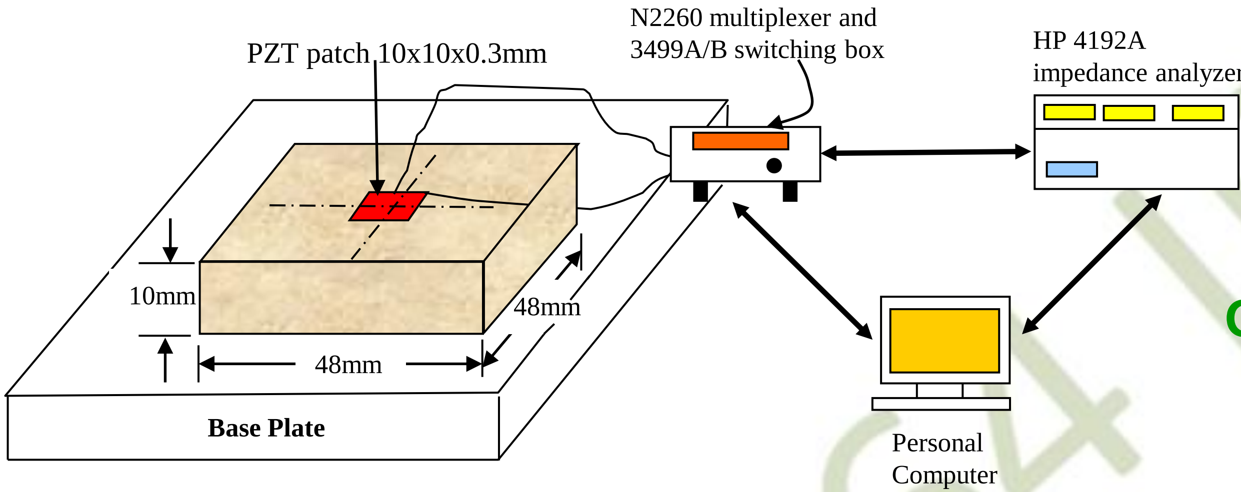
# GENERALIZED ELECTRO-MECHANICAL IMPEDANCE MODEL

$$\overline{Y} = G + Bj = 4\omega j \frac{l^2}{h}\left[\overline{\varepsilon_{33}^{T}} - \frac{2d_{31}^{2}\overline{Y^{E}}}{(1-\nu)} + \frac{2d_{31}^{2}\overline{Y^{E}}}{(1-\nu)}\left(\frac{Z_{a,eff}}{Z_{s,eff}+Z_{a,eff}}\right)\left(\frac{\tan \kappa l}{\kappa l}\right)\right]$$

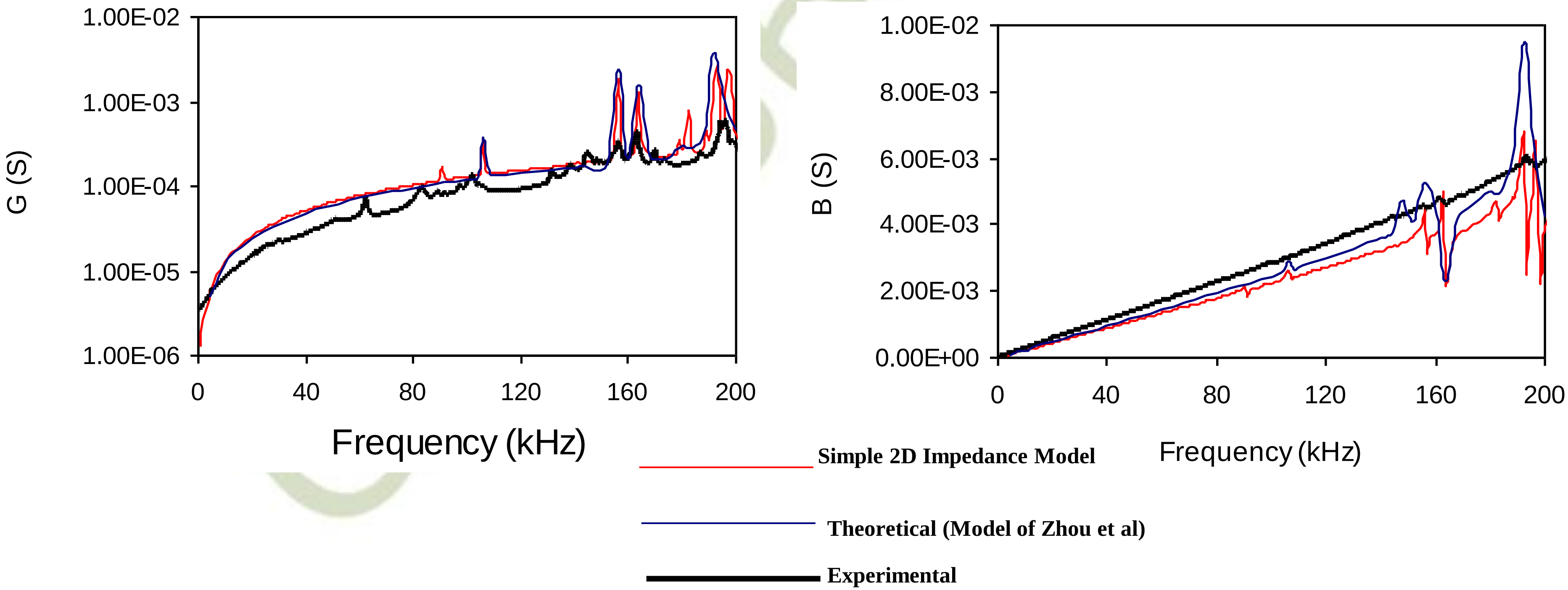
$$Z_{a,eff} = \frac{2\kappa l h \overline{Y^{E}}}{j\omega(\tan kl)(1-\nu)}$$

**A single complex term for $Z_{s,eff}$ accounts for two dimensional interaction of the PZT patch with the host structure. Thus, the mechanical impedance of the structure can be extracted from the measured electrical impedance.**

# EXPERIMENTAL VERIFICATION




**OBSERVATIONS??**



# PREVIOUS COMPARISON

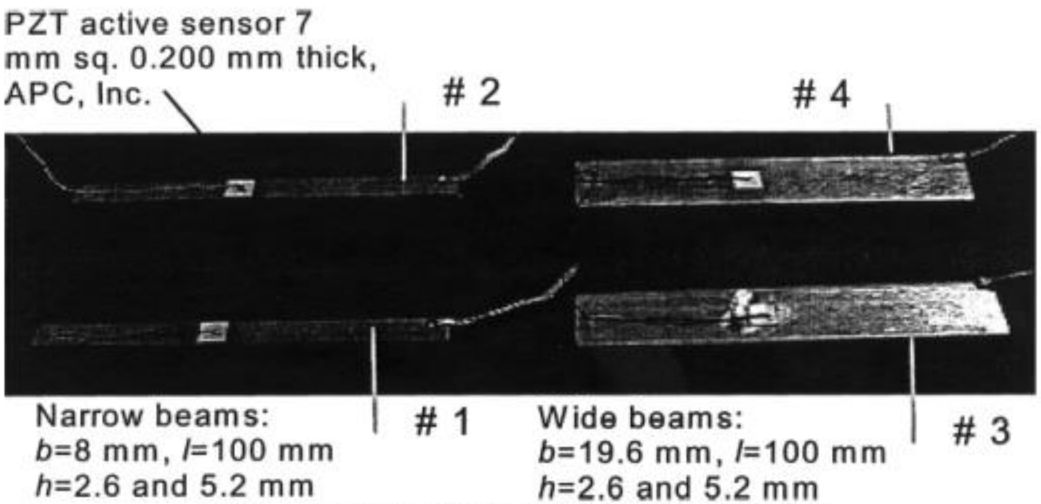



**Figure 17.** *Experimental specimens to simulate one-dimensional stucture.*

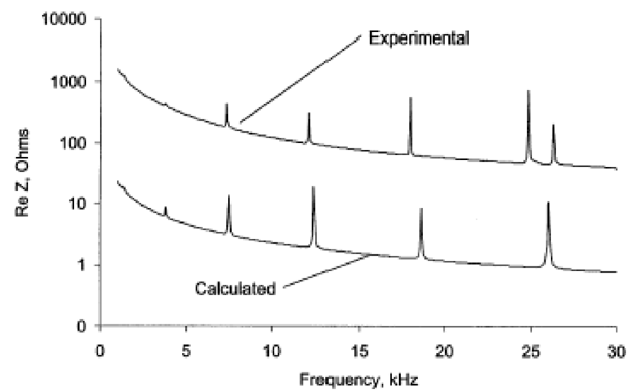



**Figure 18.** *Experimental and calculated spectra of frequencies for beam #1 of Figure 17.*

## Giurgiutiu and Zagrai (2000)

# GENERALIZED ELECTRO-MECHANICAL IMPEDANCE MODEL

$$\overline{Y} = G + Bj = 4\omega j \frac{l^2}{h}\left[\overline{\varepsilon_{33}^T} - \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)} + \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)}\left(\frac{Z_{a,eff}}{Z_{s,eff}+Z_{a,eff}}\right)\left(\frac{\tan\kappa l}{\kappa l}\right)\right]$$

$$Z_{a,eff} = \frac{2\kappa lh\overline{Y^E}}{j\omega(\tan kl)(1-\nu)}$$

**A single complex term for $Z_{s,eff}$ accounts for two dimensional interaction of the PZT patch with the host structure. Thus, the mechanical impedance of the structure can be extracted from the measured electrical impedance.**

**For free PZT patch, $Z_{s,eff} = 0$**

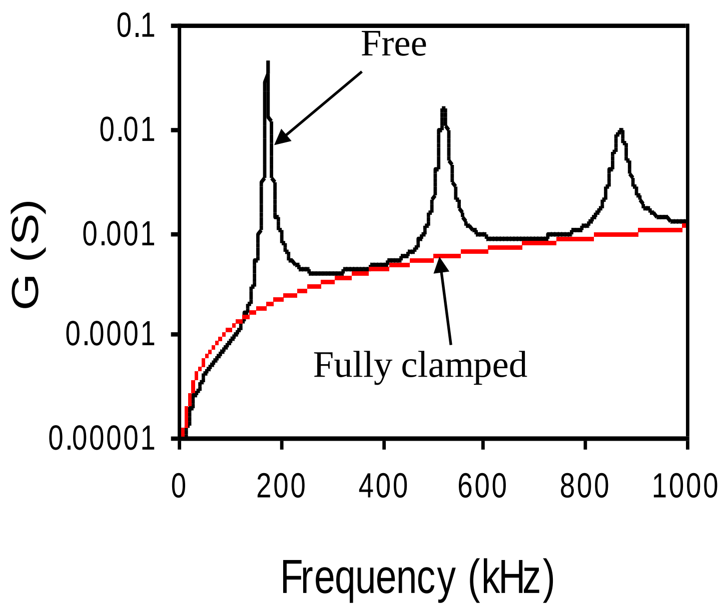
$$\overline{Y_{free}} = G + Bj = 4\omega j \frac{l^2}{h}\left[\overline{\varepsilon_{33}^T} - \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)} + \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)}\left(\frac{\tan\kappa l}{\kappa l}\right)\right]$$

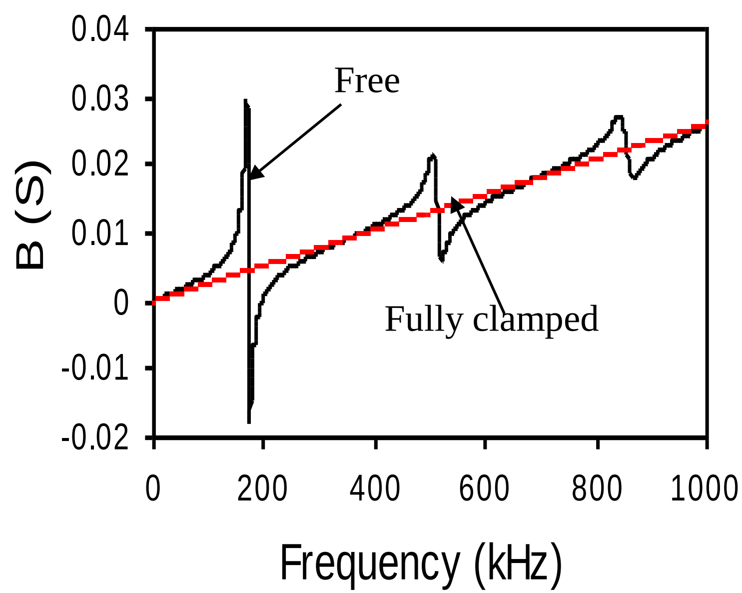
# THEORETICAL FREE PIEZO SIGNATURES

$$\bar{Y} = G + Bj = 4\omega j \frac{l^2}{h}\left[\bar{\varepsilon}_{33}^T - \frac{2d_{31}^2 \bar{Y}^E}{(1-\nu)} + \frac{2d_{31}^2 \bar{Y}^E}{(1-\nu)}\left(\frac{Z_{a,eff}}{Z_{s,eff}+Z_{a,eff}}\right)\left(\frac{\tan \kappa l}{\kappa l}\right)\right]$$

**Free: 0**

**Clamped: ∞**



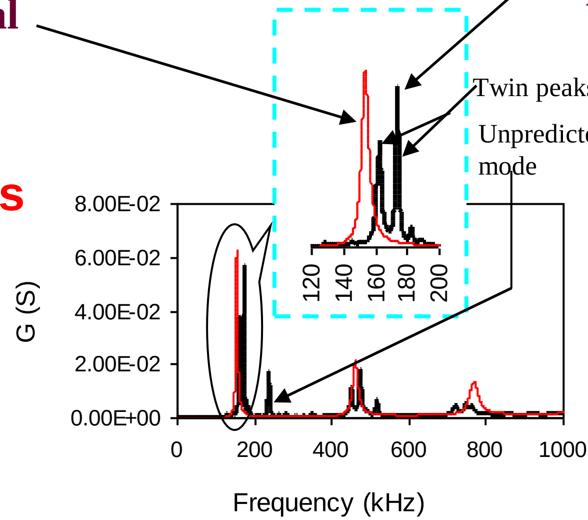
# ANALYSIS OF FREE PZT SIGNATURES

$$\overline{Y}_{free} = 4\omega j \frac{l^2}{h}\left[\overline{\varepsilon_{33}^T} + \frac{2d_{31}^2 \overline{Y^E}}{(1-\nu)}\left(\frac{\tan \kappa l}{\kappa l} - 1\right)\right] \qquad G_f = \frac{8\pi f l^2}{h}\left[\varepsilon_{33}^T \delta - \frac{2d_{31}^2 Y^E}{(1-\nu)}\{\eta(r-1)+t\}\right] \qquad B_f = \frac{8\pi f l^2}{h}\left[\varepsilon_{33}^T + \frac{2d_{31}^2 Y^E}{(1-\nu)}\{(r-1)-\eta t\}\right]$$

**For low frequencies**

Analytical

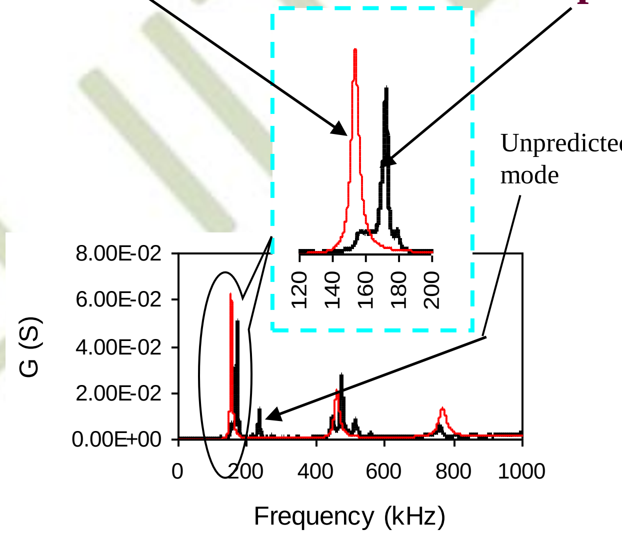
Experimental




**FREE PZT PATCH-1**

Analytical

Experimental




**FREE PZT PATCH-2**

**(1) TWIN PEAKS**

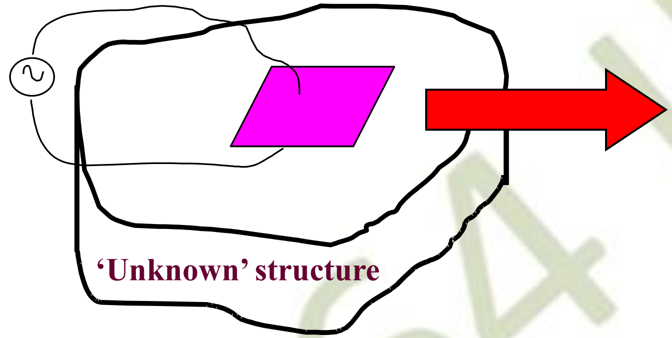
**(2) EXPERIMENTAL PEAK FREQUENCY HIGHER**

$\varepsilon_{33}^T$ ➔ **1.7919x10$^{-8}$ F/m and 1.7328x10$^{-8}$ F/m (against a value of 2.124x10$^{-8}$ F/m provided by the manufacturer).**

$\delta$ ➔ **was worked out to be 0.0238 and 0.0225 respectively, against a value of 0.015 supplied by the manufacturer.**

**Since we rely entirely on the bonded piezo- transducer for structural identification, we must accurately model the behaviour of the PZT patch**

FREE PZT PATCH




UPDATED MODEL

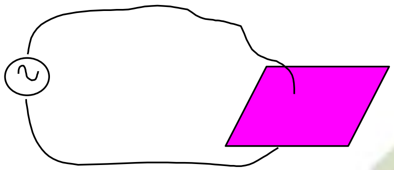
$$\bar{Y} = G + Bj = 4\omega j \frac{l^2}{h}\left[\overline{\varepsilon_{33}^{T}} - \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)} + \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)}\left(\frac{Z_{a,eff}}{Z_{s,eff}+Z_{a,eff}}\right)\bar{T}\right] \qquad Z_{a,eff} = \frac{2h\overline{Y^E}}{j\omega(1-\nu)\bar{T}}$$

$$\bar{T} = \begin{cases} \dfrac{\tan(C\kappa l)}{C\kappa l} & \text{for single-peak behaviour.} \\[2ex] \dfrac{1}{2}\left(\dfrac{\tan C_1\kappa l}{C_1\kappa l} + \dfrac{\tan C_2\kappa l}{C_2\kappa l}\right) & \text{for twin-peak behaviour.} \end{cases}$$

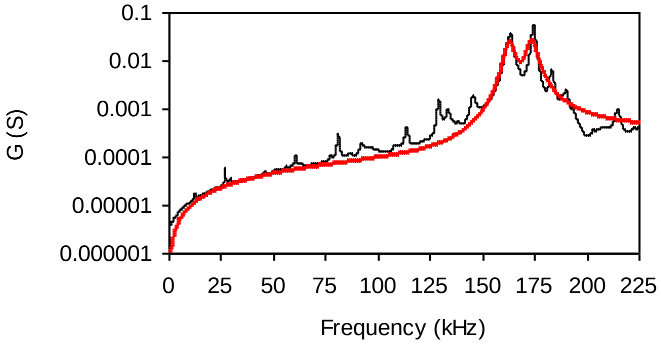
**C, $C_1$ and $C_2$ to be determined from free PZT signature**

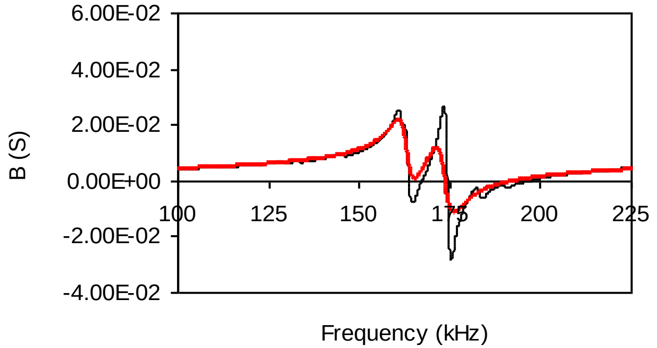
# UPDATED MODEL OF PZT PATCH

**FREE PZT PATCH**

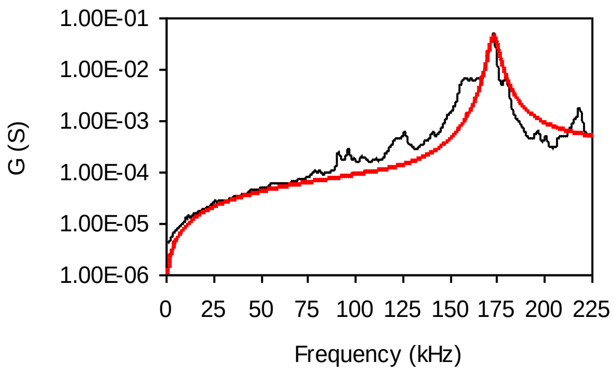


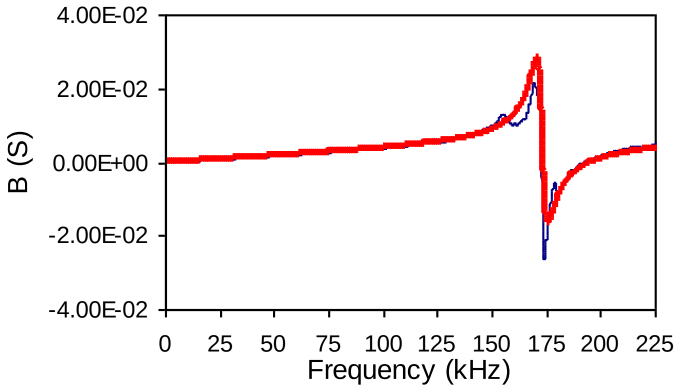
**FREE PZT PATCH-1**



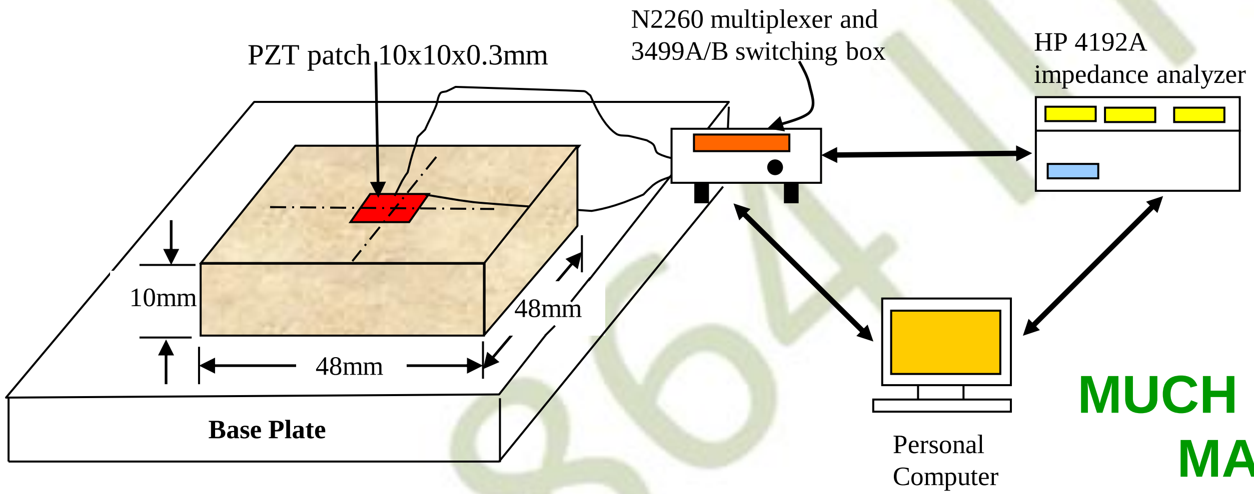

**FREE PZT PATCH-2**



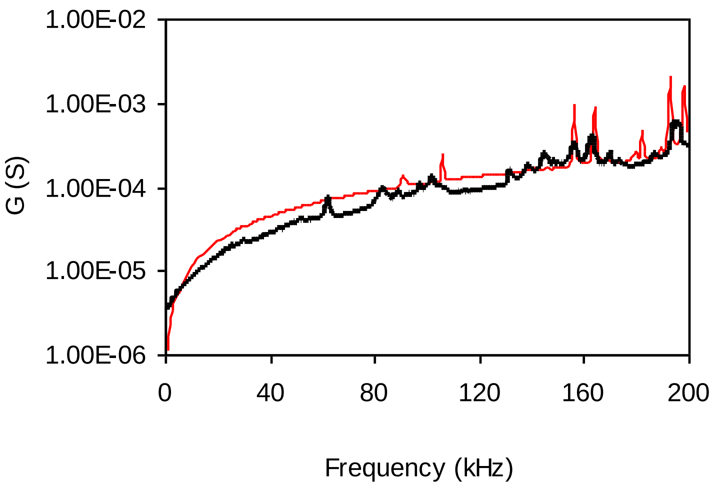



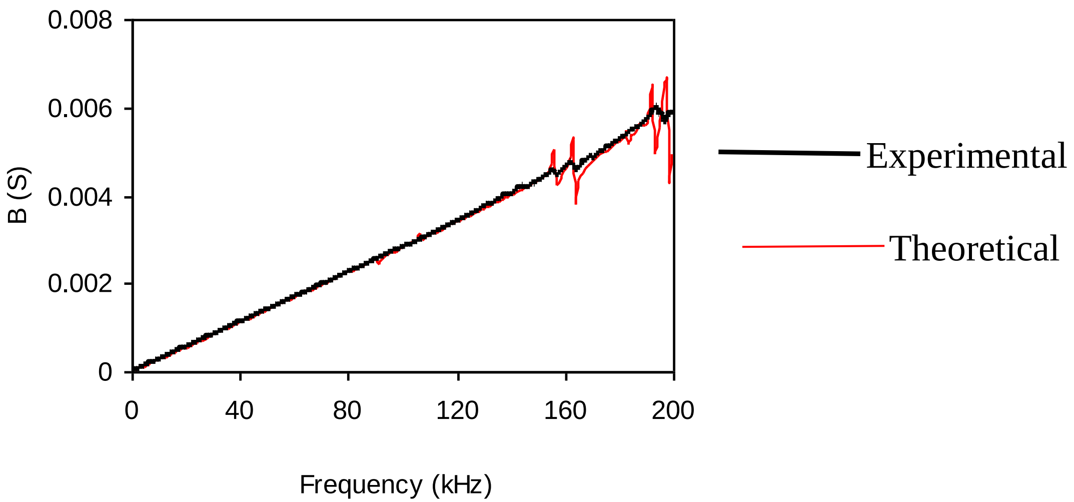

Experimental Analytical

# EXPERIMENTAL VERIFICATION FOR (UPDATED PZT MODEL)

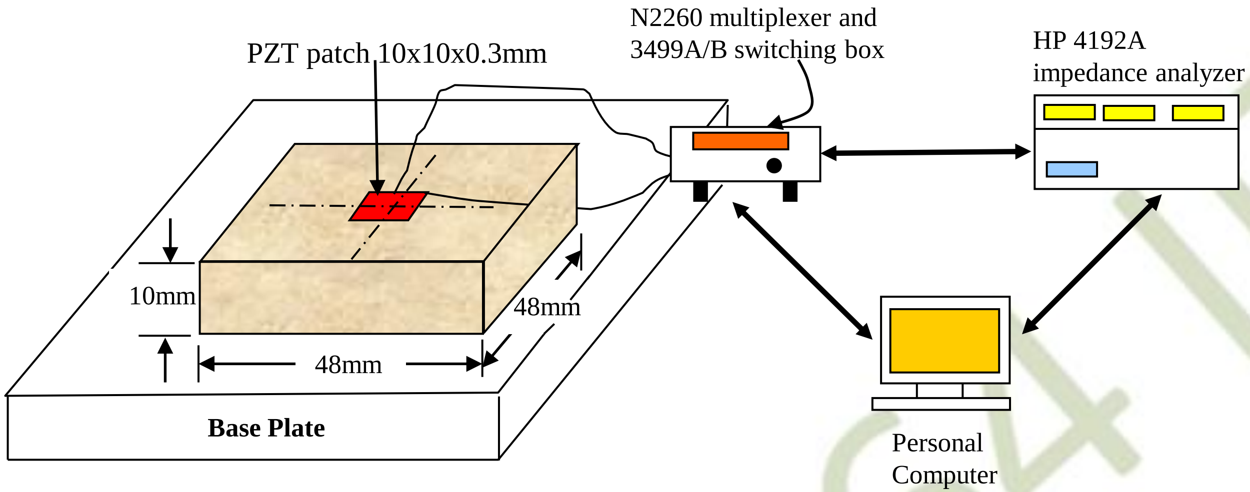



**MUCH BETTER MATCH**

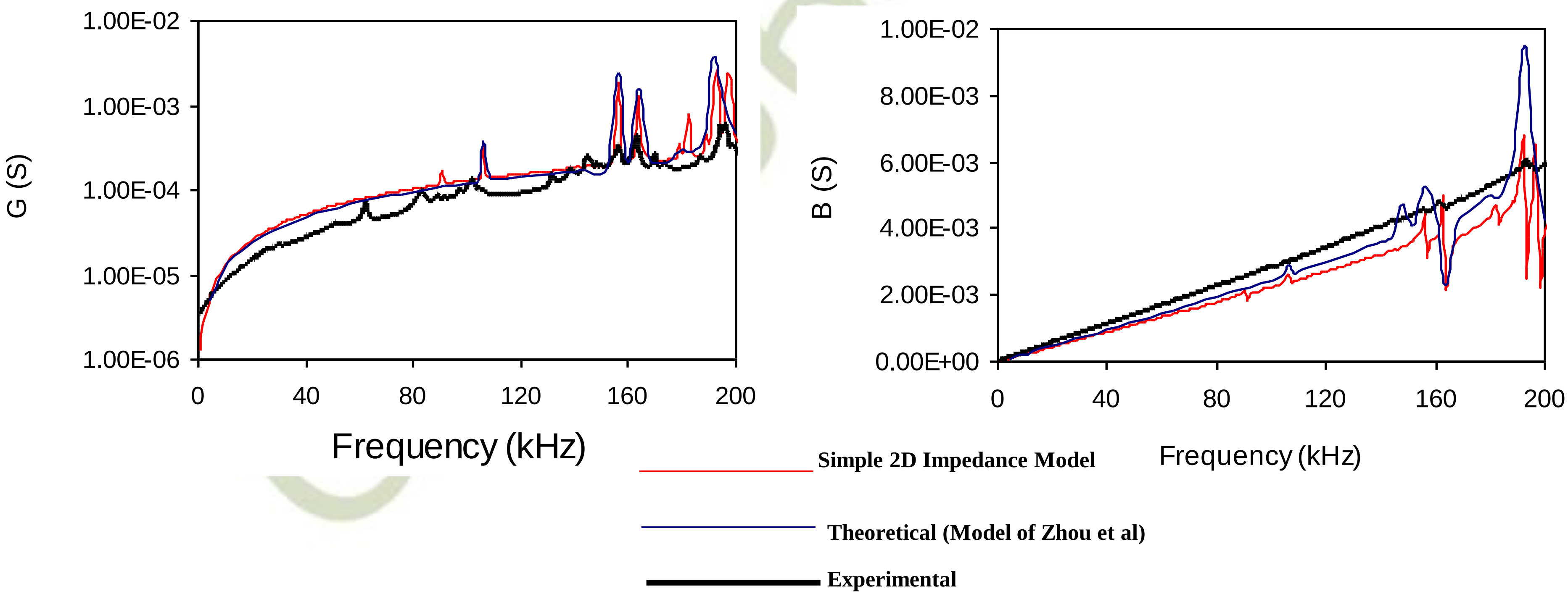


# EXPERIMENTAL COMPARISON WITHOUT UPDATION

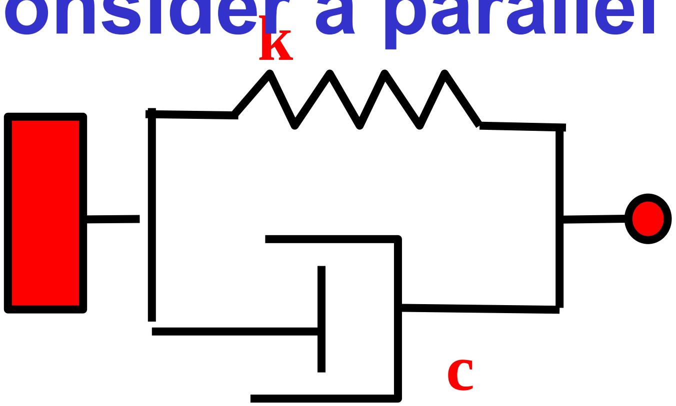



**INFERIOR MATCH WITHOUT UPDATION**



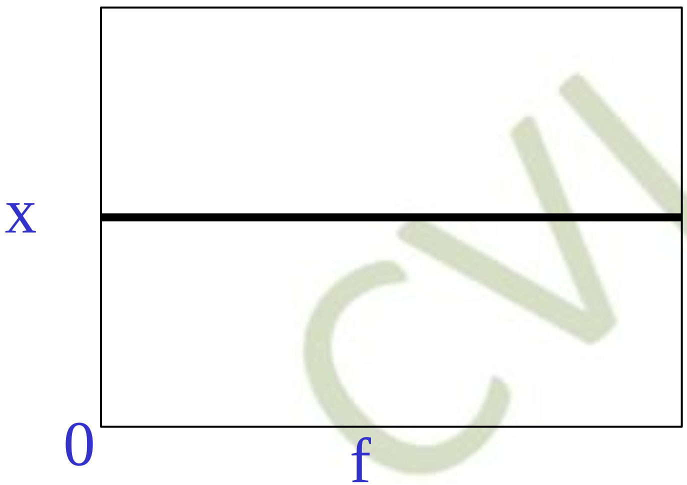
# IMPEDANCE-BASED "IDENTIFICATION"

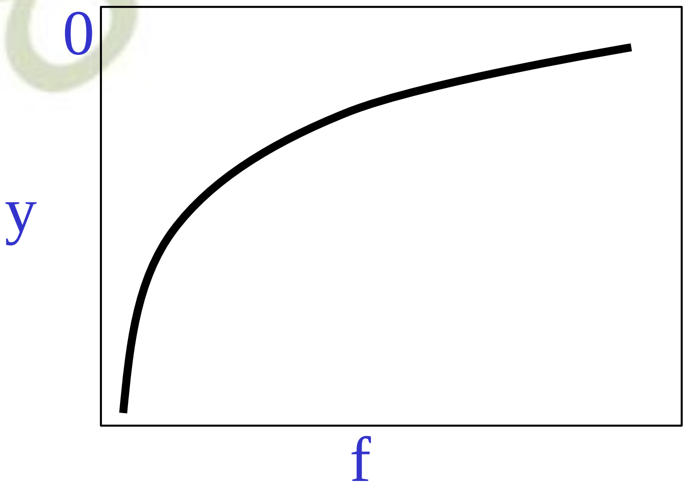
## Consider a parallel combination of "k" and "c"




$$Z = x + yj$$
$$= c - \frac{k}{\omega} j$$

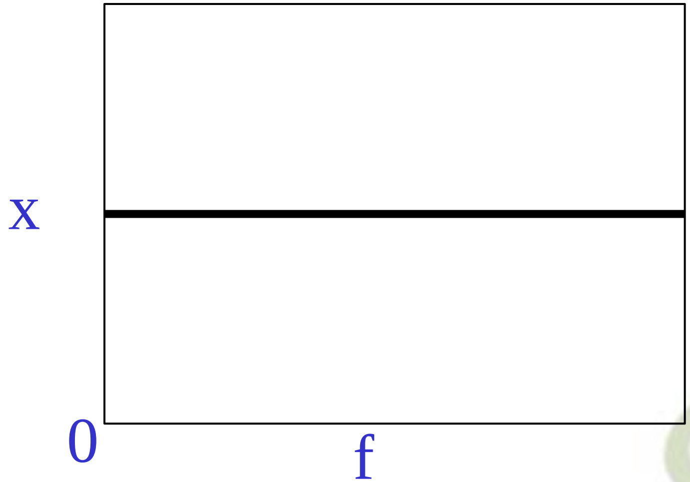
## Plot of "x" and "y" with frequency

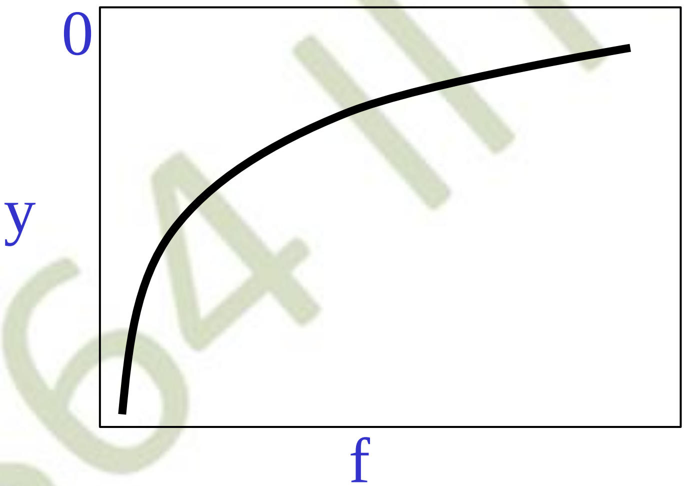


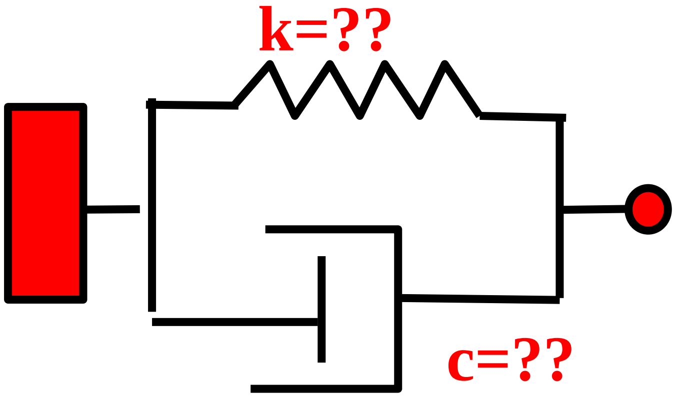
# IMPEDANCE-BASED "IDENTIFICATION"

## Identification mean solving the inverse problem



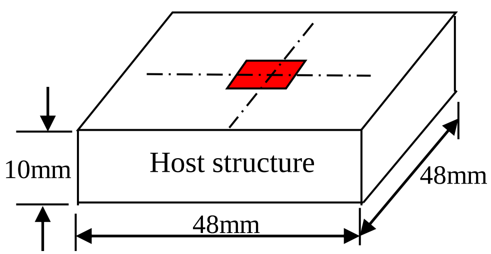

**From plots of G and B, extract "x" and "y" and then identify the combination**



# IMPEDANCE BASED "IDENTIFICATION"

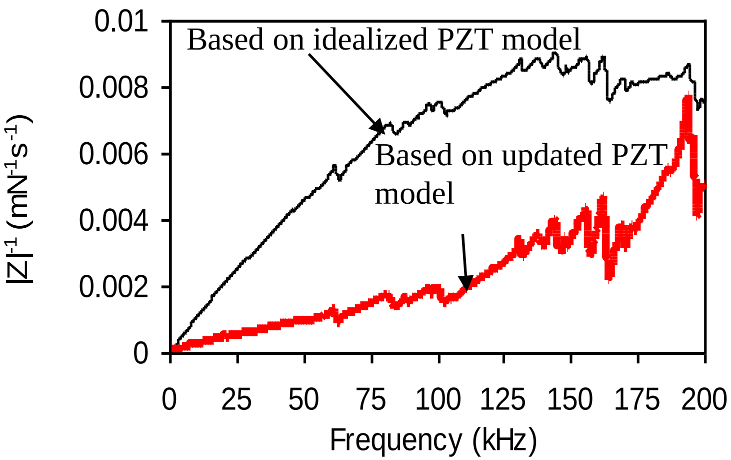
Desirable in SHM: To identify structure purely from experimental measurements without any modelling

$$\bar{Y} = G + Bj = 4\omega j \frac{l^2}{h}\left[\overline{\varepsilon_{33}^T} - \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)} + \frac{2d_{31}^2\overline{Y^E}}{(1-\nu)}\left(\frac{Z_{a,eff}}{Z_{s,eff}+Z_{a,eff}}\right)\bar{T}\right]$$



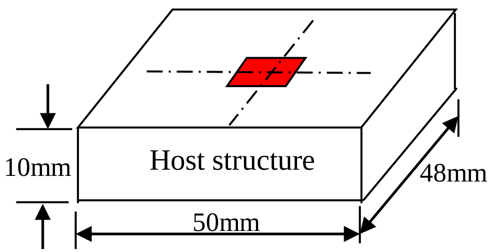

**Signature decomposition, solve equations simultaneously**

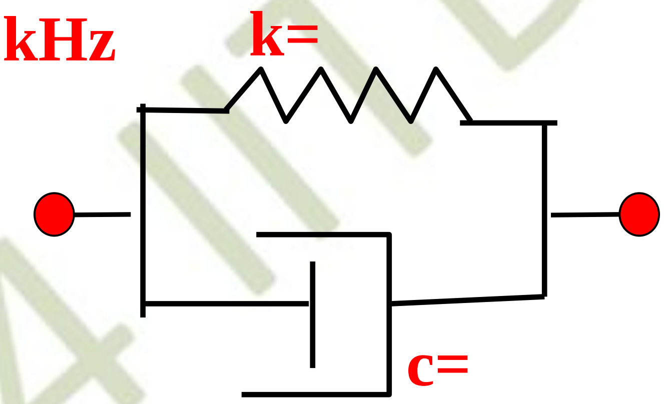
$$Z_{s,eff} = x + yj$$

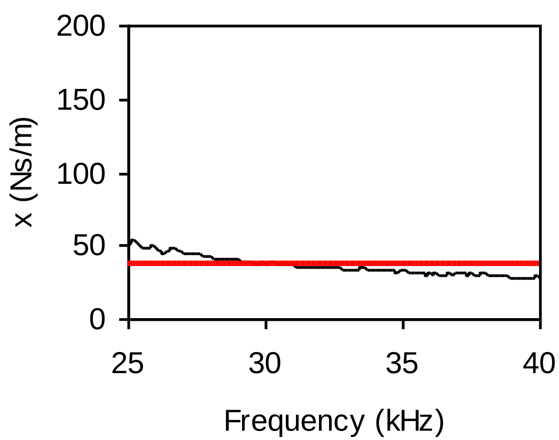


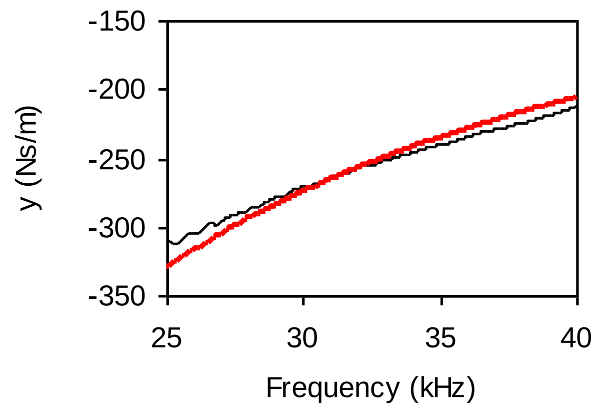
# STRUCTURAL IDENTIFICATION USING PIEZO-IMPEDANCE TRANSDUCERS

## Identification: 25-40kHz



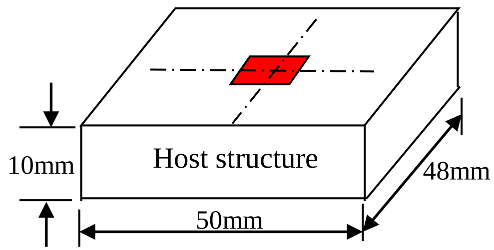



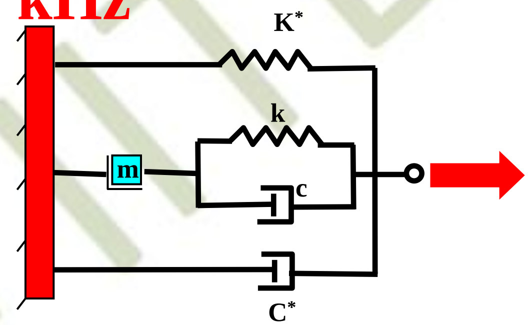



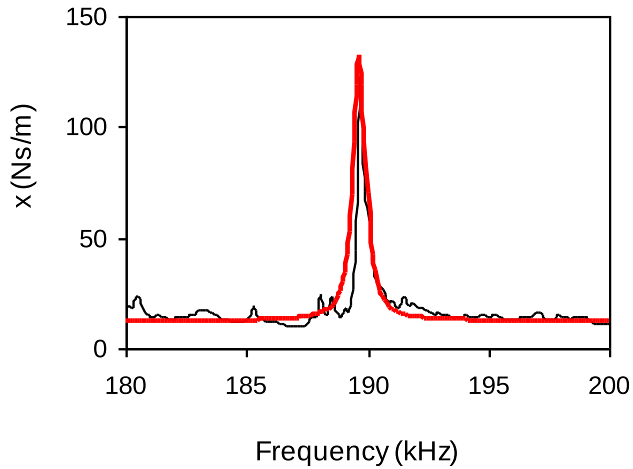

Experimental    Equivalent system

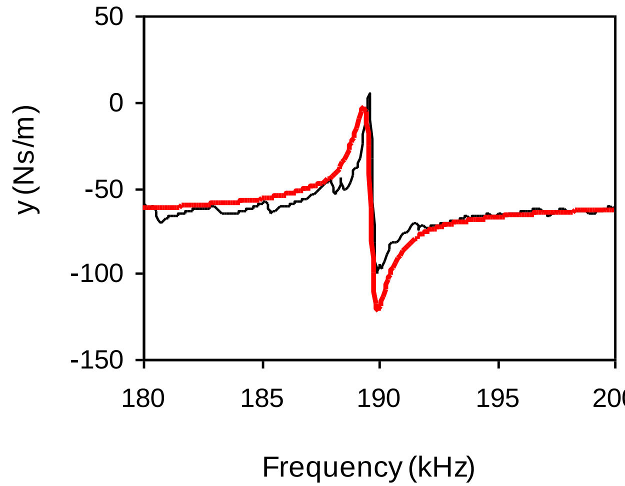
# STRUCTURAL IDENTIFICATION USING PIEZO-IMPEDANCE TRANSDUCERS

## Identification: 180-200 kHz



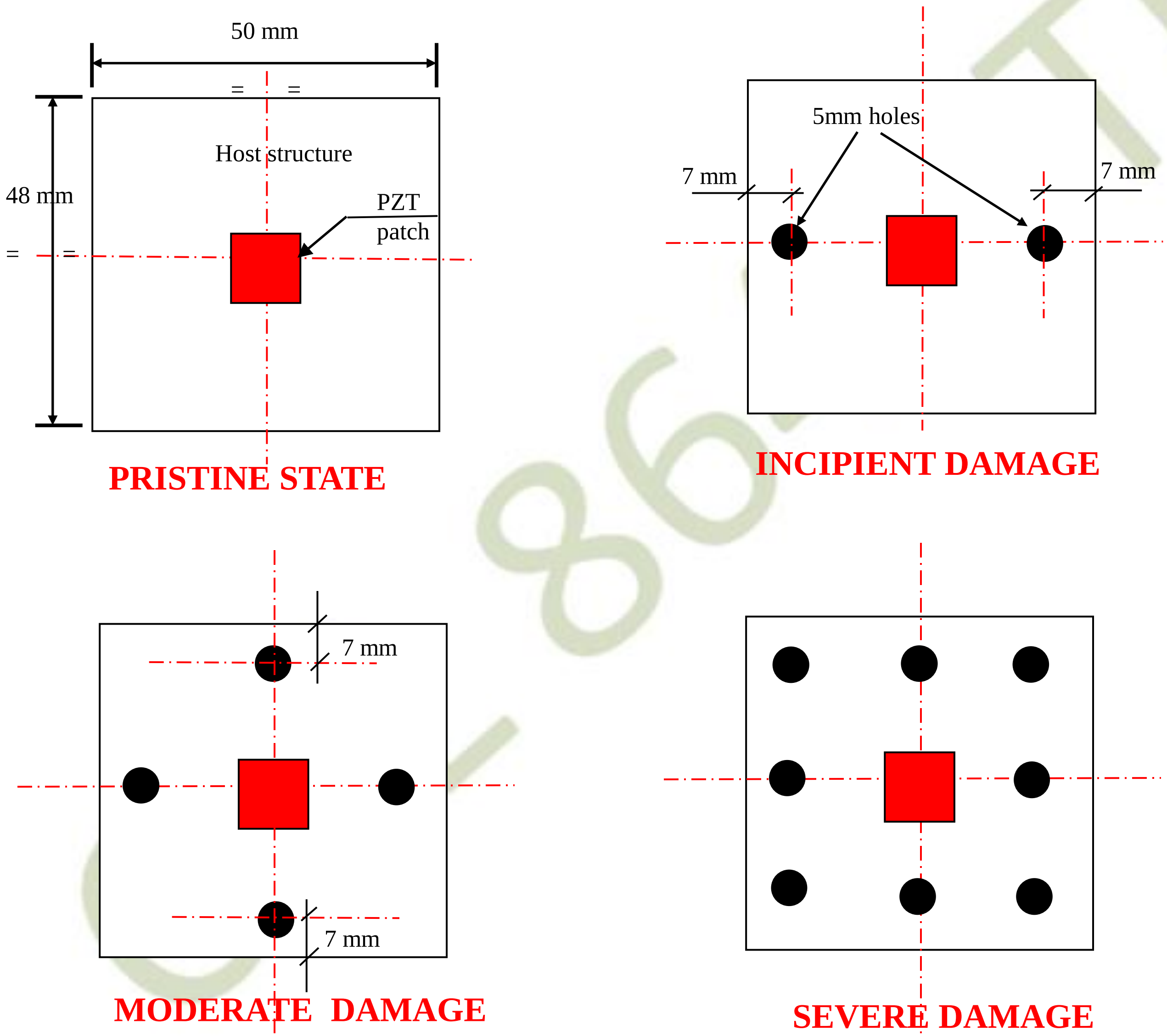




Experimental

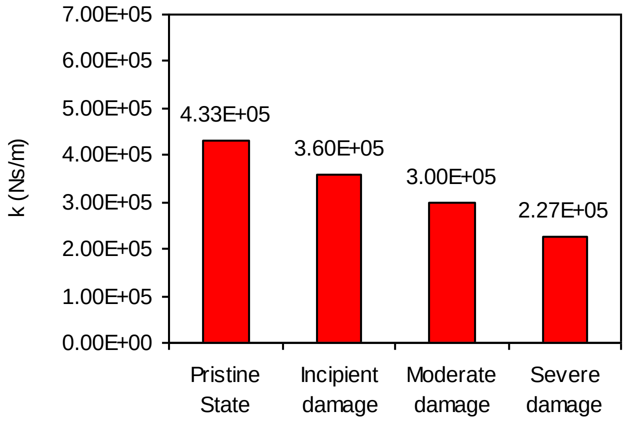
Equivalent system

# STRUCTURAL IDENTIFICATION

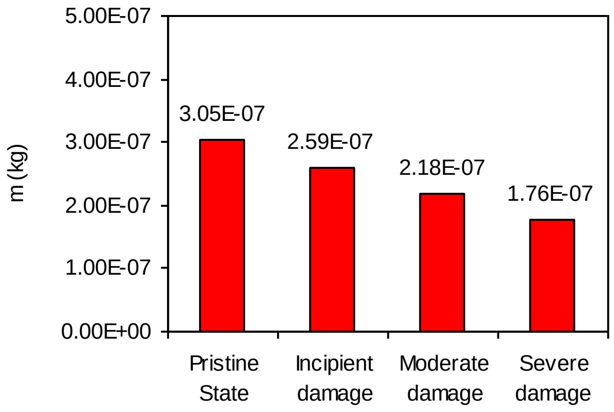


# EFFECT OF DAMAGE

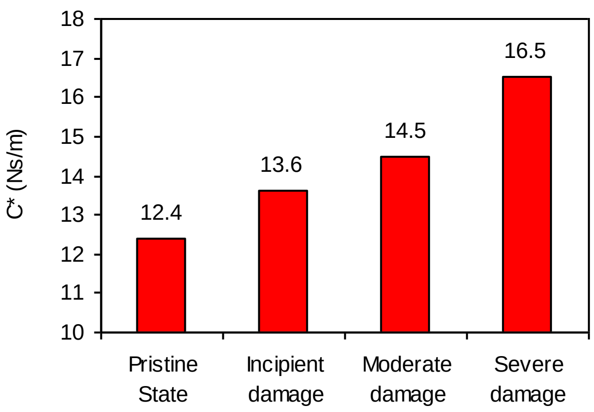
## IDENTIFIED STIFFNESS




## IDENTIFIED MASS



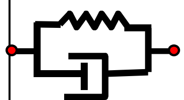
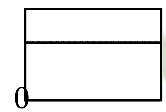
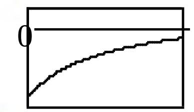
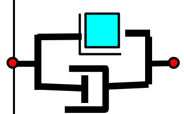
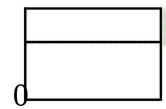
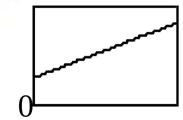
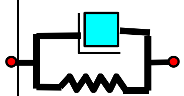
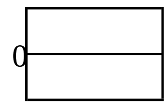
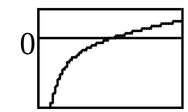
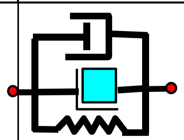
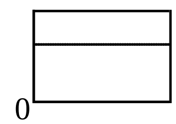
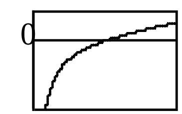
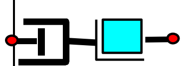
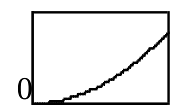
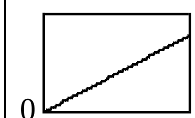
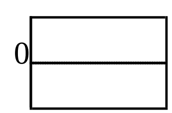
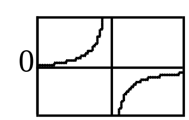
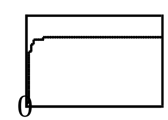
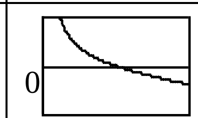

## IDENTIFIED DAMPING



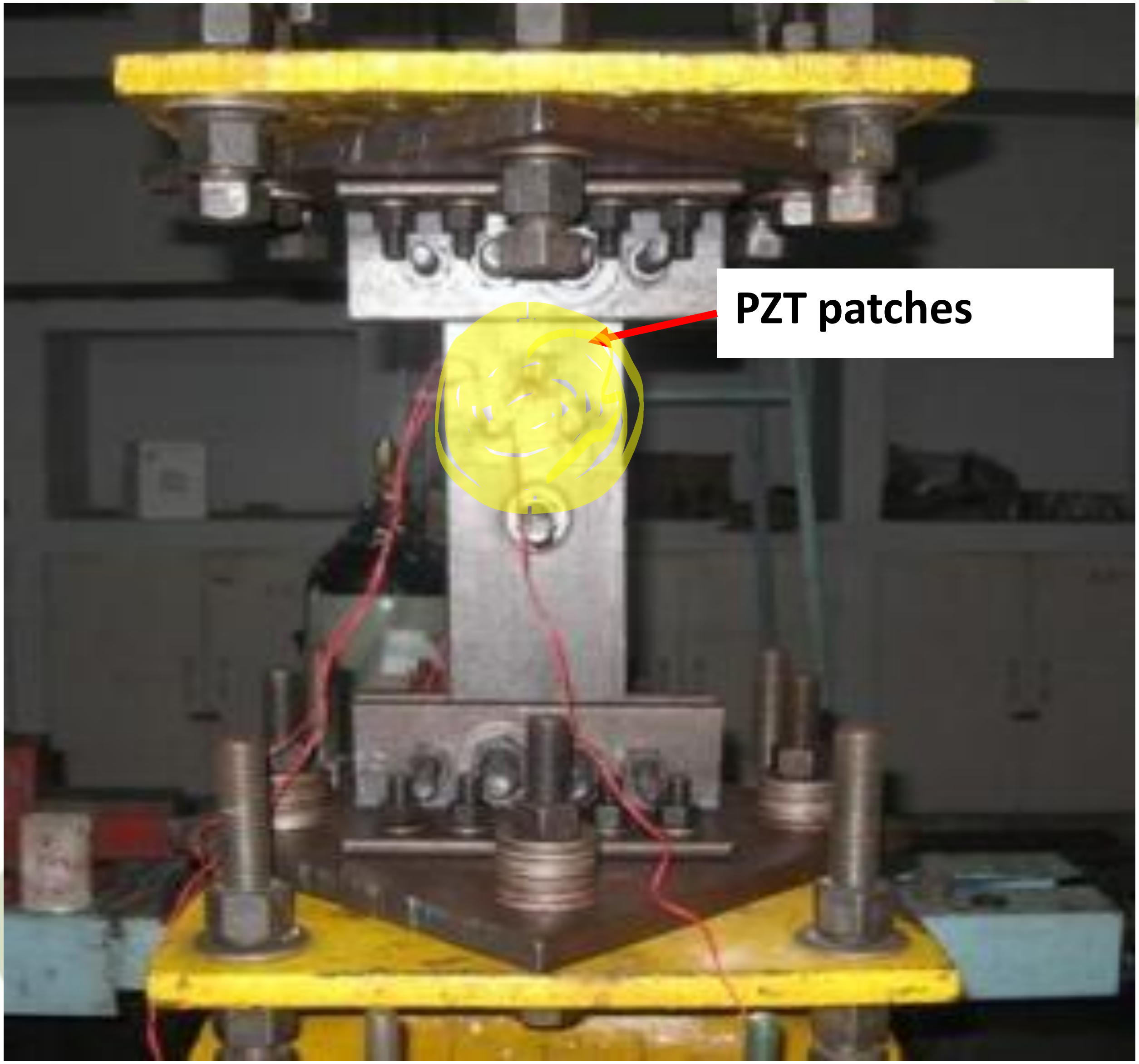

**Identification purely EXPERIMENTAL, by the bonded PZT patch**

**NO *a priori* structural information required**

# STANDARD PLOTS TO FACILITATE IMPEDANCE-BASED "IDENTIFICATION"

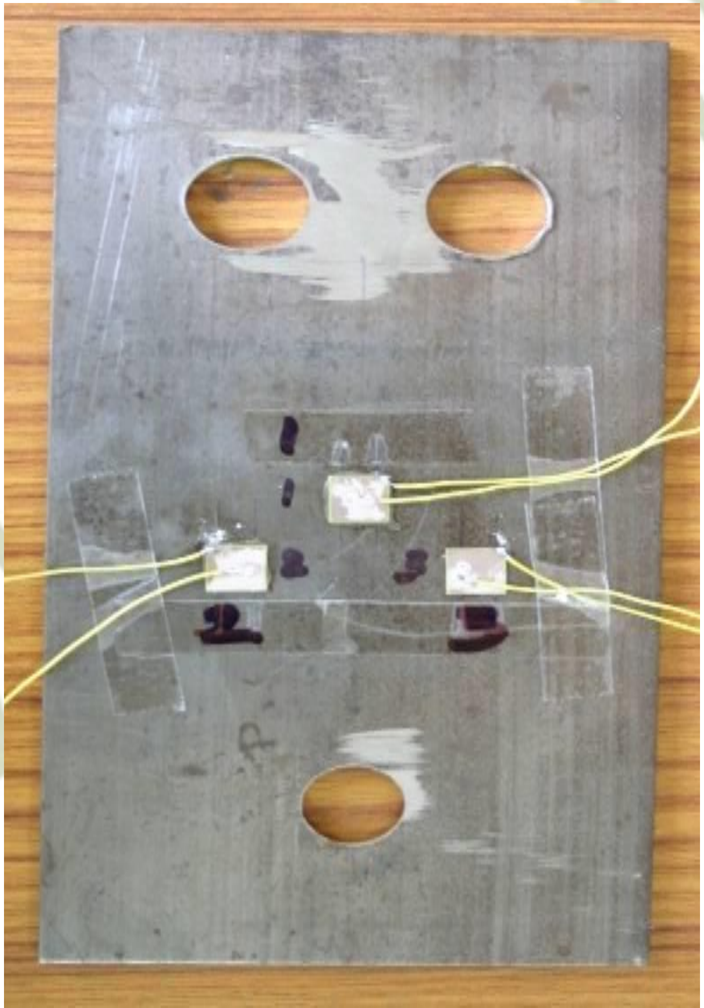
| No. | COMBIN—ATION | x | y | x vs Freq. | Y vs Freq. |
|---|---|---|---|---|---|
| 1 |  | c | $-\frac{k}{\omega}$ |  |  |
| 2 |  | c | $m\omega$ |  |  |
| 3 |  | 0 | $m\omega - \frac{k}{\omega}$ |  |  |
| 4 |  | c | $m\omega - \frac{k}{\omega}$ |  |  |
| 5 |  | $\frac{c^{-1}}{c^{-2}+(\omega m)^{-2}}$ | $\frac{(\omega m)^{-1}}{c^{-2}+(\omega m)^{-2}}$ |  |  |
| 6 |  | 0 | $\frac{-1}{(\omega / k)-(\omega m)^{-1}}$ |  |  |
| 7 |  | $\frac{c^{-1}}{c^{-2}+(\omega / k - 1/\omega m)^{2}}$ | $\frac{-(\omega / k - 1/\omega m)}{c^{-2}+(\omega / k - 1/\omega m)^{2}}$ |  |  |

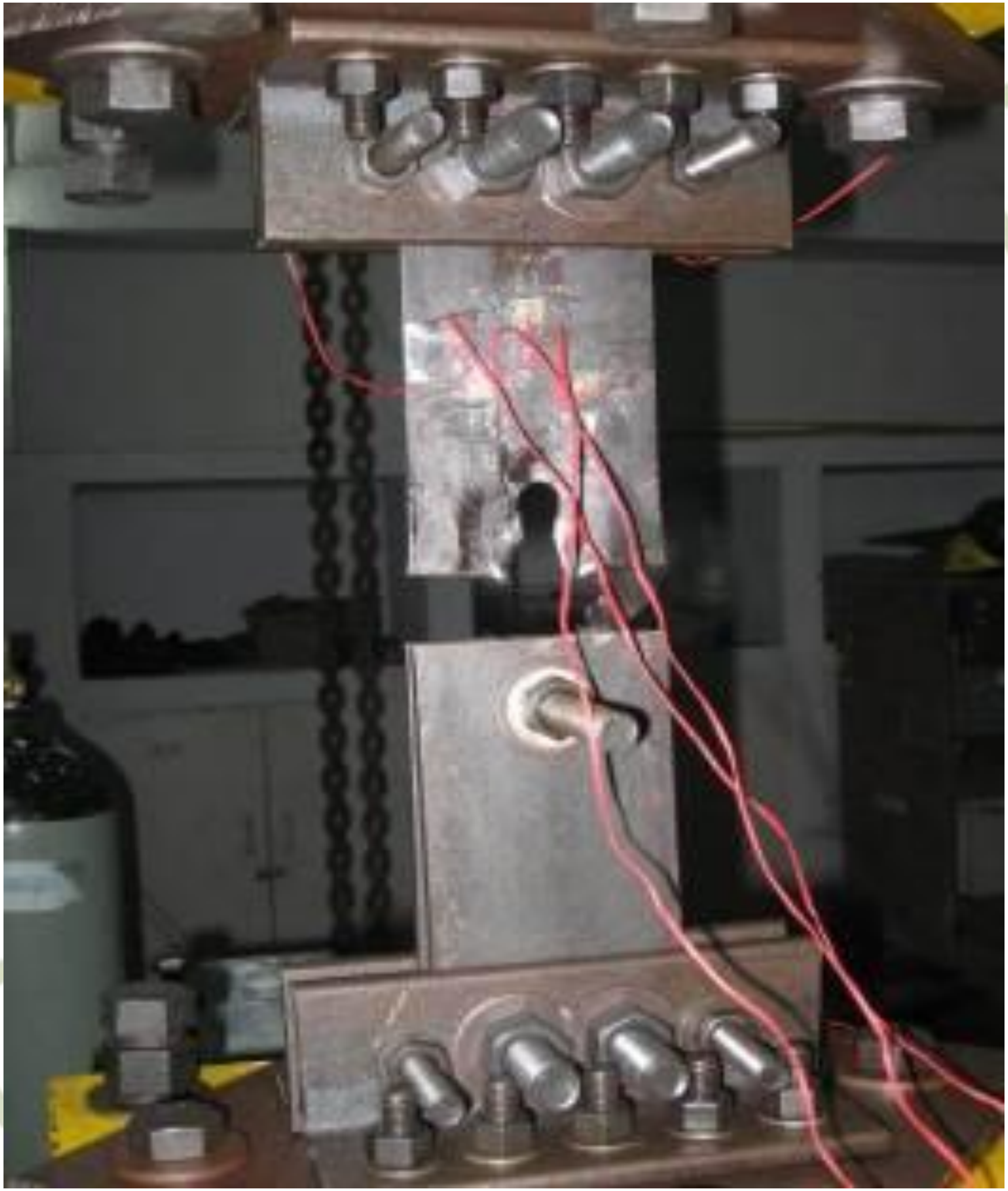
# TEST SPECIMEN



# TEST SPECIMEN

| **CYCLES** | **CYCLE RATIO CR (=** ***N/N$_o$*****)** | **OPERATING FREQUENCY** | **STRESS EXTREMES** |
|---|---|---|---|
| 0-26000 | 0-0.409 | 3 Hz | $0.08f_y$-$0.64f_y$ |
| 26001-58500 | 0.409-0.92 | 4 Hz | $0.216f_y$-$0.608f_y$ |
| 58501-63586 | 0.92-1 | 5 Hz | $0.314f_y$-$0.664f_y$ |





# CONVENTIONAL DAMAGE QUANTIFICATION

Root Mean Square Deviation (RMSD)= $\sqrt{\dfrac{\sum_{i=1}^{i=N}\left(w_i - u_i\right)^2}{\sum_{i=1}^{i=N} u_i^{\,2}}}$

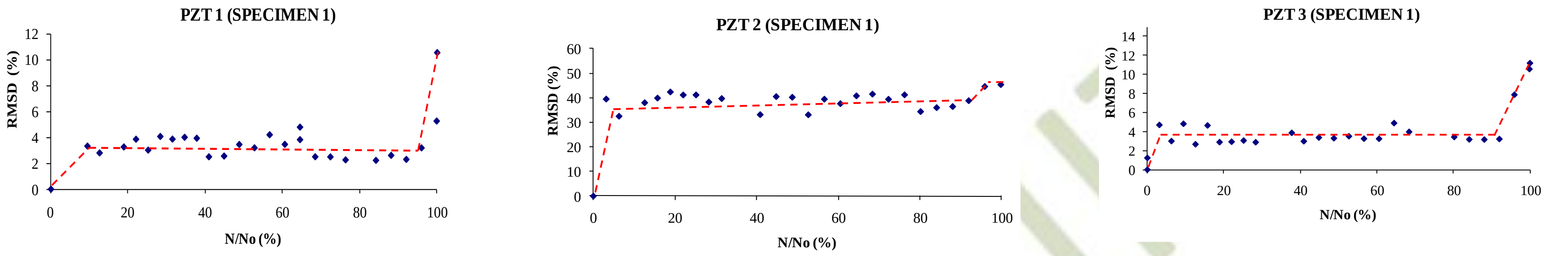
**Signature before damage**= $[u_i]$,

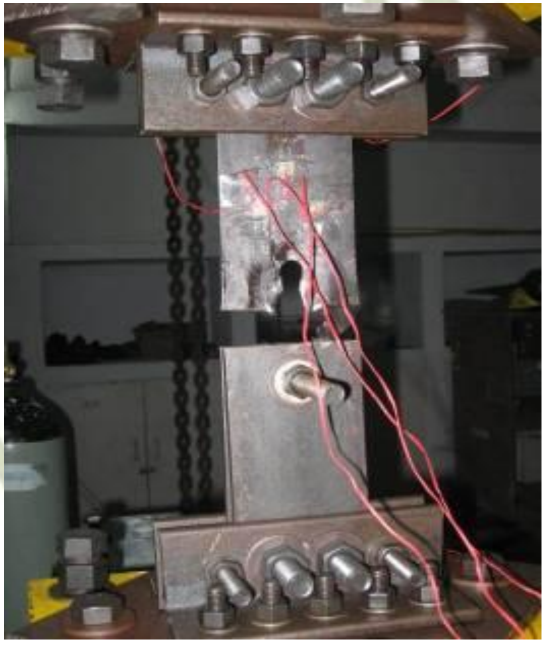
**Signature after damage** = $[w_i]$ ;

i= 1,2,3,….

**Non- Parametric Damage Quantification**

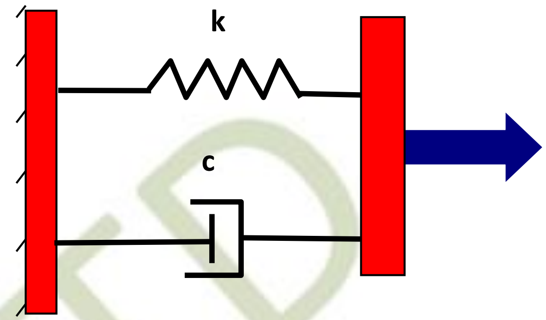
# RMSD DEVIATION

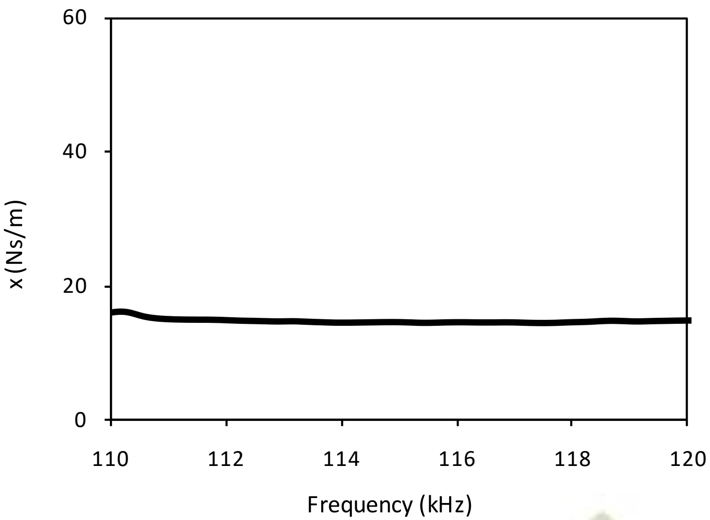


**Conclusion: Largely scatter, no good correlation with damage, no uniform scale**

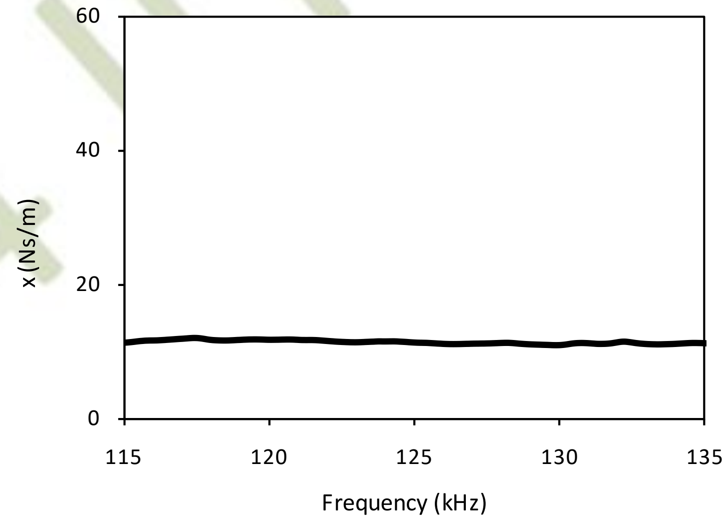
# MECHANICAL IMPEDANCE APPROACH…..HOW BETTER?

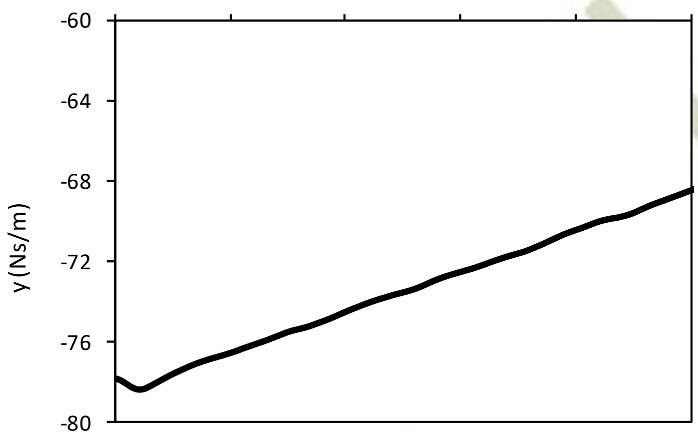


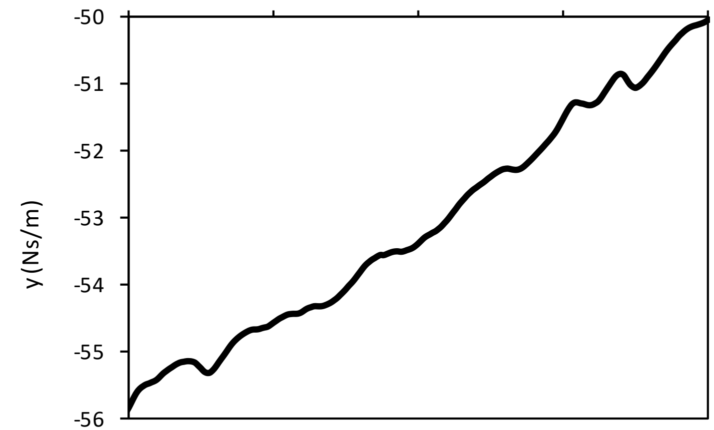




$$x = c$$

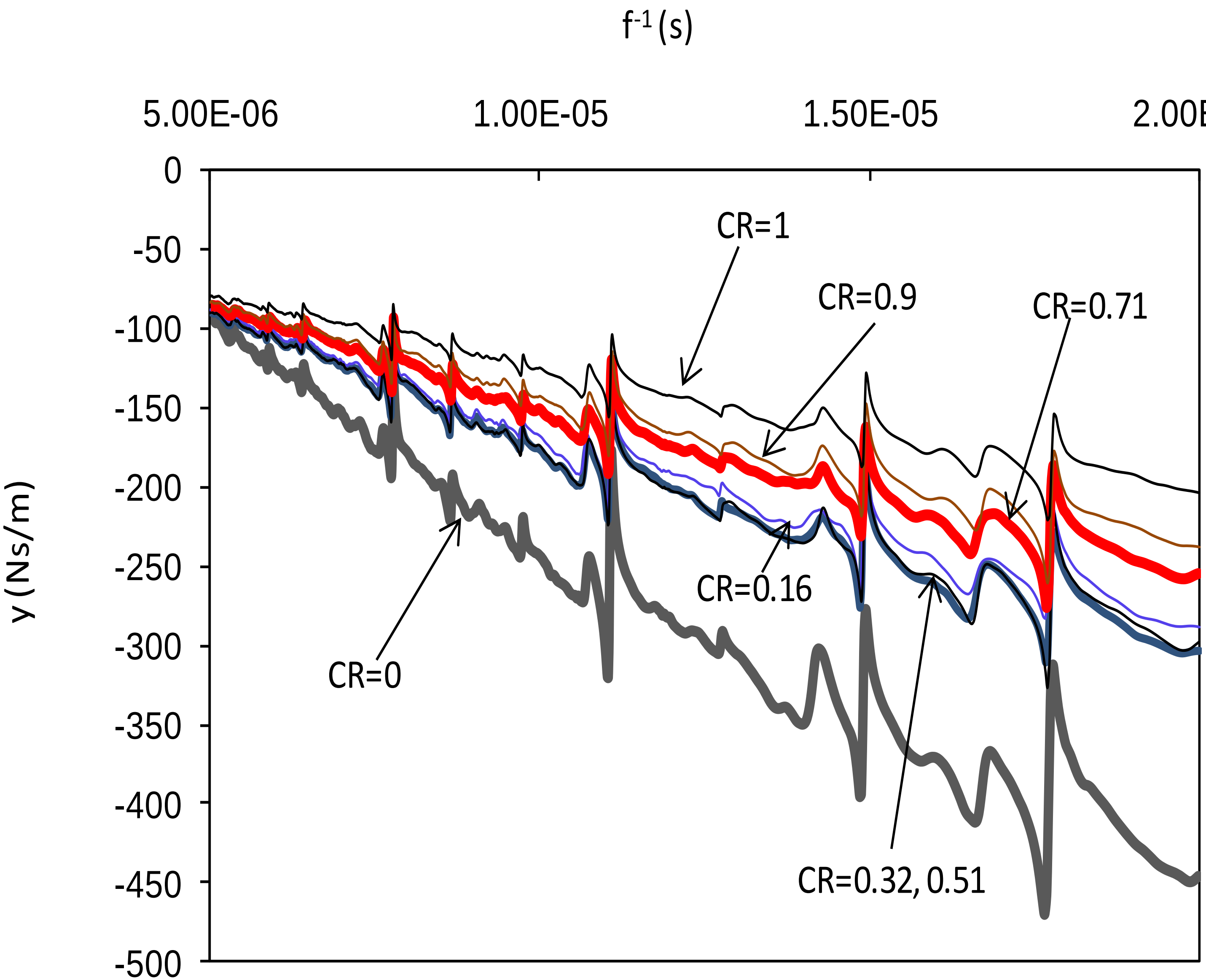
$$y = \frac{-k}{\omega}$$

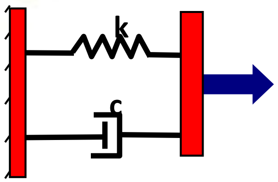


# IDENTIFIED PARAMETERS




$$x = c$$

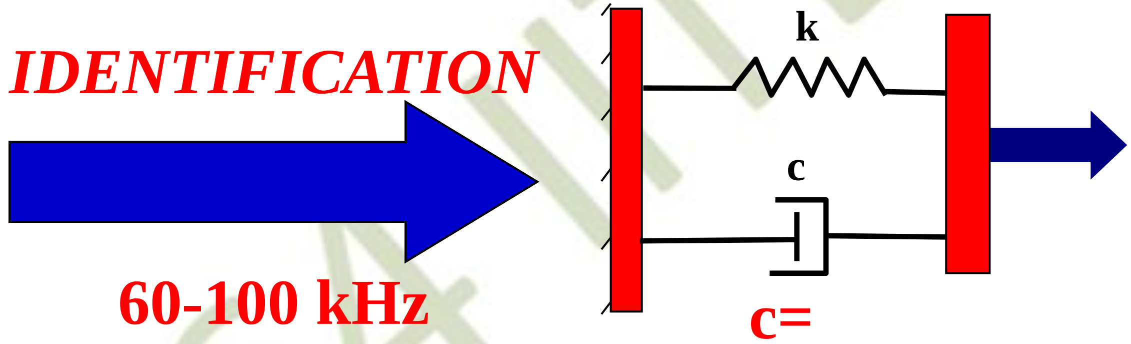
$$y = \frac{-k}{\omega}$$

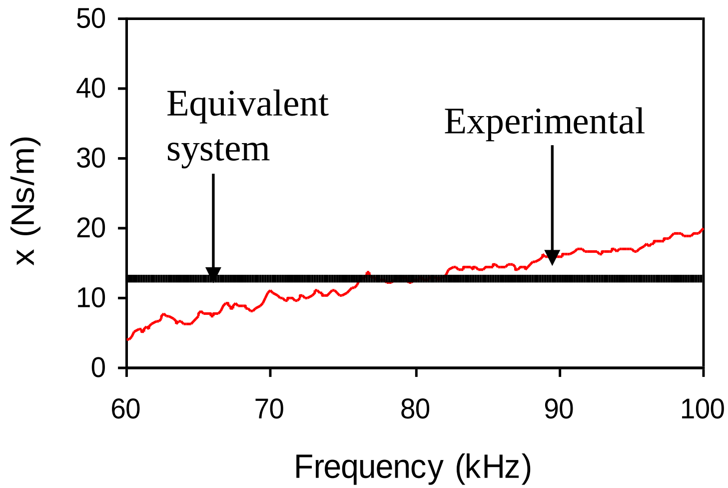
$$k = avg(-2\pi f y)$$

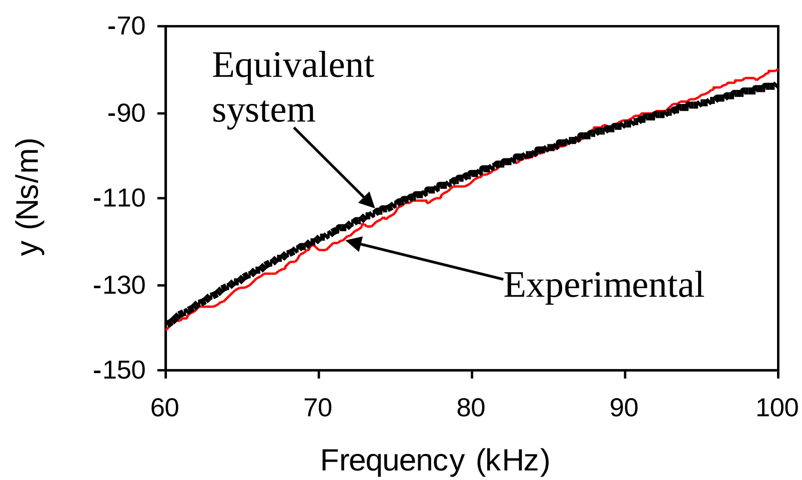
$$c = avg(x)$$

PZT IDENTIFIED STIFNESS CORRELATES WELL WITH ACTUAL STIFFNESS, MUCH BETTER VARIATION THAN RMSD

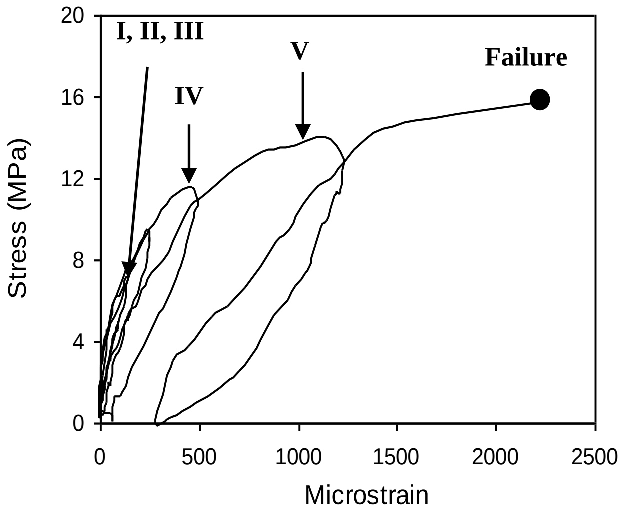
# CONCRETE STRUCTURAL IDENTIFICATION BY PIEZO-TRANSDUCERS

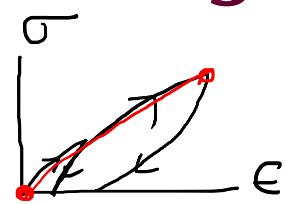



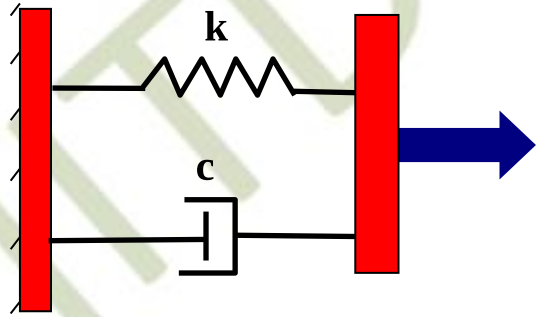




Load cell
Concrete cube
Data logger
PC 2
Universal Testing Machine (UTM)
PC 1
Impedance Analyzer

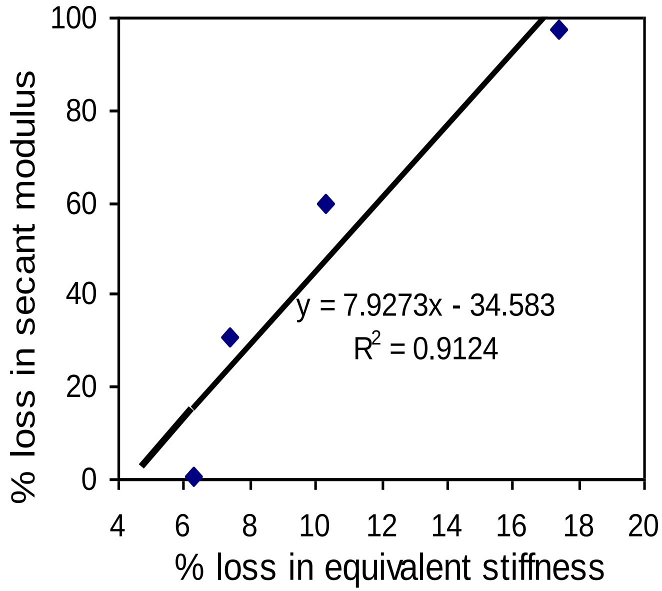


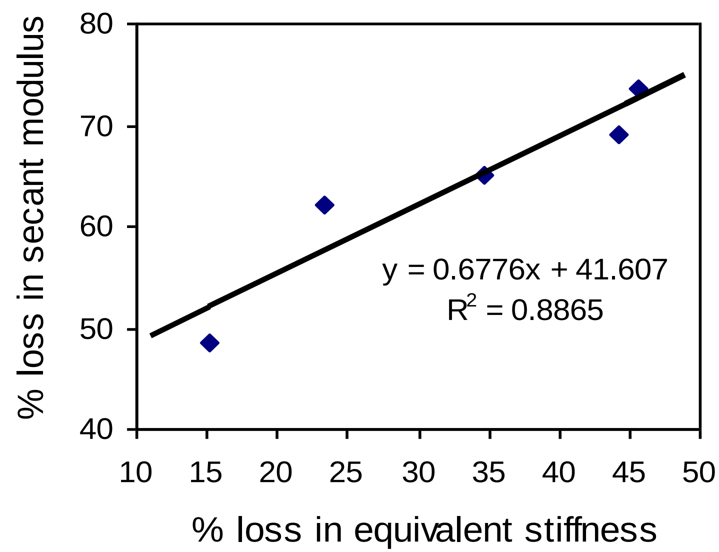

# *Strength: 17 MPa 86 MPa*




## *CORRELATION BETWEEN IDENTIFIED STIFFNESS AND SECANT MODULUS*




PZT IDENTIFIED STIFNESS CORRELATES WELL WITH ACTUAL STIFFNESS FOR CONCRETE ALSO

# THANK YOU

## Suggested reading:

## Soh and Bhalla (2005)

**(downloadable links at:**
**http://web.iitd.ac.in/~sbhalla/journals.pdf )**