DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

LEC 4

LOW-FREQUENCY VIBRATION (GLOBAL DYNAMIC) TECHNIQUES FOR STRUCTURAL HEALTH MONITORING

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SHM TECHNIQUES

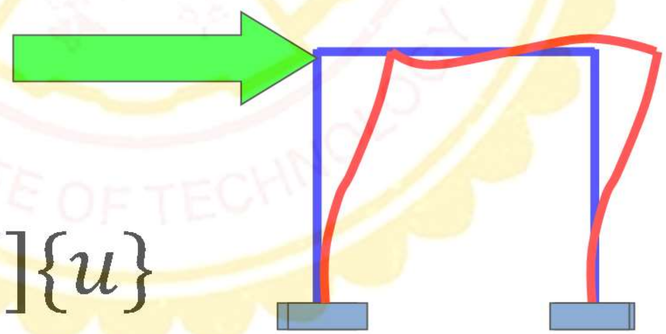
LOCAL

- EMI technique
- Involve application of some NDE technique

GLOBAL

- Global Static Response
- Global Dynamic Response

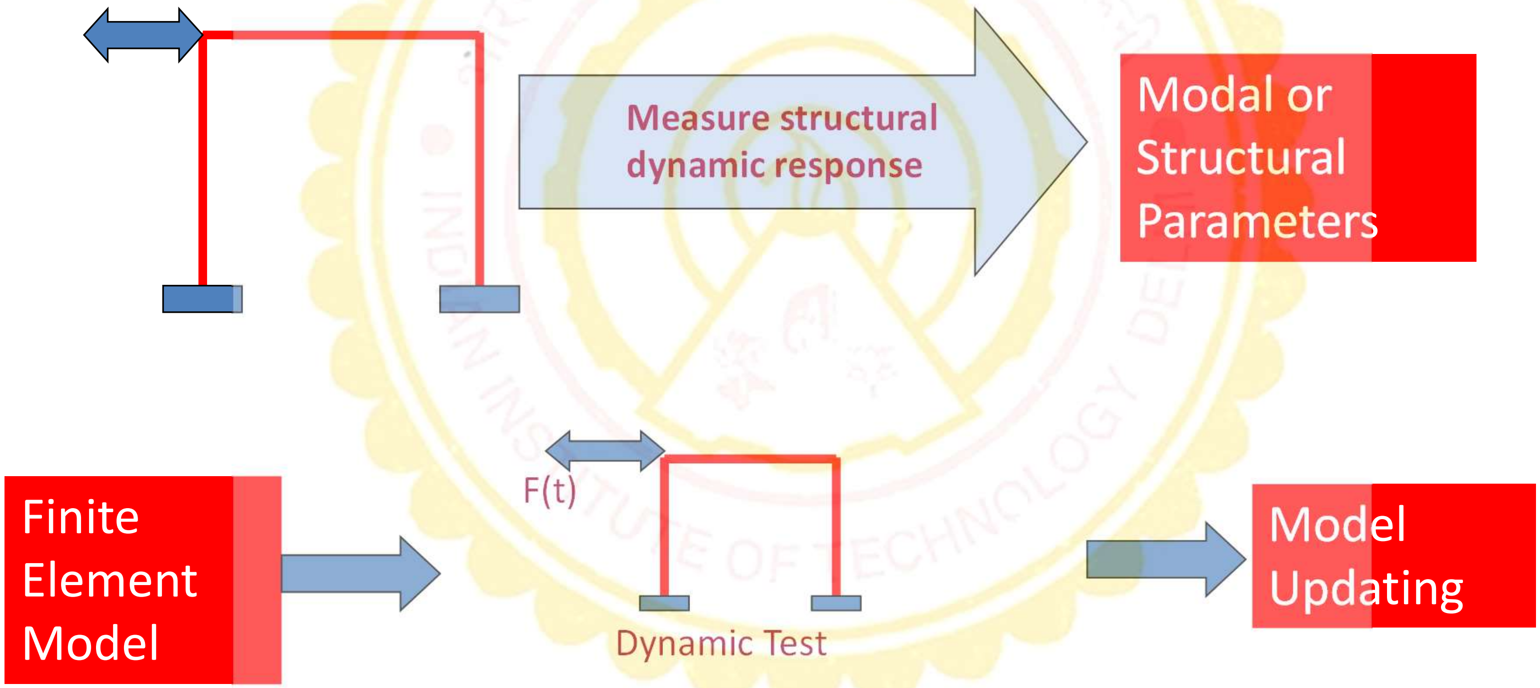
$$\{P\} = [\overset{?}{K}]\{u\}$$



GLOBAL SHM TECHNIQUES

Global Dynamic Techniques

(Dynamic Response Based Techniques)

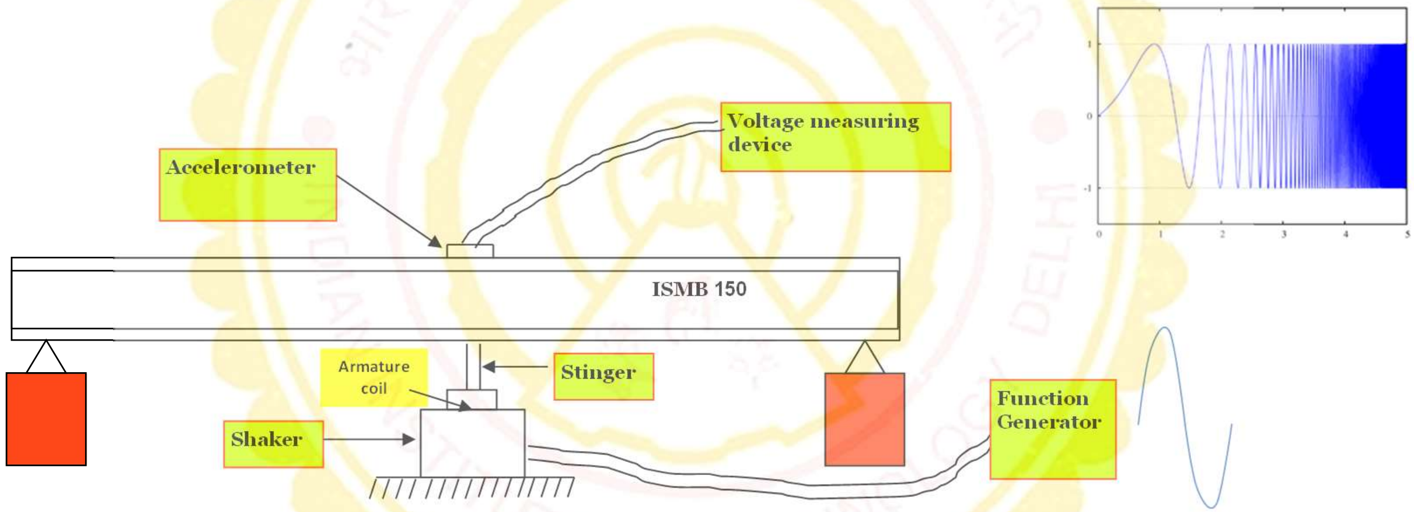


CONTENTS

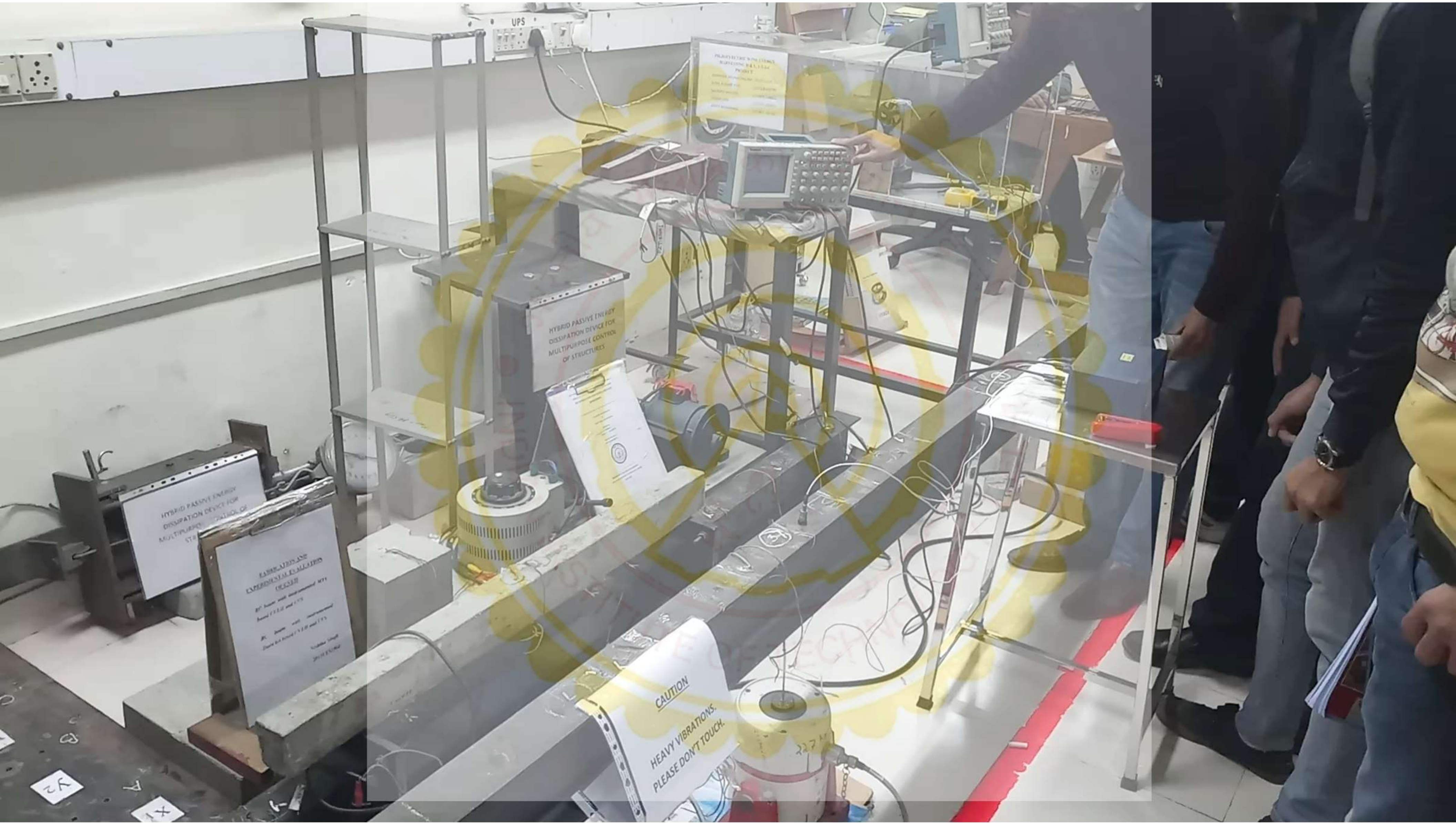
- EXPERIMENTAL MODAL ANALYSIS
  (NON MATHEMATICAL TREATMENT)
- GLOBAL VIBRATION TEHNIQUES FOR STRUCTURAL HEALTH MONITORING
- USE OF MODERN SMART SENSORS IN GLOBAL VIBRATION TECHNIQUES

EXPERIMENTAL MODAL ANALYSIS

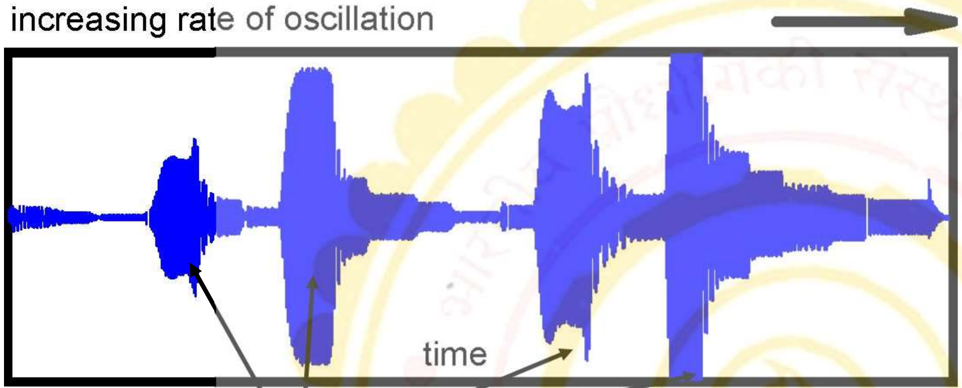
Modal analysis is a process whereby the structure is described in terms of its natural dynamic characteristics, namely the natural frequencies, mode shapes and damping, by applying an external excitation and measuring structure's response at representative locations.



Let us apply a sinusoidal signal of constant amplitude, but vary the frequency gradually (This is technically called as **SWEEP** signal).

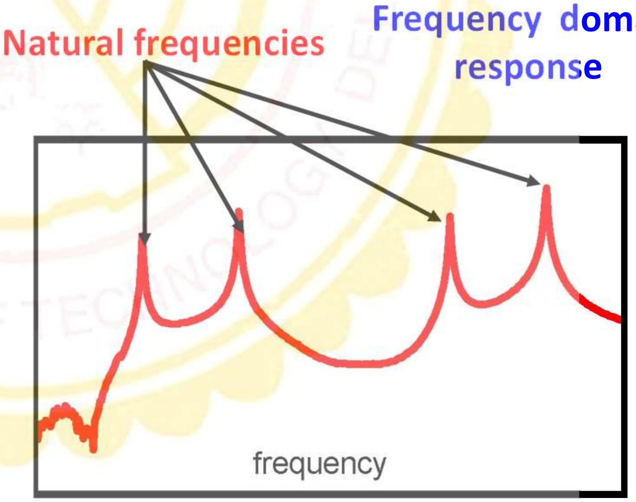

UPS
HYBRID PASSIVE ENERGY DISSIPATION DEVICE FOR MULTIPURPOSE CONTROL OF STRUCTURES
CAUTION
HEAVY VIBRATIONS. PLEASE DON'T TOUCH.

TIME AND FREQUENCY DOMAINS



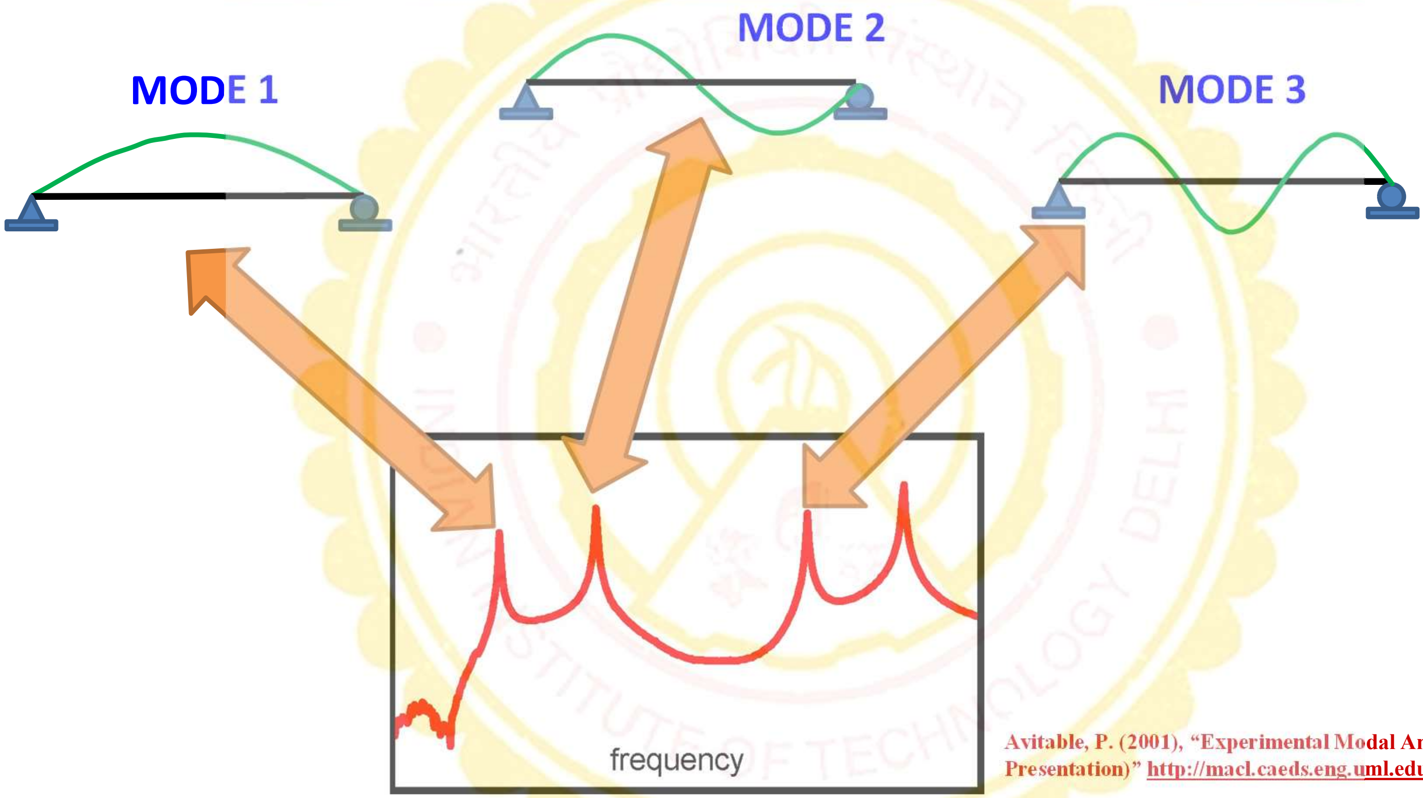
Avitable, P. (2001), "Experimental Modal Analysis (A Simple Non-mathematical Presentation)" http://macl.caeds.eng.uml.edu/umlspace/mspace.html

Fast Fourier Transform (FFT)



The frequency domain response is much easier to evaluate as compared to the time domain.

MODE SHAPES



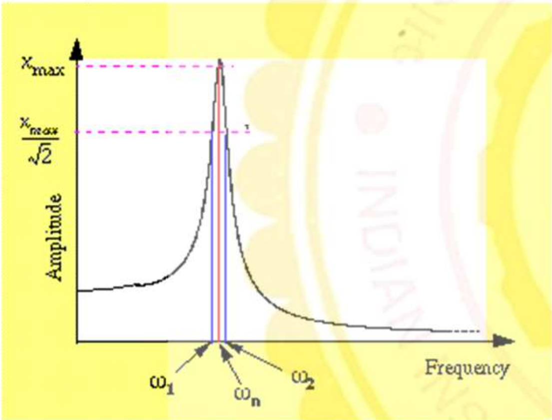
Avitable, P. (2001), "Experimental Modal Analysis (A Simple Non-mathematical Presentation)" http://macl.caeds.eng.uml.edu/umlspace/mspace.html

Mode shape is the deformation pattern of the structure when **excited** at a particular natural frequency.

DAMPING

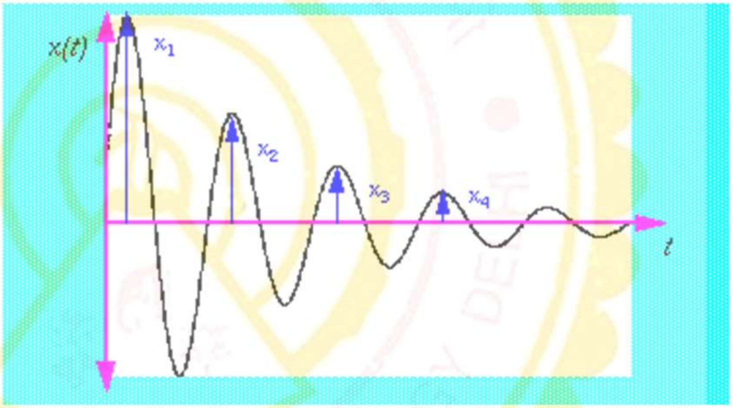
Damping is the energy dissipation property of a material or system under vibration/ dynamic loading.

Half power band width method



$$\xi = \frac{\omega_2 - \omega_1}{2\omega_n}$$

Logarithmic decrement method

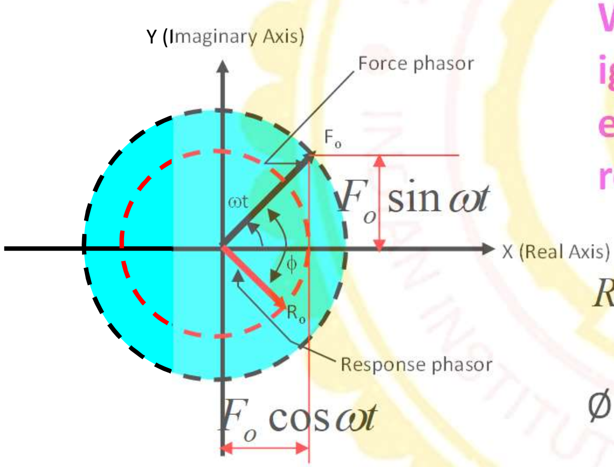


$$\delta = \ln\left(\frac{x_n}{x_{n-1}}\right) \qquad \xi = \frac{\delta}{2\pi}$$

FREQUENCY RESPONSE FUNCTION (FRF)

It is the ratio of the output response (displacement/ velocity/ acceleration) of a structure (undergoing vibration) at a point to the force applied at the same or other point, at a particular frequency. Both the force and the response are simultaneously measured.

$$F(t) = F_o \cos \omega t + jF_o \sin \omega t = F_o e^{j\omega t}$$



We may just consider the real component and ignore the imaginary, or vice versa, use of exponential function makes mathematical representation simpler and compact…..

$$R = R_o \cos(\omega t - \phi) + jR_o \sin(\omega t - \phi) = u_o e^{j(\omega t - \phi)}$$

Ø = Phase difference between force and response

Why???

FRF (ω) : $$h(\omega) = \frac{R}{F} = \frac{R_o e^{j(\omega t - \phi)}}{F_o e^{j\omega t}} = \frac{R_o}{F_o} e^{-j\phi}$$

Unique function of structural stiffness, damping and mass

FREQUENCY RESPONSE FUNCTION (FRF)

Respnose could be displacement/ velocity/ acceleration/ strain etc…

Being sinusoidal in nature, displacement, velocity and acceleration are mathematically related and can easily substitute one another

Displacement $u = u_o e^{j(\omega t - \phi)}$

Velocity $\dot{u} = \dfrac{du}{dt} = u_o j\omega e^{j(\omega t - \phi)}$

$$= u_o e^{\pi j/2} \omega e^{j(\omega t - \phi)}$$

$$= u_o \omega e^{j(\omega t - \phi + \pi/2)}$$

Leads displacement by (π/2)

Acceleration $\ddot{u} = \dfrac{d\dot{u}}{dt} = -u_o \omega^2 e^{j(\omega t - \phi)} = u_o \omega^2 e^{j(\omega t - \phi + \pi)}$

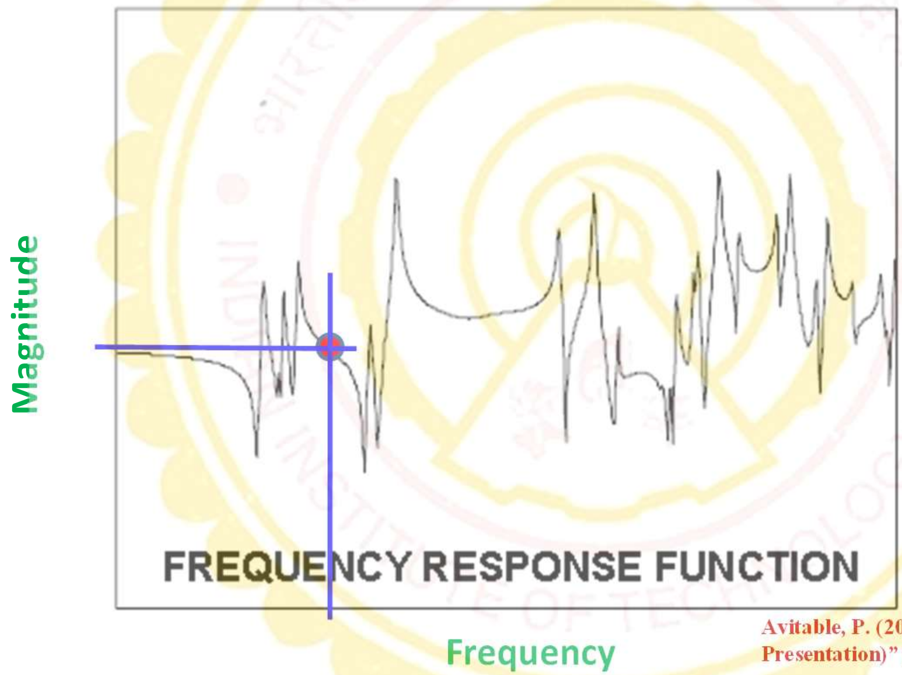
Leads displacement by π

Acceleration is generally measured in majority of the modal tests.

In summary, $\dot{u} = j\omega u \qquad \ddot{u} = -\omega^2 u$

FREQUENCY RESPONSE FUNCTION (FRF)

In general, FRF is a complex number, consisting of real and imaginary parts. It is generally plotted over a frequency range



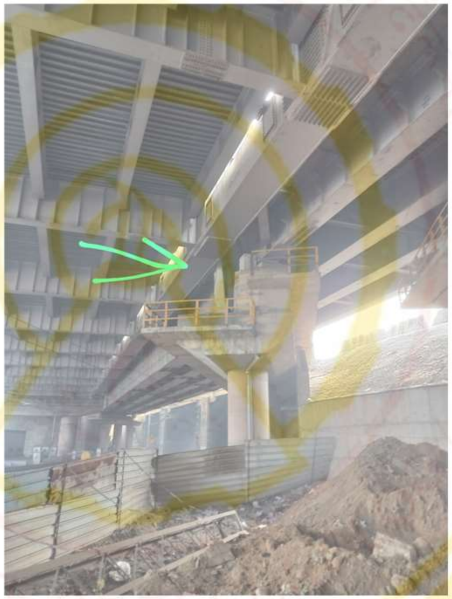
(Magnitude/ Real/ Imaginary)

Avitable, P. (2001), "Experimental Modal Analysis (A Simple Non-mathematical Presentation)" http://macl.caeds.eng.uml.edu/umlspace/mspace.html

FRF can be used to predict the response of the structure against a known force excitation.

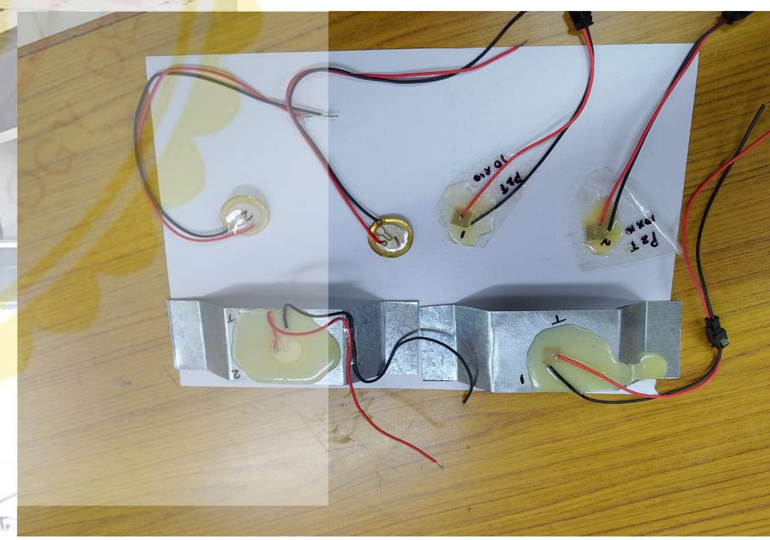
HANDS ON EXPERIENCE WITH FRF

- Laboratory data
- Field data

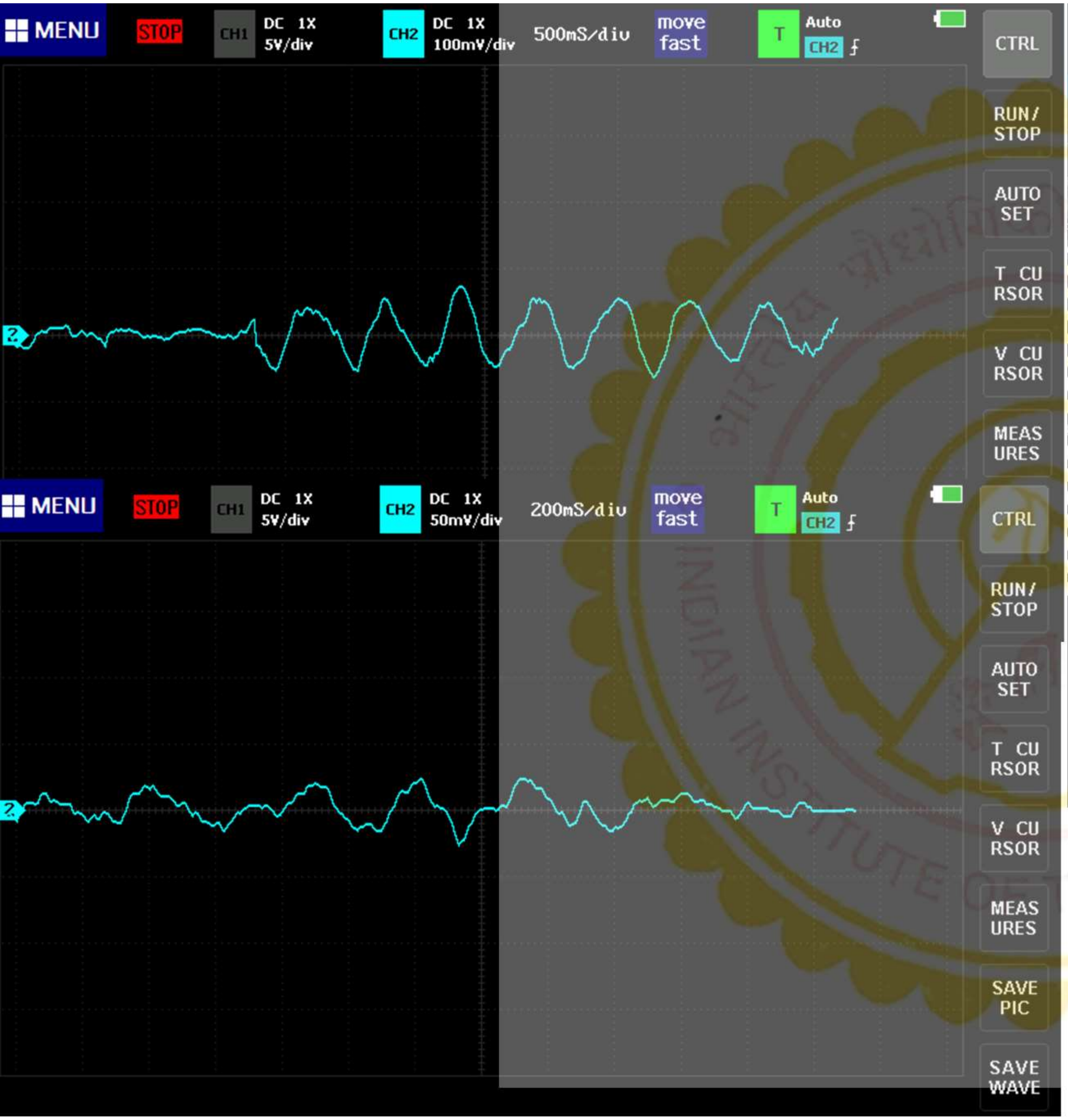


CHIPIYANA ROB (GHAZIABAD)

Joint IITD-CRRI Research



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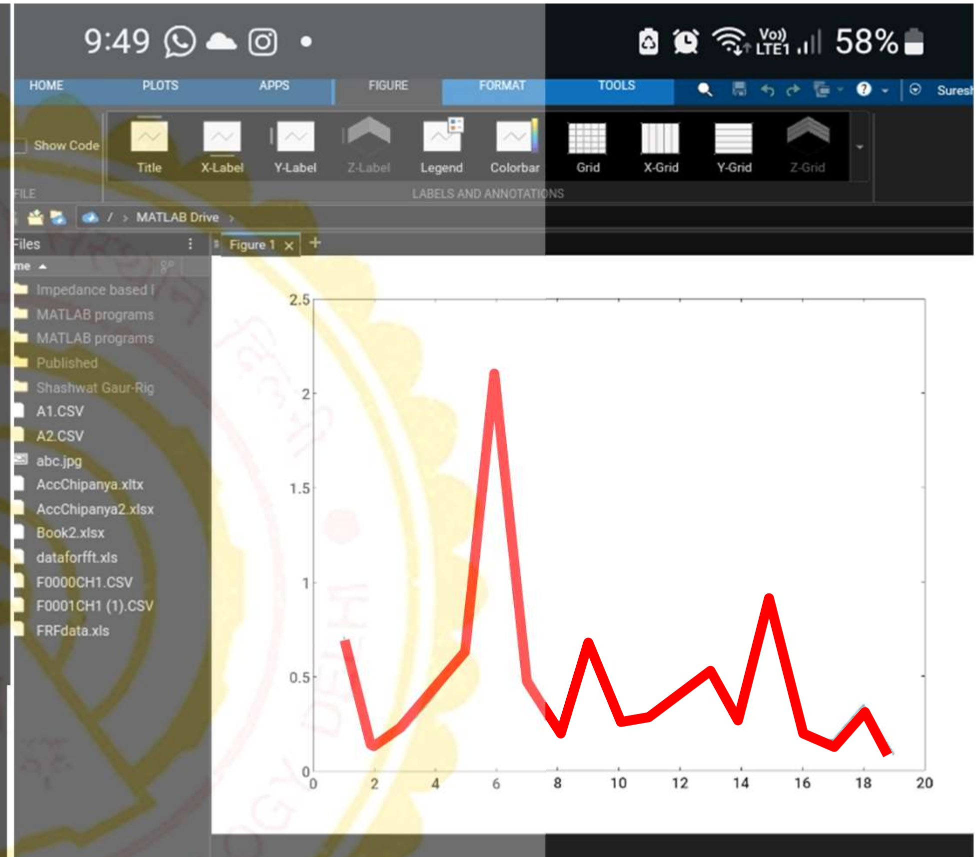
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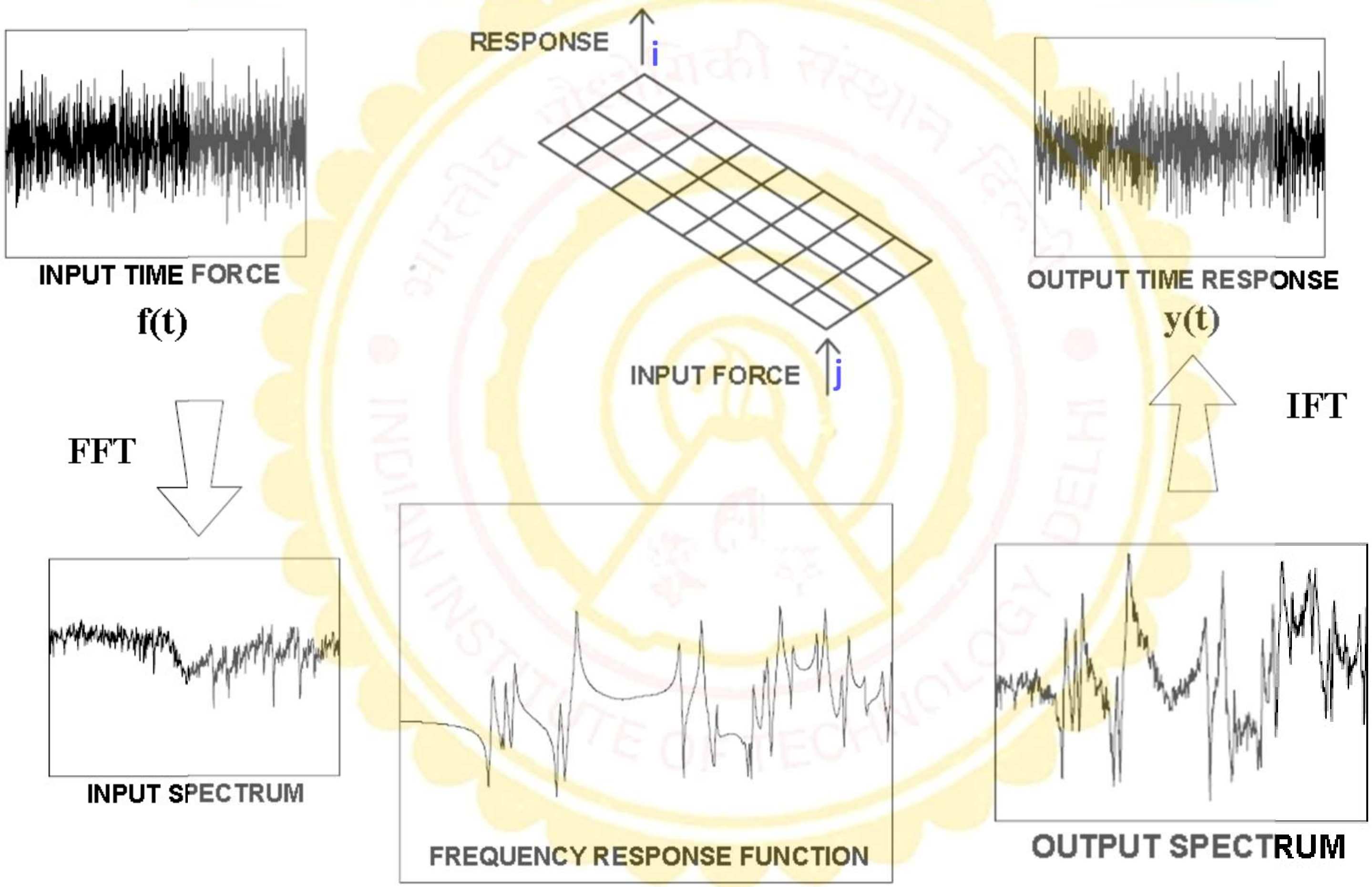
FREQUENCY RESPONSE FUNCTION (FRF)



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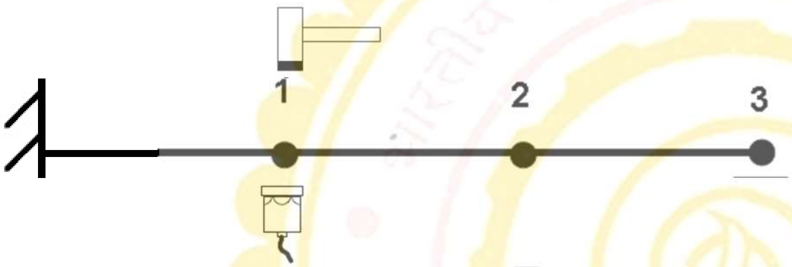
MODE SHAPES FROM FRF

h_{ij} = Response at 'i' due to force applied at 'j'.

i, j = 1, 2, 3.



h_{11} = Response at "1' due to unit force at '1'

h_{12} = Response at "1' due to unit force at '2'

h_{13}

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MODE SHAPES FROM FRF

Similarly, we can experimentally derive:

h_{21} h_{22} h_{23} Response at 2, point of impact varied.

and also

h_{31} h_{32} h_{33} Response at 3, point of impact varied.

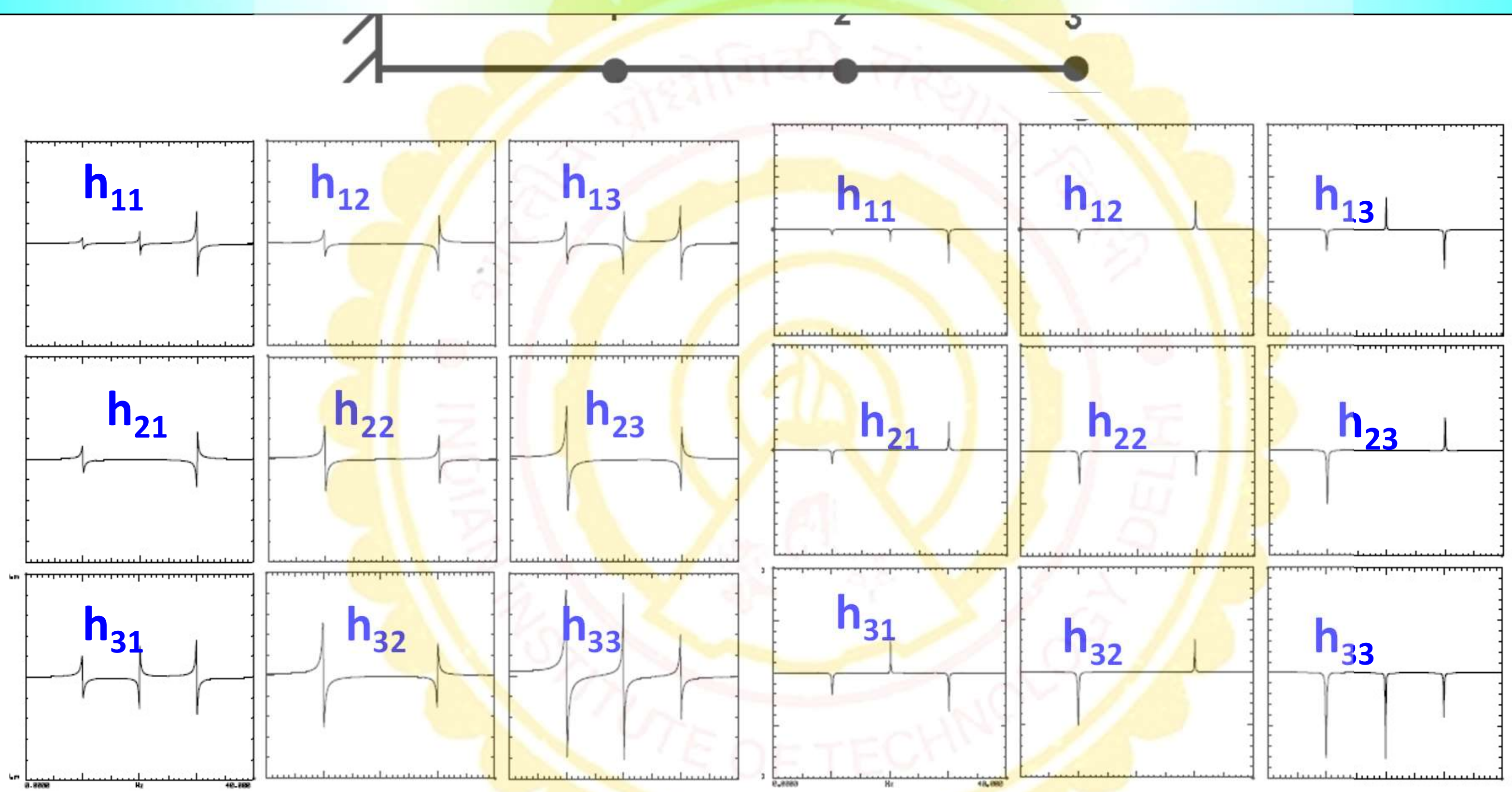
IF THE STRUCTURE IS LINEARLY ELASTIC,

WHAT IS THE RELATION BETWEEN: h_{ij} AND h_{ji}

MUST BE EQUAL SINCE MAXWELL-BETTI'S THEOREM STATES:

The displacement at a point due to unit load at another point is equal to that at the other point due to unit load applied at the first point.

MODE SHAPES FROM FRF

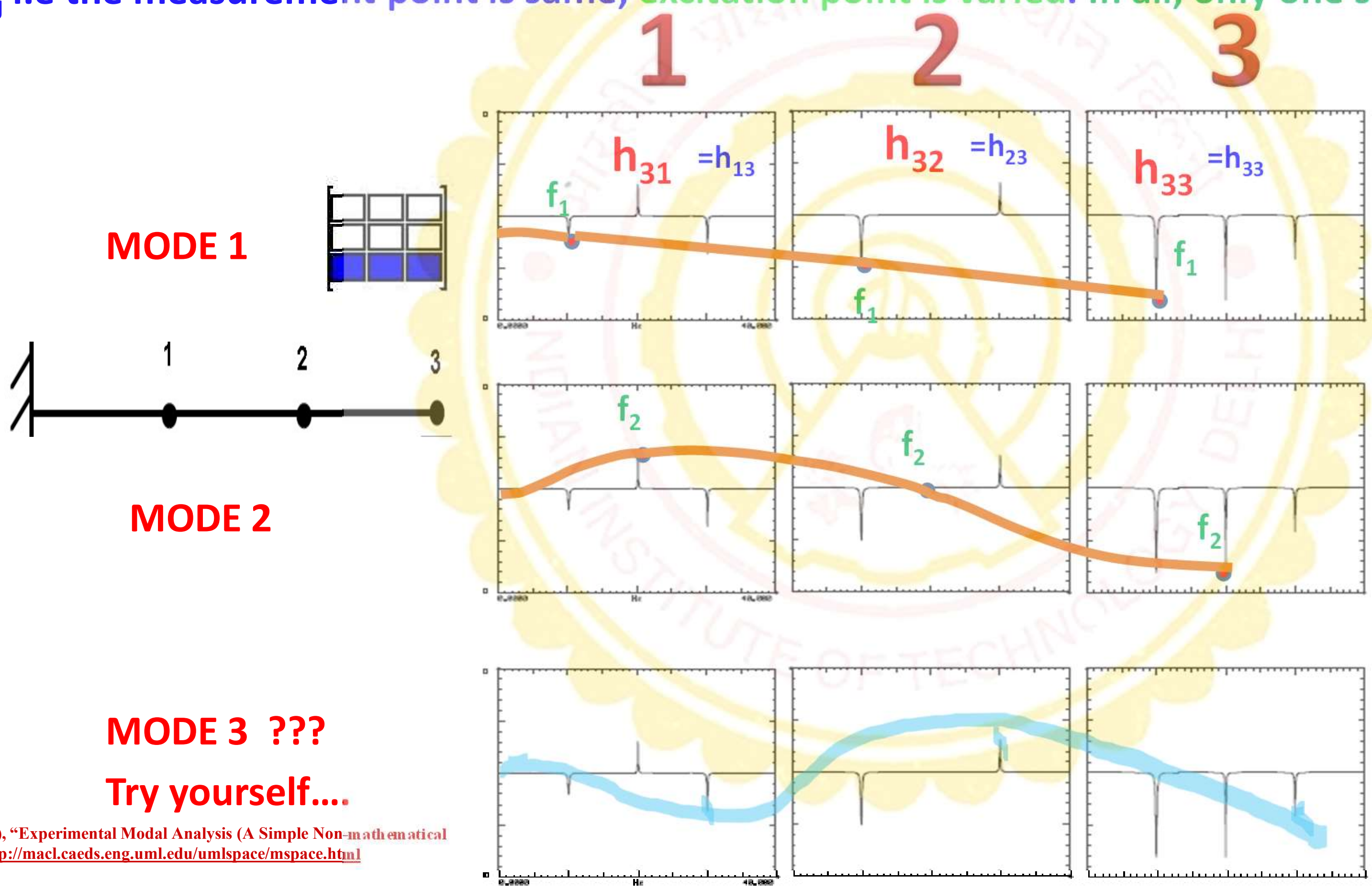


Real part of FRF **Imaginary part of FRF**

Avitable, P. (2001), "Experimental Modal Analysis (A Simple Non-mathematical Presentation)" http://macl.caeds.eng.uml.edu/umlspace/mspace.html

MODE SHAPES FROM FRF

In general, any one row can provide us all the mode shapes. Let us use the imaginary part of the third row i.e h_{3j} i.e the measurement point is same, excitation point is varied. In all, only one sensor required.



PEAK PICKING METHOD

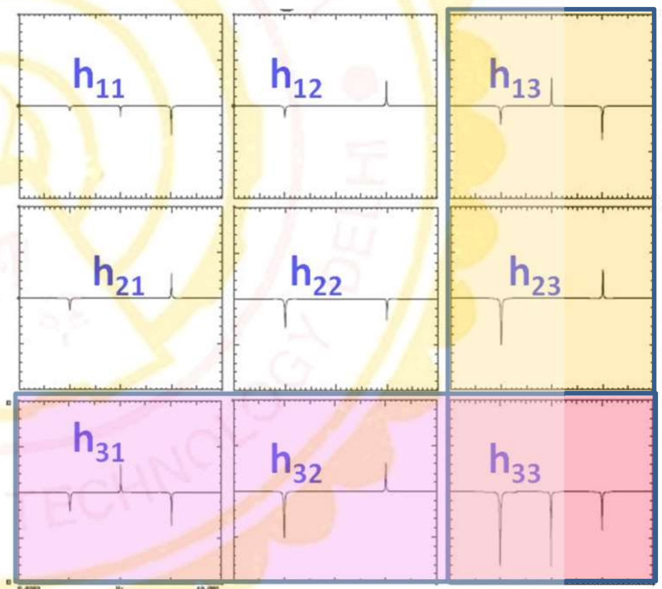
Avitable, P. (2001), "Experimental Modal Analysis (A Simple Non-mathematical Presentation)" http://macl.caeds.eng.uml.edu/umlspace/mspace.html

MODE SHAPES FROM FRF

Since $h_{ij} = h_{ji}$

Any one column can also be used in place of a row.

If say the third column is used i.e h_{j3} i.e measurement point varies, excitation point remains same. We can use three sensors with simultaneous measurements and thus acquire all data in a single go.



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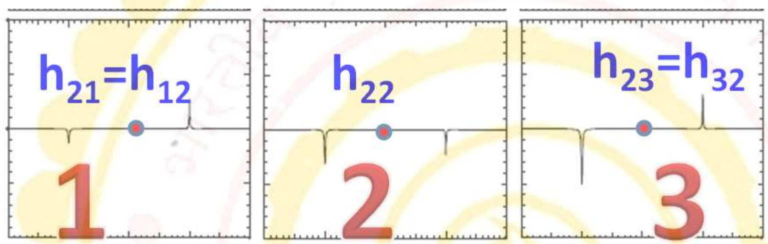
Or can also repeat three times with one sensor, each time changing position of the sensor. This gives third column…which is same as third row.

A WORD OF CAUTION!

Let us try row 2 to obtain a mode shape, say the second mode



We cannot!

We do not see any peak at the frequency corresponding to mode 2. Why?

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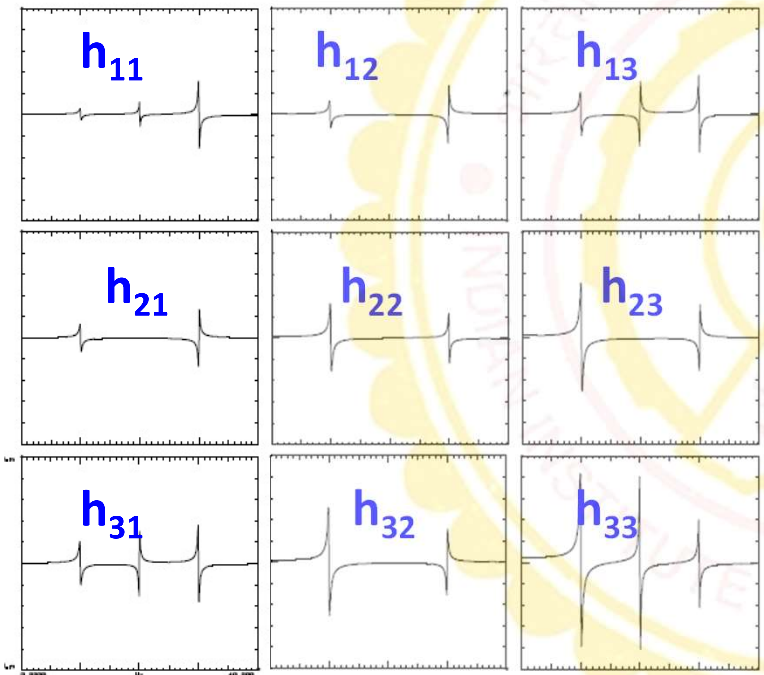
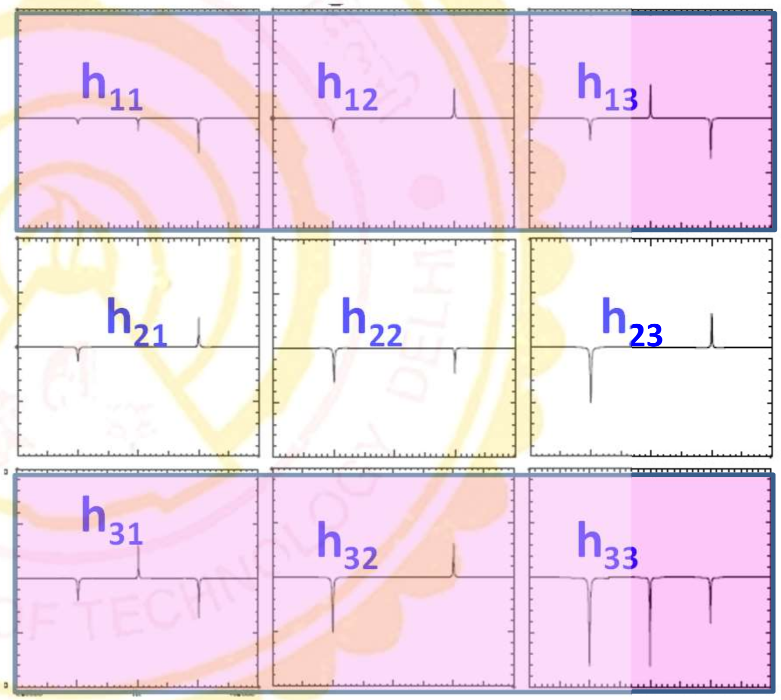
Measurement point 2 is a nodal point for mode 2 i.e. ZERO amplitude of vibration.

Hence, nodal points should be preferably avoided as measurement or excitation points.

BUT ROWS 1 AND 3 ARE FINE!



Real part of FRF



Imaginary part of FRF

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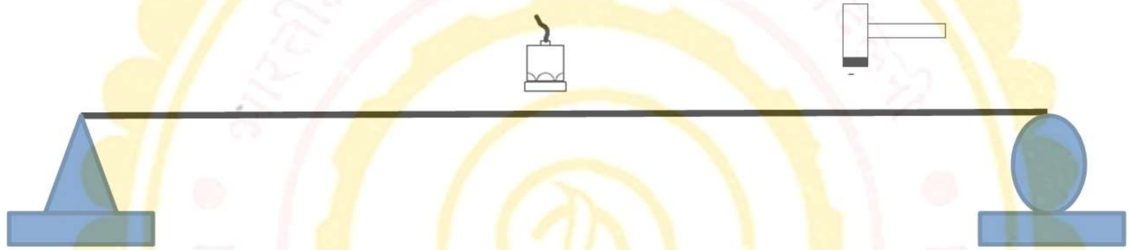
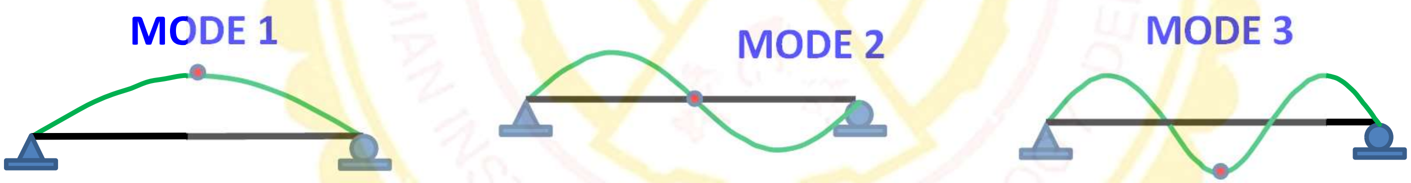
PRACTICAL RELEVANCE

Measurement at the mid point of a simply supported beam.

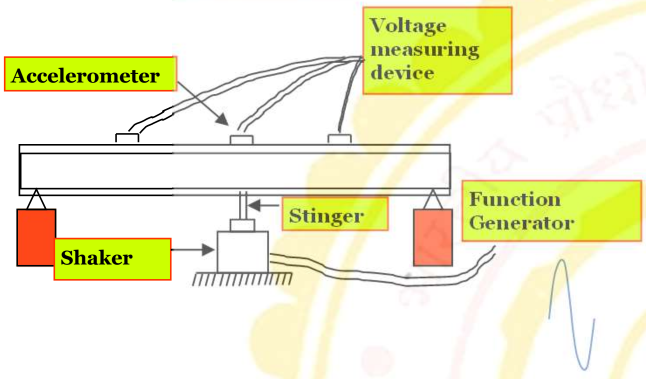


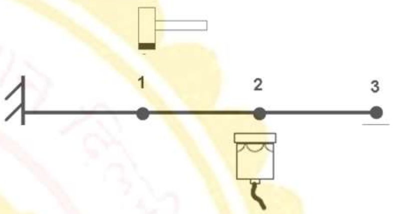


Modes 1, 3, 5, 7….. will be identified. In fact very good point

Modes 2, 4, 6, 8….. will be missed out.

SHAKER vs IMPACT TEST





Single point excitation, multiple response….hence a column of FRF

Generally one point for response, multiple points for excitation……hence a row of FRF

Theoretically no difference….but practically…..

Need to make elaborate arrangements

Very quick

Frequency of excitation can be very well controlled.

Input excitation frequency dependent on the hardness of the hammer tip, harder the tip, wider the frequency range

Potential stiffening effect of shaker and stinger system in shaker test

Effect of location of accelerometers, especially when same set is moved around the structure, applicable in both impact test and shaker test, especially for a very light structure. Can be minimized by "dummy accelerometers"

Suggested reading

Avitable (2001) pp. 1 to 6, Farrar and Jauregui (1998)

CONTENTS

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- **GLOBAL VIBRATION TEHNIQUES FOR STRUCTURAL HEALTH MONITORING**
- USE OF MODERN SMART SENSORS IN GLOBAL VIBRATION TECHNIQUES

CHANGE IN STIFFNESS METHOD

(Zimmerman and Kaouk, 1994)

Submitted in June 1992

Based on the principle that damage alters the stiffness matrix of the structure

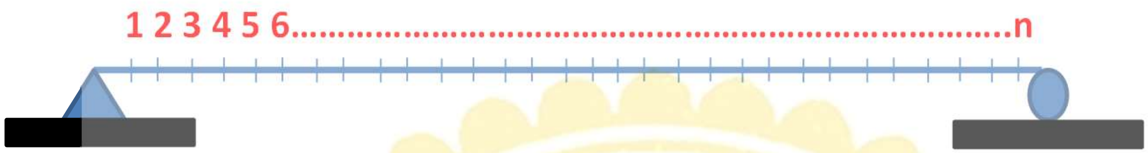
The stiffness matrix is derived from the measured natural frequencies and the mode shapes of the structure.

The stiffness matrix [**K**] was expressed by the researchers in terms of the mode shape matrix **[ϕ]**, the mass matrix [**M**], and the modal stiffness matrix [**Ω**] (diagonal ω_i^2) as

$$[K] = [M][\Phi][\Omega][\Phi]^T[M] \;\; = [M]\left(\sum_{i=1}^{n} \omega_i^2 \phi_i \phi_i^T\right)[M]$$

Mass normalized

where "n" is the total number of modes considered.



Consider a "n" degree of freedom structure. For sake of derivation, let us assume that all "n" mode shapes are available

For the i^{th} mode, the eigenvalue problem can be written as

$$\phi_i^T[K]\phi_i = \omega_i^2 \ \phi_i^T[M]\phi_i$$

$$\phi_i^T[M]\phi_i = m_i$$ i^{th} modal mass

Let us define complete "modal matrix of the structure (nxn) by arranging all "n" mode shapes vertically

$$[\Phi] = [\phi_1\phi_2\phi_3\ldots\phi_i\ldots\phi_n]$$

The total eigenvalue problem can be written as

$$[\Phi]^T[K][\Phi] = [\Omega][\Phi]^T[M][\Phi]$$

$$[\Omega] = [diag\ \omega_i^2]$$

$$[\Phi]^T[M][\Phi] = ?? \quad = [diag\ \ m_i\ ] = [M_p]$$

$$[\Phi]^T[K][\Phi] = [\Omega][M_p]$$

Pre-multiply both sides by $\left\{[\Phi]^T\right\}^{-1}$ Post-multiply by $\left\{[\Phi]\right\}^{-1}$

$$\left\{[\Phi]^T\right\}^{-1}[\Phi]^T[K][\Phi][\Phi]^{-1} = \left\{[\Phi]^T\right\}^{-1}[\Omega][M_p][\Phi]^{-1}$$

$$[K] = \left\{[\Phi]^T\right\}^{-1}[\Omega][M_p][\Phi]^{-1}$$

$$[K] = \left\{[\Phi]^T\right\}^{-1}[\Omega][M_p][\Phi]^{-1}$$

$$[\Phi]^T[M][\Phi] = [M_p]$$

Pre-multiply both sides by $[M_p]^{-1}$

$$[M_p]^{-1}[\Phi]^T[M][\Phi] = I$$

Post-multiply both sides by $[\Phi]^{-1}$

$$[\Phi]^{-1} = [M_p]^{-1}[\Phi]^T[M]$$

$$[\Phi]^{-1} = [M_p]^{-1}[\Phi]^T[M]$$

$$\left\{[\Phi]^T\right\}^{-1} = ?? = [M_p]^{-1}[\Phi]\,[M]$$

Replace Phi by Phi (transpose) in top equation

If mode shapes are mass normalized

$$[\Phi]^T[M][\Phi] = [M_p] = [I]$$

$$[\Phi]^{-1} = [\Phi]^T[M]$$

$$\left\{[\Phi]^T\right\}^{-1} = [\Phi]\,[M]$$

Summary: $[K] = \left\{[\Phi]^T\right\}^{-1} [\Omega][M_p][\Phi]^{-1}$

(For mass normalized case)

$[\Phi]^{-1} = [\Phi]^T[M]$ $\quad \left\{[\Phi]^T\right\}^{-1} = [\Phi]\,[M]$ $\quad [M_p] = [I]$

$$[K] = [\Phi]\;[M][\Omega][\Phi]^T[M]$$

$$[K] = [M][\Phi]\;[\Omega][\Phi]^T[M]$$

Even if we have incomplete modal information, in terms of "m" modes only, we still obtain [K] matrix "nxn"

CHANGE IN STIFFNESS METHOD
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Submitted in June 1992

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$$[K]=[M][\Phi][\Omega][\Phi]^T[M] \quad =[M]\left(\sum_{i=1}^{n}\omega_i^2\phi_i\phi_i^T\right)[M]$$

Mass normalized

where "n" is the total number of modes considered.

The main disadvantage of this method is that the higher modes are more important in the estimation of the stiffness matrix, since the modal contribution to [**K**] increases as the modal frequency increases. The dynamic vibration testing, on the other hand, can yield only the **first few mode shapes**.

CHANGE IN FLEXIBILITY METHOD
(Pandey and Biswas, 1994)

Paper was submitted on 15 Feb 1991

Based on the principle that damage alters the flexibility matrix of the structure, that is the inverse of the stiffness matrix

$$[F] = [\Phi][\Omega]^{-1}[\Phi]^T \quad = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T$$

Mass normalized

$$[K] = \{[\Phi]^T\}^{-1} [\Omega][M_p][\Phi]^{-1}$$

Just take inverse of both sides to obtain the flexibility matrix **(HW)**

CHANGE IN FLEXIBILITY METHOD (Pandey and Biswas, 1994)

Paper was submitted on 15 Feb 1991

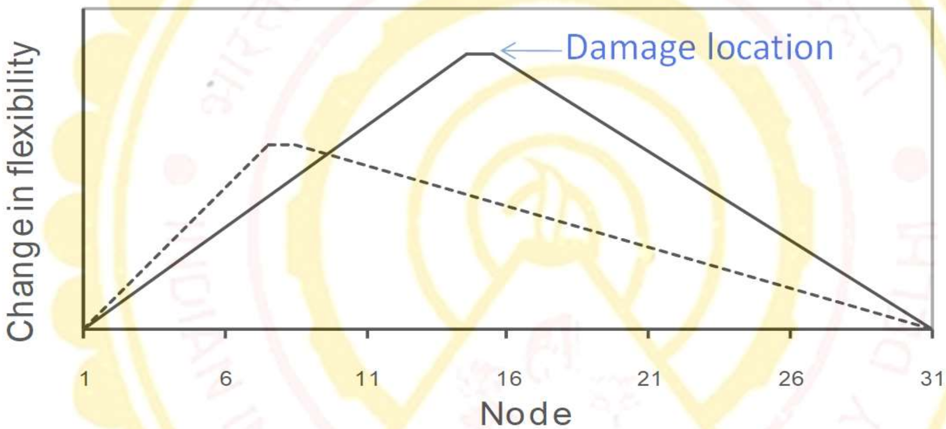
Based on the principle that damage alters the flexibility matrix of the structure, that is the inverse of the stiffness matrix

$$[F] = \underset{\substack{\uparrow \\ \text{Mass normalized}}}{[\Phi]}[\Omega]^{-1}[\Phi]^T \quad = \sum_{i=1}^{n} \frac{1}{\omega_i^2}\phi_i\phi_i^T$$

As can be seen from the equation, [**F**] is proportional to the square of the inverse of the modal frequencies.

Therefore, it converges rapidly with increasing frequencies. **Hence, only a few lower modes are sufficient for an accurate estimation of [F]**. The technique is thus an improvement over the change in stiffness method.

CHANGE IN FLEXIBILITY METHOD
(Pandey and Biswas, 1994)



Identification of damage location from a plot of the change in flexibility for a beam structure

CHANGE IN MODE SHAPE CURVATURE METHOD (Pandey et al. , 1991)

The curvature at any location of the beam subjected to a bending moment M (x), could be expressed as (***v*** is vertical deflection)

$$v''(x) = \frac{M(x)}{EI} = \frac{\varepsilon}{y}$$

Curvature mode shape is obtained as the second derivative of the displacement mode shape

where E is the Young's modulus of elasticity and *I* the moment of inertia of the section.

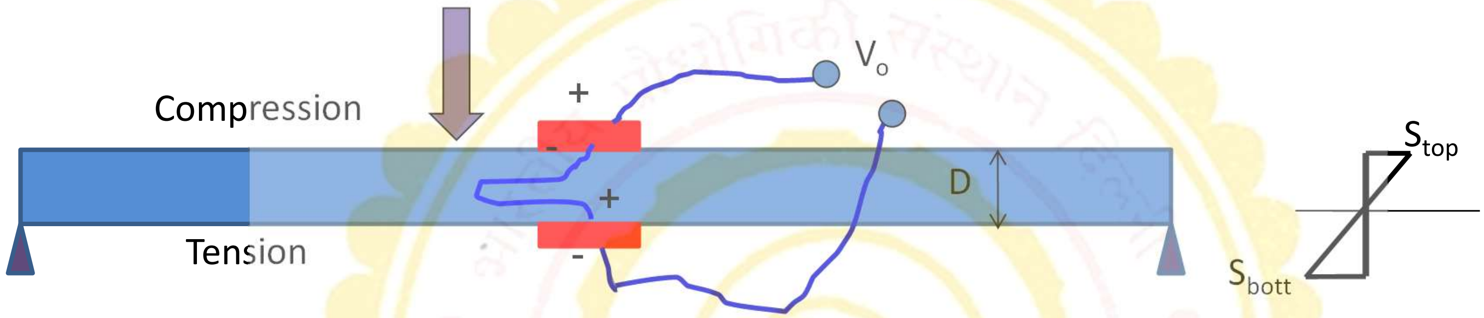
A reduction in stiffness associated with the damage will lead to an increase in curvature.

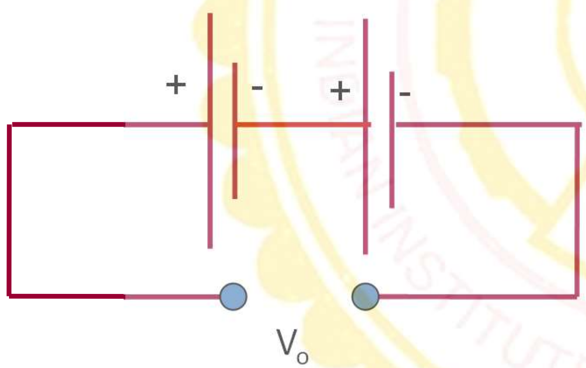
In this method, the pre and post damage displacement mode shapes are first extracted by experimental modal analysis.

Differences in pre and post damage curvature mode shapes are maximum in the vicinity of the damaged region. This method is computationally simpler as compared to the change in stiffness method and the change in flexibility method.

The method is expected to be very much expedient with piezo sensors bonded on extreme fibres since that can enable curvature mode shape directly.

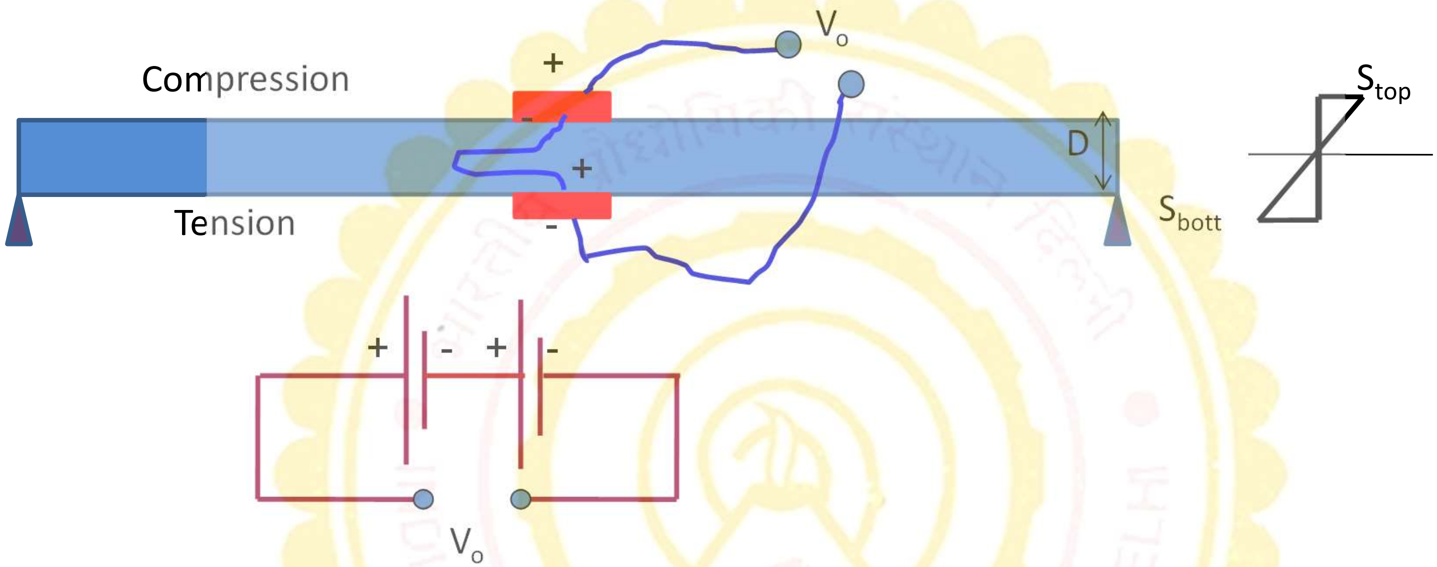
SENSING OF FLEXURE





$$V_o = k(S_{top} + S_{bott})$$

$$\phi = \frac{S_{top} + S_{bott}}{D}$$



$$V_o = k(S_{top} + S_{bott}) \qquad \phi = \frac{S_{top} + S_{bott}}{D}$$

Piezo patches enable obtaining curvature mode shapes directly

CHANGE IN UNIFORM LOAD SURFACE CURVATURE METHOD

(Zhang and Aktan, 1995)

The coefficients of the J^{th} column of the flexibility matrix represent the deflected shape assumed by a structure when a unit load is applied at the J^{th} degree of freedom.

The sum of all the columns of the flexibility matrix represent the deformed shape if a unit load is applied at each degree of freedom.

Zhang and Aktan used the curvature of this uniform load surface to determine the location of damage. The curvature change at the i^{th} location was expressed as

$$\Delta F_i'' = \left| F_i^{*''} - F_i'' \right|$$

Post-damage curvature   Pre-damage curvature

The damage location is identified as the element with maximum value of ΔF

DAMAGE INDEX METHOD
(Stubbs and Kim, 1994)

Strain energy

$$U = \frac{1}{2}\int_V \sigma\varepsilon dV$$

$$= \frac{1}{2}\int_L EI(v'')^2\, dx$$

Modal strain energy

$$U = \frac{1}{2}\int_L EI(\phi'')^2 dx$$

DAMAGE INDEX METHOD
(Stubbs and Kim, 1994)

The damage index β is calculated based on the strain energy stored in the structure when it deforms in a particular mode shape. For the j^{th} location and the i^{th} mode, the damage index β_{ij} can be defined as

$$\beta_{ij} = \frac{\left(\int_a^b [\phi_i^{*''}(x)]^2 dx + \int_0^L [\phi_i^{*''}(x)]^2 dx \right) \int_0^L [\phi_i^{''}(x)]^2 dx}{\left(\int_a^b [\phi_i^{''}(x)]^2 dx + \int_0^L [\phi_i^{''}(x)]^2 dx \right) \int_0^L [\phi_i^{*''}(x)]^2 dx}$$

$\phi_i^{''}(x)$ **Undamaged structure**

$\phi_i^{*''}(x)$ **Damaged structure**

DAMAGE IDEX METHOD (Stubbs and Kim, 1994)

$$\beta_{ij} = \frac{\left(\int_a^b [\phi_i^{*''}(x)]^2\,dx + \int_0^L [\phi_i^{*''}(x)]^2\,dx\right)\int_0^L [\phi_i^{''}(x)]^2\,dx}{\left(\int_a^b [\phi_i^{''}(x)]^2\,dx + \int_0^L [\phi_i^{''}(x)]^2\,dx\right)\int_0^L [\phi_i^{*''}(x)]^2\,dx}$$

Here, 'a' and 'b' are the limits of a segment of the beam where damage is being evaluated.

Strain energy $U = \frac{EI}{2}\int_L (y'')^2\,dx$

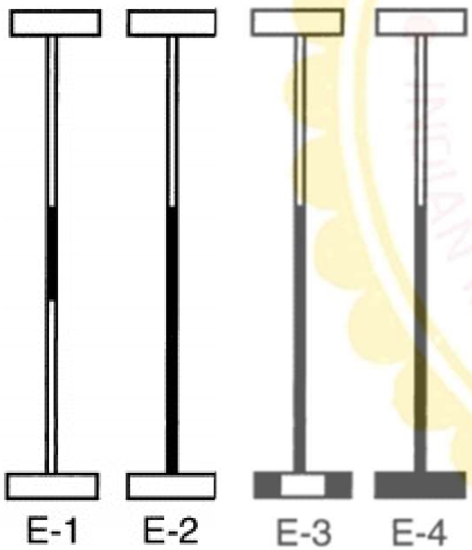
Modal Strain energy $U_\phi = \frac{EI}{2}\int_L (\phi'')^2\,dx$

This technique has an advantage over the previous ones in that it has a specific criterion for determining whether damage has occurred at a particular location. Other techniques look for the largest change in a particular parameter. It is therefore suitable for multiple damage locations.

COMPARATIVE STUDY OF DIFFERENT ALGORITHMS

Farrar and Jauregui (1998)

Farrar and Jauregui (1998) did a comparative study of the techniques on a single structure, the I-40 bridge over Rio Grande in Albuquerque, New Mexico, USA.



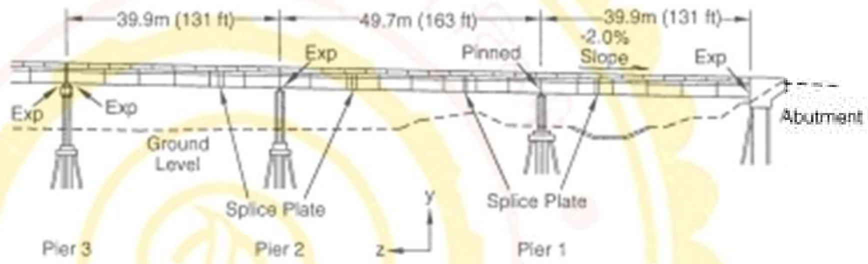


Figure 1. Elevation view of the portion of the I-40 bridge that was tested.

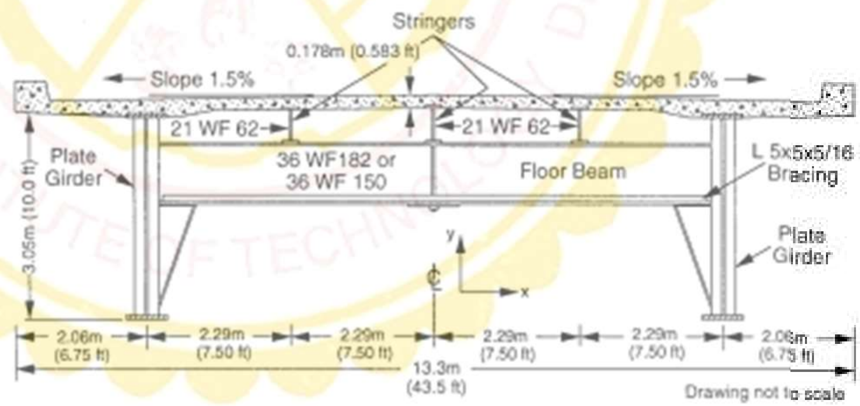


Figure 2. Cross-section geometry of the I-40 bridge.

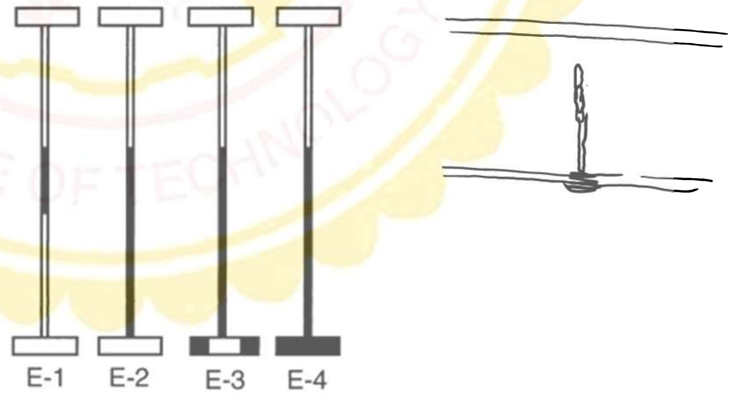
COMPARATIVE STUDY OF DIFFERENT ALGORITHMS

Farrar and Jauregui (1998)

The bridge had a maximum span length of about 50m and a girder depth of about 3m.

Damage was induced by means of a torch cut, at the middle of the 50m span and was started from the mid depth of web as a 0.6m long by 10mm wide crack .

It was then extended in three stages to the entire bottom half of the web and the bottom flange. The bridge was excited by a hydraulic shaker (both in undamaged and damaged conditions) and the modal data was extracted.



COMPARATIVE STUDY OF DIFFERENT ALGORITHMS

Farrar and Jauregui (1998)

The major conclusions drawn based on the analysis of the response data by various methods on a real bridge being decommissioned, are as follows :

1. **Standard modal properties such as mode shapes and resonant frequencies are poor indicators of damage. No noticeable change in the measured resonant frequencies and mode shapes was observed until the final level of damage.**

2. **All the methods identified the damage location correctly for the most severe damage case, i.e a cut through the entire bottom half of the beam.**

3. **In some of the methods, human judgment was required to correctly identify the location of damage. If they were applied blindly, it would have been difficult to locate the damage.**

4. **When the entire set of tests are considered, the damage index method was found to be the one with the best performance.**

SUMMARY

1. Curvature mode shape and modal energy approach more sensitive to damage.

2. Obtaining curvature mode shapes from displacement mode shapes implies numerical differentiation……

3. Accuracy of mode shape very much depend upon the spatial resolution of measurements.

4. Is it possible to derive the curvature mode shape directly?

5. How about issues with real-life structures?

CONTENTS

- EXPERIMENTAL MODAL ANALYSIS
  (NON MATHEMATICAL TREATMENT)

- GLOBAL VIBRATION TEHNIQUES FOR STRUCTURAL HEALTH MONITORING

- USE OF MODERN SMART SENSORS IN GLOBAL VIBRATION TECHNIQUES

EXPERIMENTAL VS OPERATIONAL MODAL ANALYSIS

Experimental Modal Analysis (EMA):

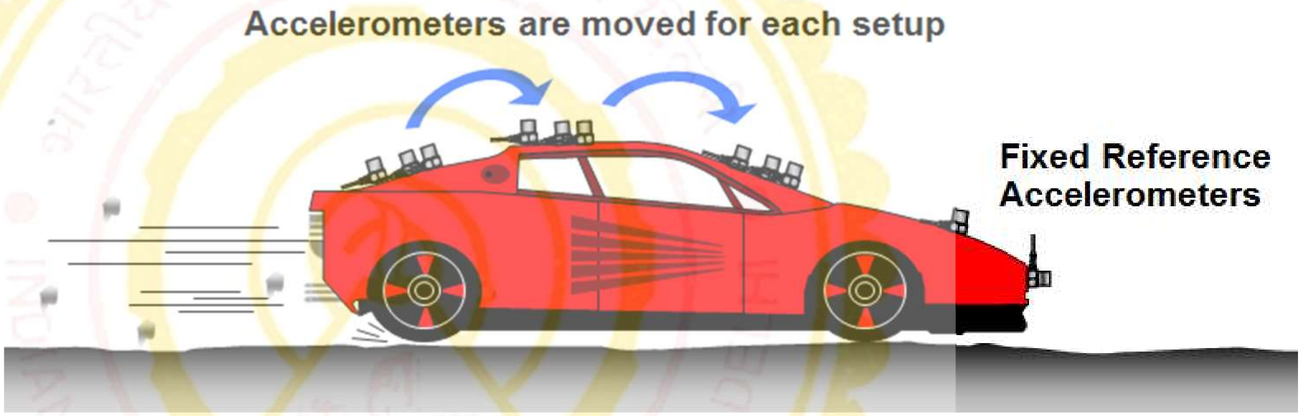
EMA typically involves subjecting the structure to controlled inputs, such as impact testing (using hammers or shakers) or controlled loads, and then measuring the response of the structure using accelerometers or other sensors. The input is known and controlled, and measurements are taken under laboratory conditions.

Operational Modal Analysis (OMA):

OMA, on the other hand, involves measuring the structural response while the structure is under its normal operating conditions. The input to the structure is not controlled but arises naturally from ambient excitation, such as wind, traffic, or machinery vibrations.

OMA: HOW RESPONSE IS NORMALIZED?

EMA: External force measured



OMA: External force not measured

A few high quality sensors are placed in positions where the modes of interest are having a good response level. These sensors are called reference sensors and are fixed in the same position when moving from one Test Setup to another. The rest of the sensors are placed in the DOF positions where mode shapes is wanted.

https://www.svibs.com/resources/ARTeMIS_Modal_Help/Operational%20Modal%20Analysis.html

EXPERIMENTAL VS OPERATIONAL MODAL ANALYSIS

Experimental Modal Analysis (EMA):

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Operational Modal Analysis (OMA):

OMA, on the other hand, involves measuring the structural response while the structure is under its normal operating conditions. The input to the structure is not controlled but arises naturally from ambient excitation, such as wind, traffic, or machinery vibrations.

OPERATIONAL MODAL ANALYSIS

Pros:

1. **Does not require controlled excitation:** OMA can be performed using ambient vibrations or operational loads, which means that testing can be conducted without interrupting normal operations.
2. **Cost-effective:** Since OMA does not require specialized equipment for excitation, it can be more cost-effective than EMA.
3. **Can be applied to large structures:** OMA is particularly useful for large and complex structures where it may be difficult to apply controlled excitation.
4. **Real-world conditions:** OMA provides modal parameters under actual operating conditions, which can be valuable for understanding the dynamic behavior of a structure in real-world scenarios.

Cons:

1. **Less accurate:** OMA typically provides less accurate results compared to EMA, especially for lightly damped structures or in situations where the excitation is not well-defined.
2. **Limited frequency range:** OMA is often limited in the frequency range over which accurate modal parameters can be obtained.
3. **Less control over testing conditions:** Since OMA relies on ambient excitation, there is less control over the testing conditions, which can affect the quality of the results.

EXPERIMENTAL MODAL ANALYSIS

Pros:

1. **High accuracy:** EMA is generally considered more accurate than OMA, especially for structures with well-defined excitation.
2. **Wide frequency range:** EMA can cover a wide frequency range, making it suitable for a variety of structures and dynamic phenomena.
3. **More control over testing conditions:** EMA allows for more control over the excitation and testing conditions, which can lead to more reliable results.
4. **Quantitative assessment:** EMA provides quantitative modal parameters that can be directly used in structural analysis and model updating.

Cons:

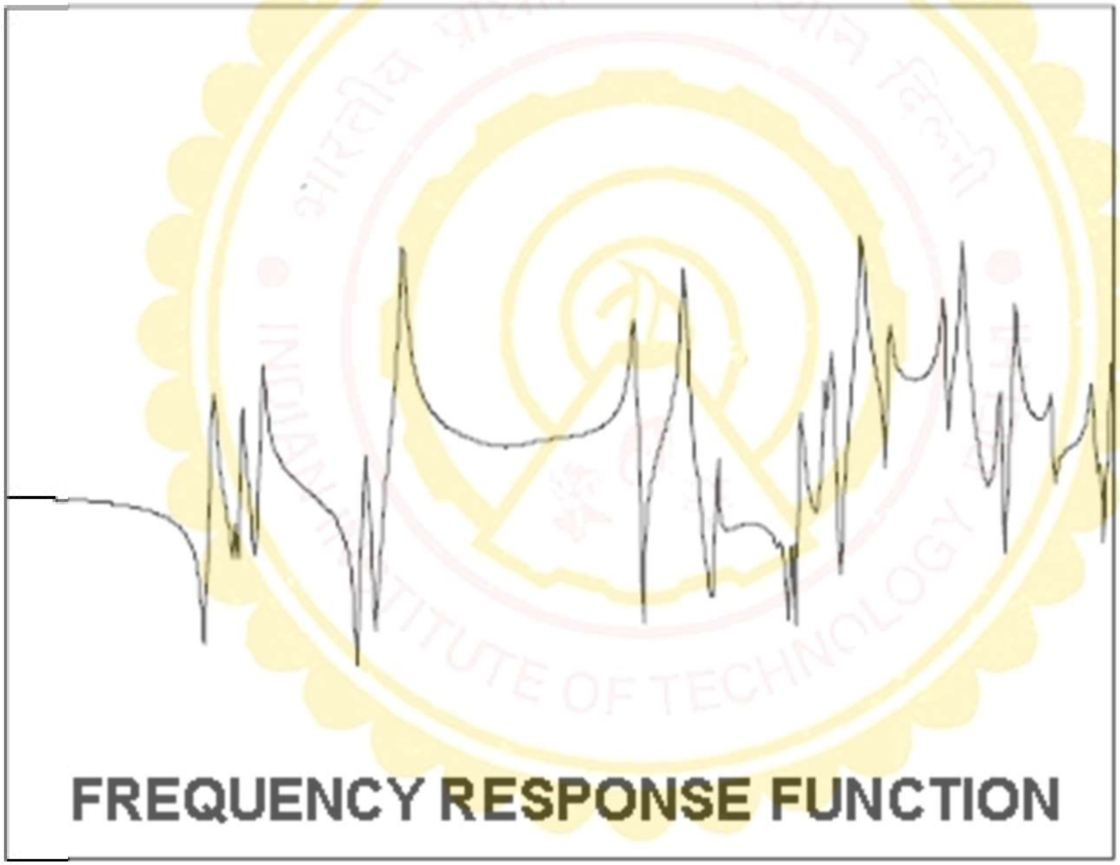
1. **Disruptive testing:** EMA requires controlled excitation, which can be disruptive to normal operations and may require the structure to be taken out of service temporarily.
2. **Cost and complexity:** EMA typically requires specialized equipment for excitation and data acquisition, which can make it more expensive and complex than OMA.
3. **Limited applicability to large structures:** EMA may be less practical for large structures where it is difficult to apply controlled excitation.

EXPERIMENTAL VS OPERATIONAL MODAL ANALYSIS

In summary, O ... to work with ambient vibrat ... ol over testing conditions.

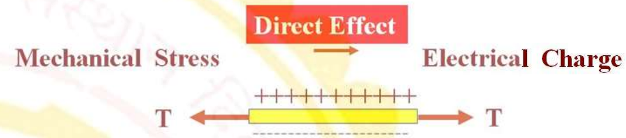
In Operational ... cally not mass normalized. Sin ... of the structure, the mode shap ... ode shapes are typically norma ... e shape is 1.

Mass normaliz ... s (EMA), where the mass distri ... is normalization allows for easi ... ir use in modal analysis and m...



PZT PATCH AS STRAIN SENSOR

Using $Q = D_3 A = CV$

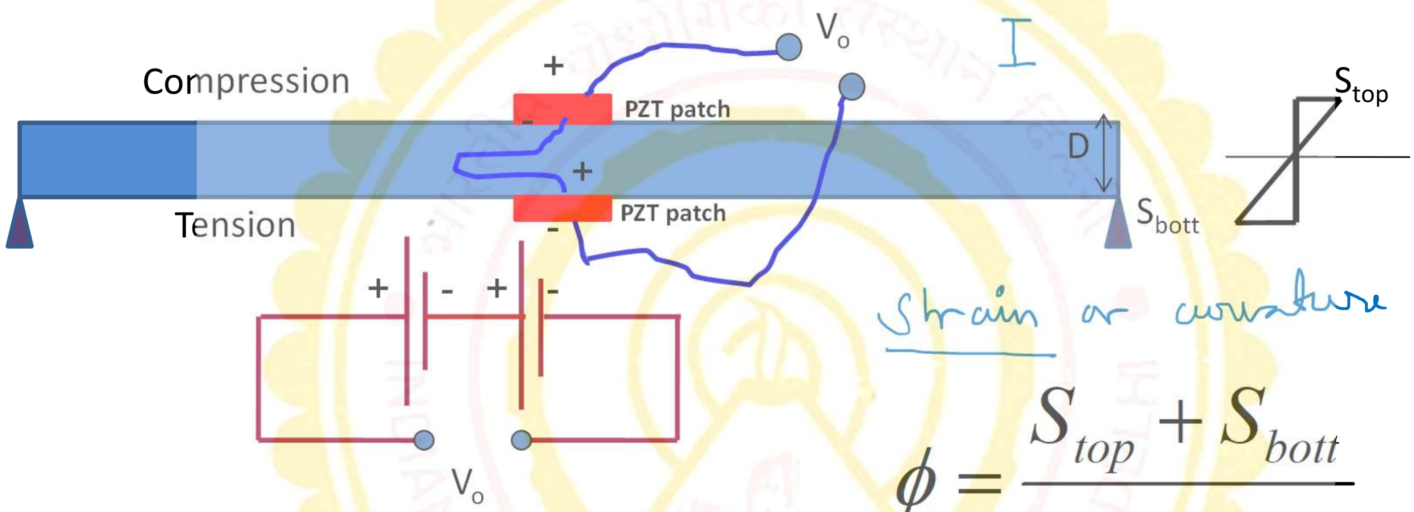


$$V = \left(\frac{d_{31} h \overline{Y^E}}{\overline{\varepsilon_{33}^T}} \right) S_1 = k S_1$$

Voltage α Strain

Sensitivity = 200mV/μm/m

PZT PATCH AS CURVATURE SENSOR



I

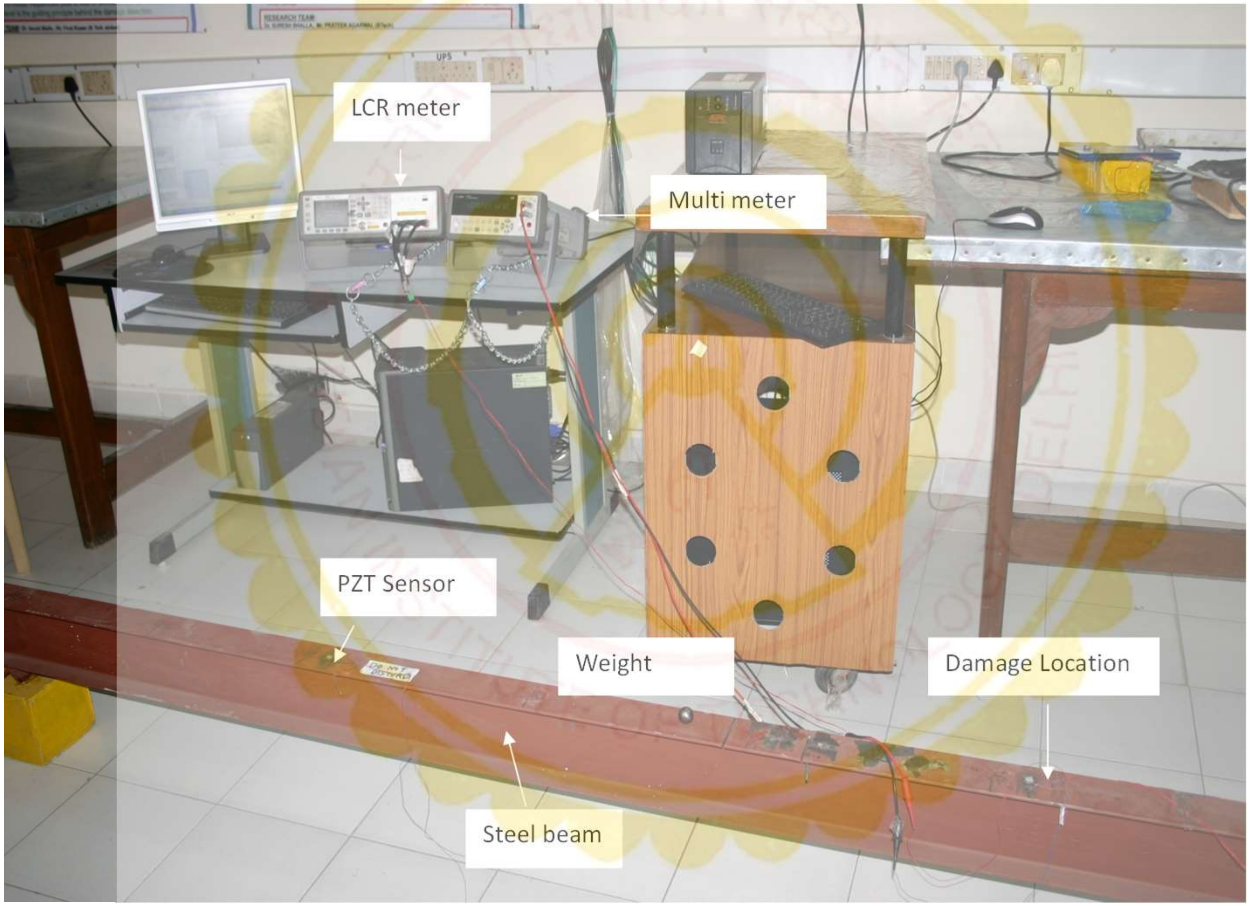
Strain or curvature mode shape

$$\phi = \frac{S_{top} + S_{bott}}{D}$$

$$V_o = k(S_{top} + S_{bott})$$

Curvature $\phi = \dfrac{S_{top} + S_{bott}}{D} = \dfrac{V_o}{kD}$

EXPERIMENTAL SETUP

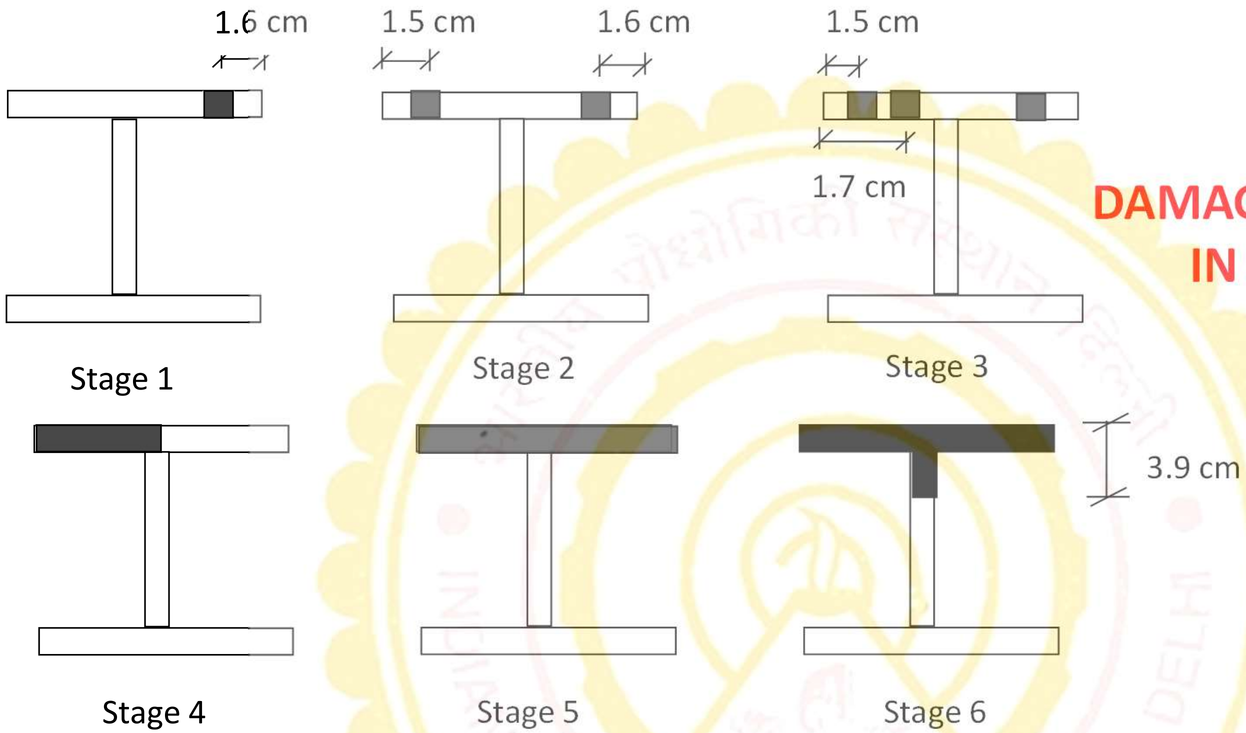


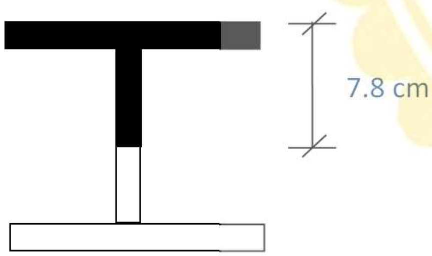
Artificial damaged steel beam

DETECTION OF INCIPIENT DAMAGE

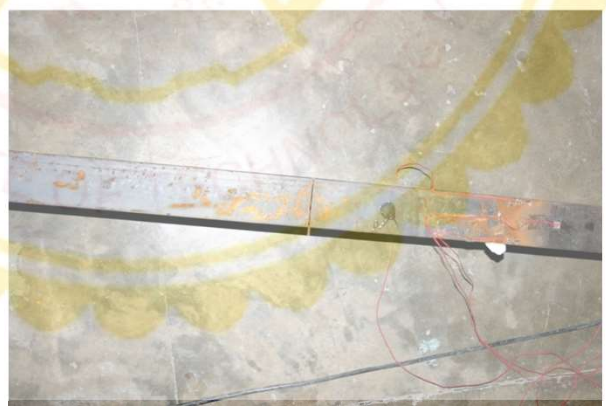
Experimental test setup

DAMAGE INDUCED IN STAGES





Stage 7



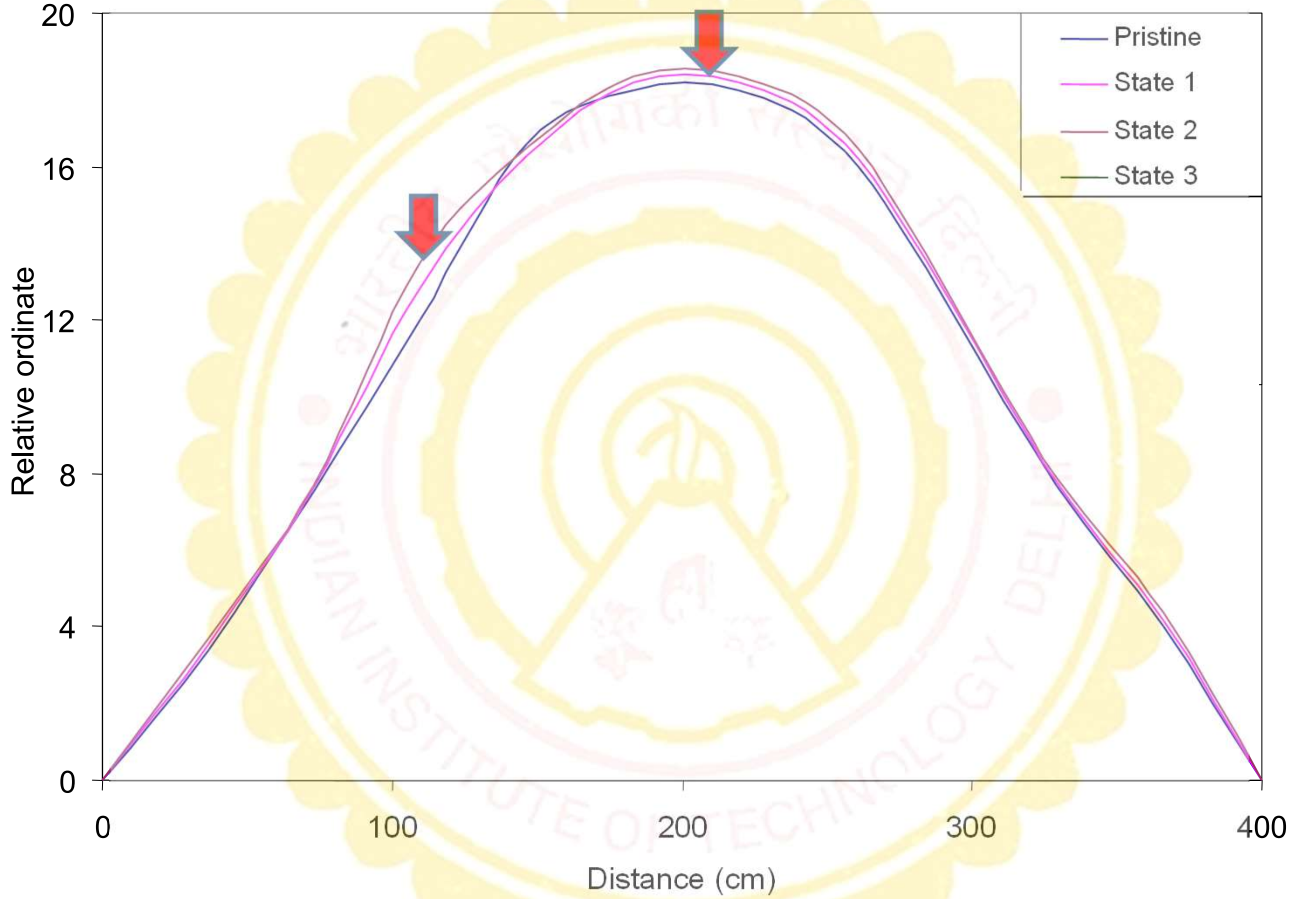
PROCEDURE

- Beam was divided into 12 nodes.
- Sensor response was measured corresponding to weight (200g) dropped at each point from 1.5m height.
- Time domain data was transformed to frequency domain.
- Three measurements were made at each point and average values obtained.
- Ordinates (real part) corresponding to first three modes were utilized to obtain the strain mode shapes.

NATURAL FREQUENCIES

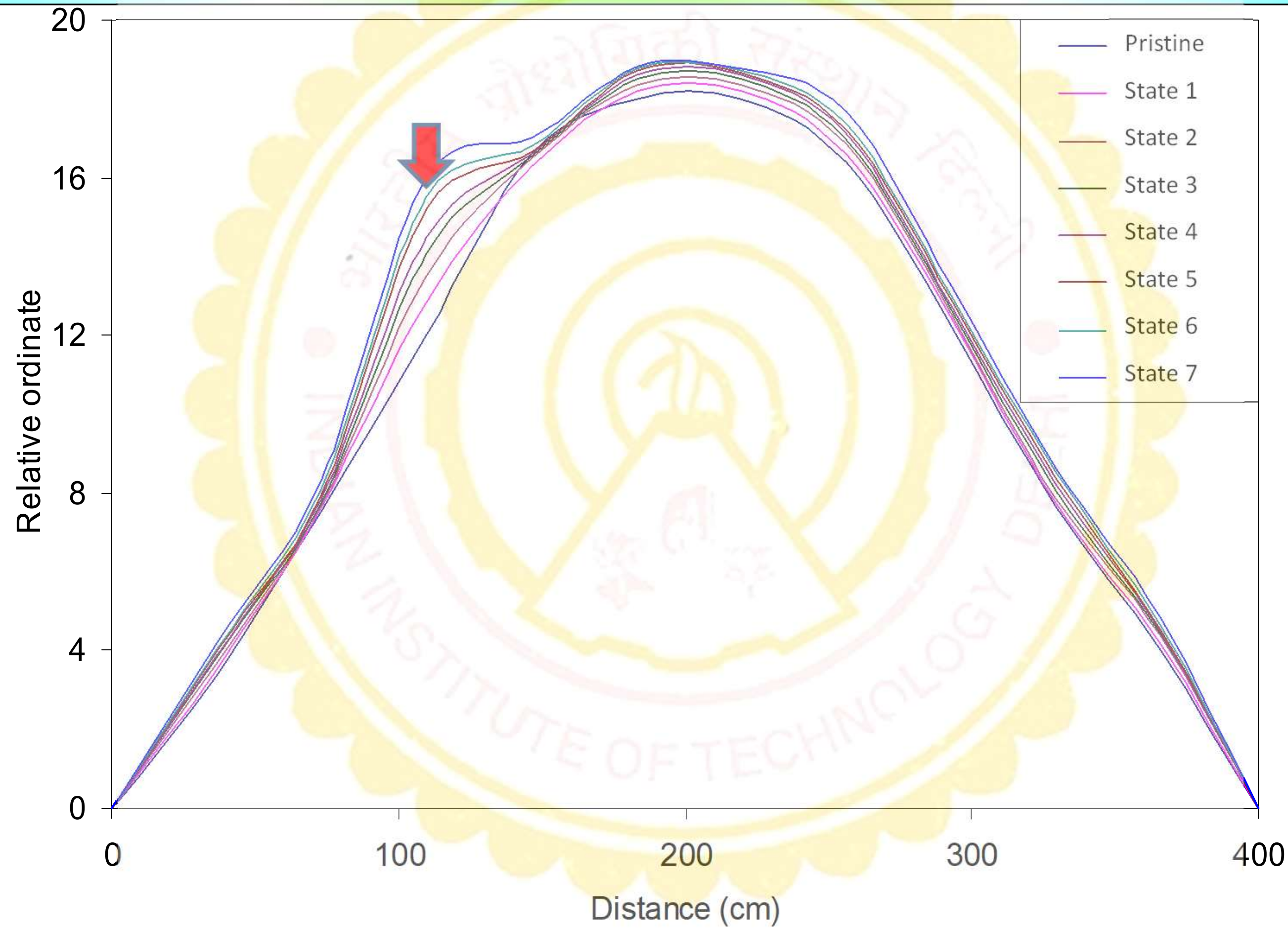
S.No.	Damage state	First natural frequency / (% change)	Second natural frequency / (% change)	Third natural frequency / (% change)
1	Undamaged	45 (-)	190 (-)	410 (-)
2	State-1	45 (0.0%)	189 (0.52%)	410 (0.00%)
3	State-2	45 (0.0%)	189 (0.52%)	408 (0.49%)
4	State-3	41 (8.9%)	185 (0.97%)	402 (1.95%)
5	State-4	39 (13.3%)	182 (4.21%)	399 (2.68%)
6	State-5	37 (17.8%)	179 (5.79%)	392 (4.39%)
7	State-6	36 (20%)	174 (8.42%)	387 (5.61%)
8	State-7	34 (24.4%)	170 (10.52%)	380 (7.32%)

Poor damage indicators in beginning!!

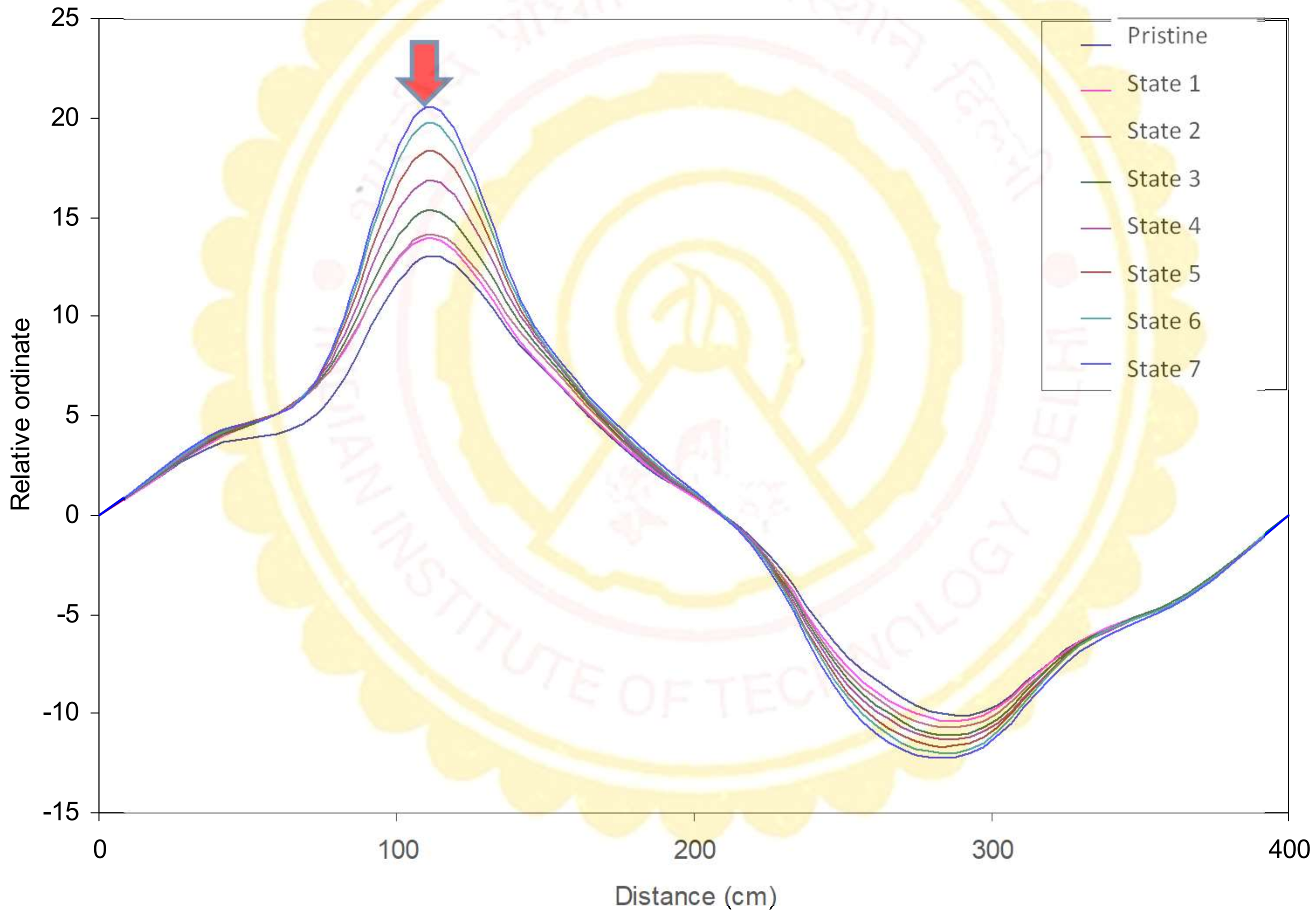


Curvature mode shape from piezo patches

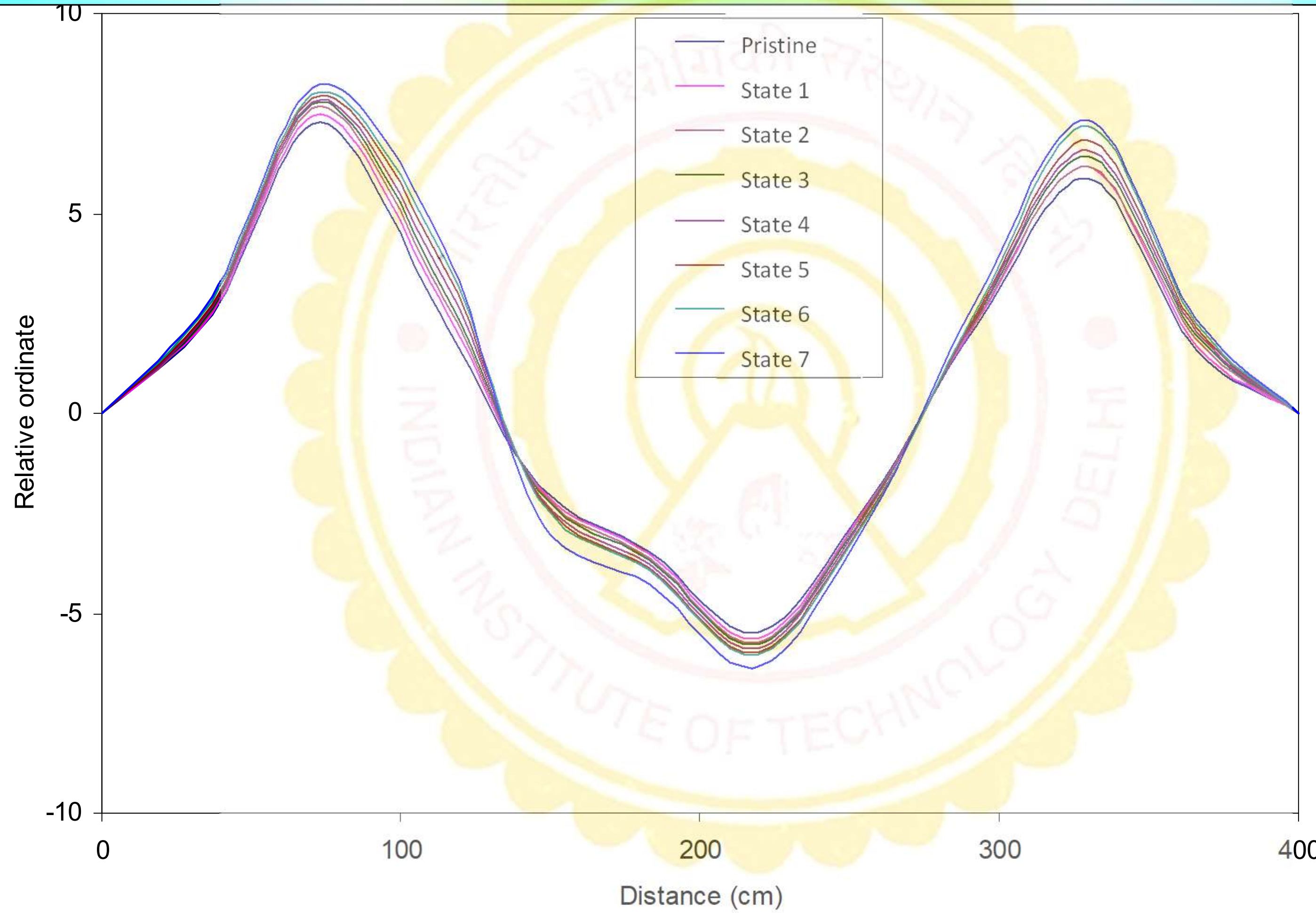
FIRST MODE SHPAE

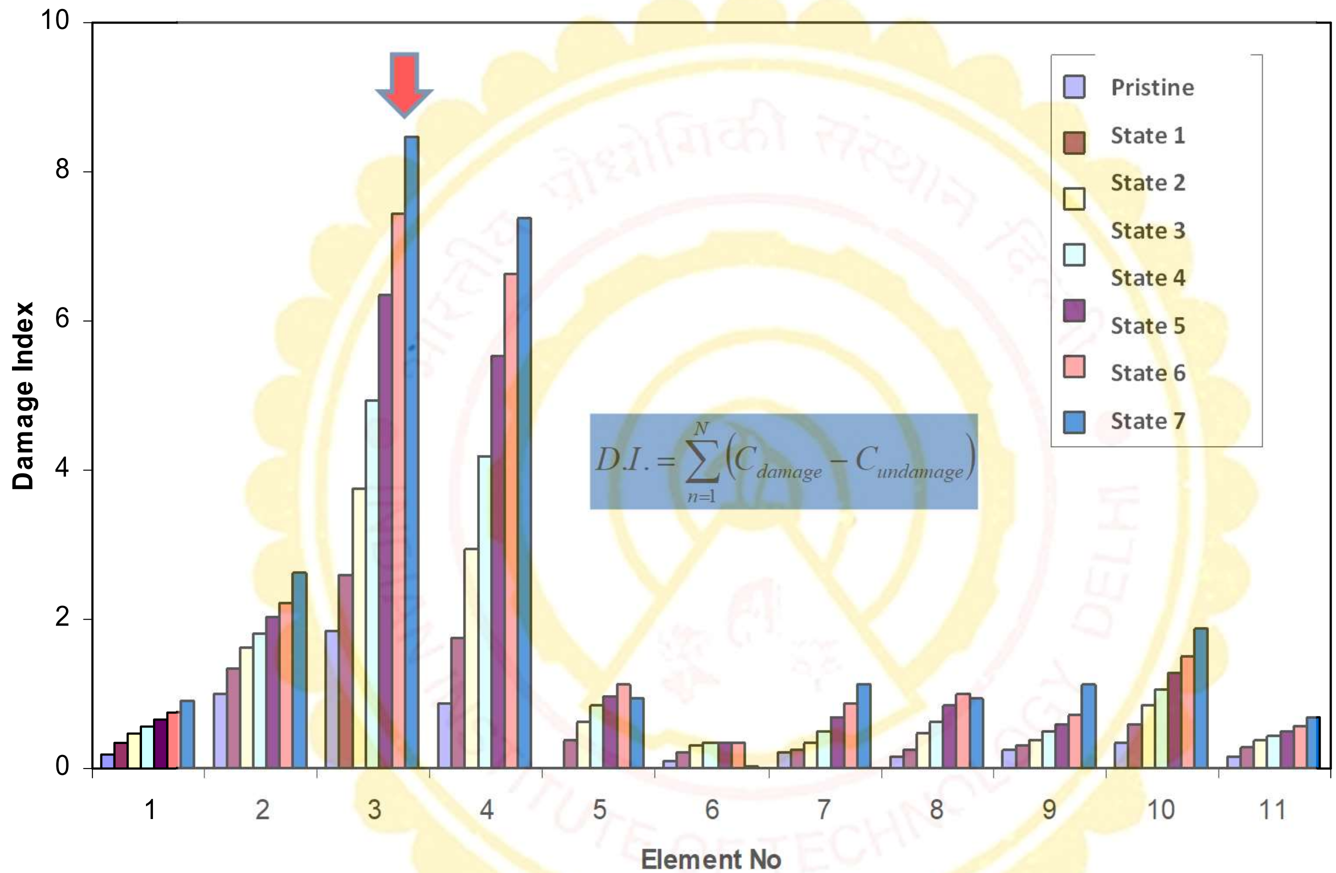


SECOND MODE SHPAE



THIRD MODE SHPAE





Damage index at an element: Sum of damage indices of two nodes of the element

ESTIMATION OF SEVERITY OF DAMAGE QUANTITATIVELY

$$\eta = \frac{(EI(x))_{damaged}}{(EI(x))_{undamaged}} = \frac{\left(Z_0^{\,2}\right)_{undamaged}\left(\int_0^L \Phi''^2(x)dx\right)_{undamaged}}{\left(Z_0^{\,2}\right)_{damaged}\left(\int_0^L \Phi''^2(x)dx\right)_{damaged}}$$

This gives reduction of stiffness averaged over entire length of beam

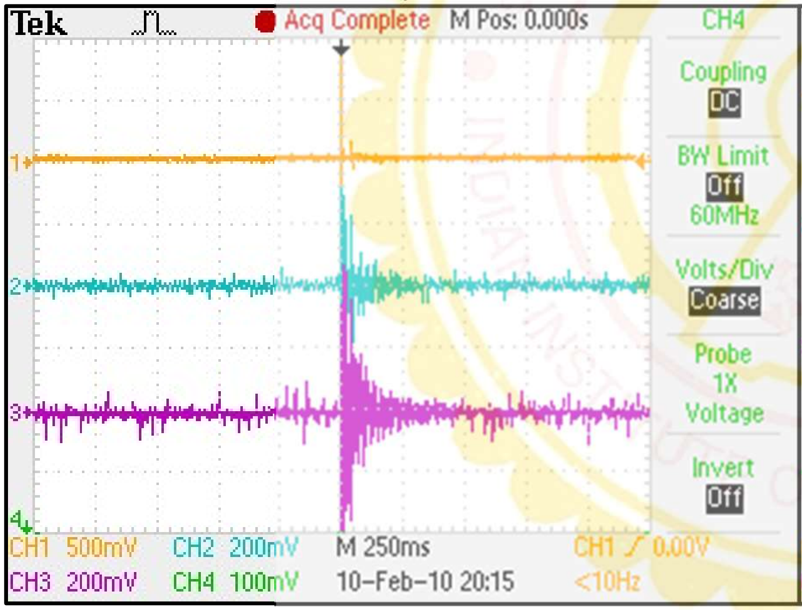
VARIATION OF STIFFNESS RATIO WITH DAMAGE FOR STEEL BEAM

S. No	Damage State	η =ratio of current stiffness to undamaged stiffness
1	State-1	0.97
2	State-2	0.94
3	State-3	0.92
4	State-4	0.88
5	State-5	0.85
6	State-6	0.83
7	State-7	0.81

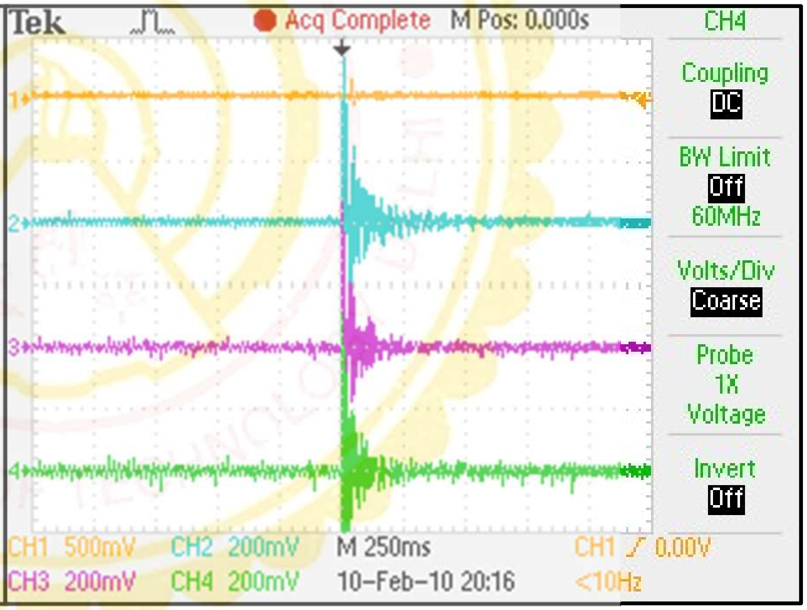
STRAIN MODE SHAPES OF SIMPLY SUPPORTED BEAM

B. Tech. Project (CHAKRABARTY, 2010)

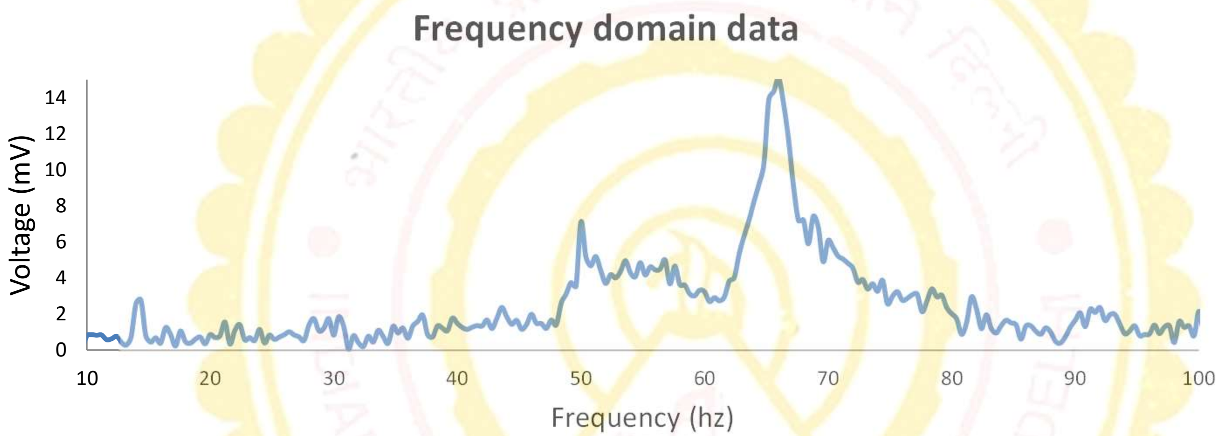
Oscilloscope 1

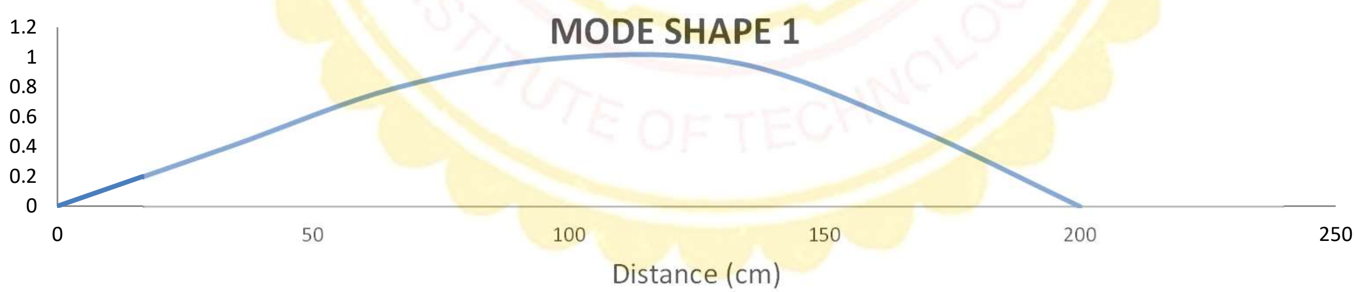


Oscilloscope 2

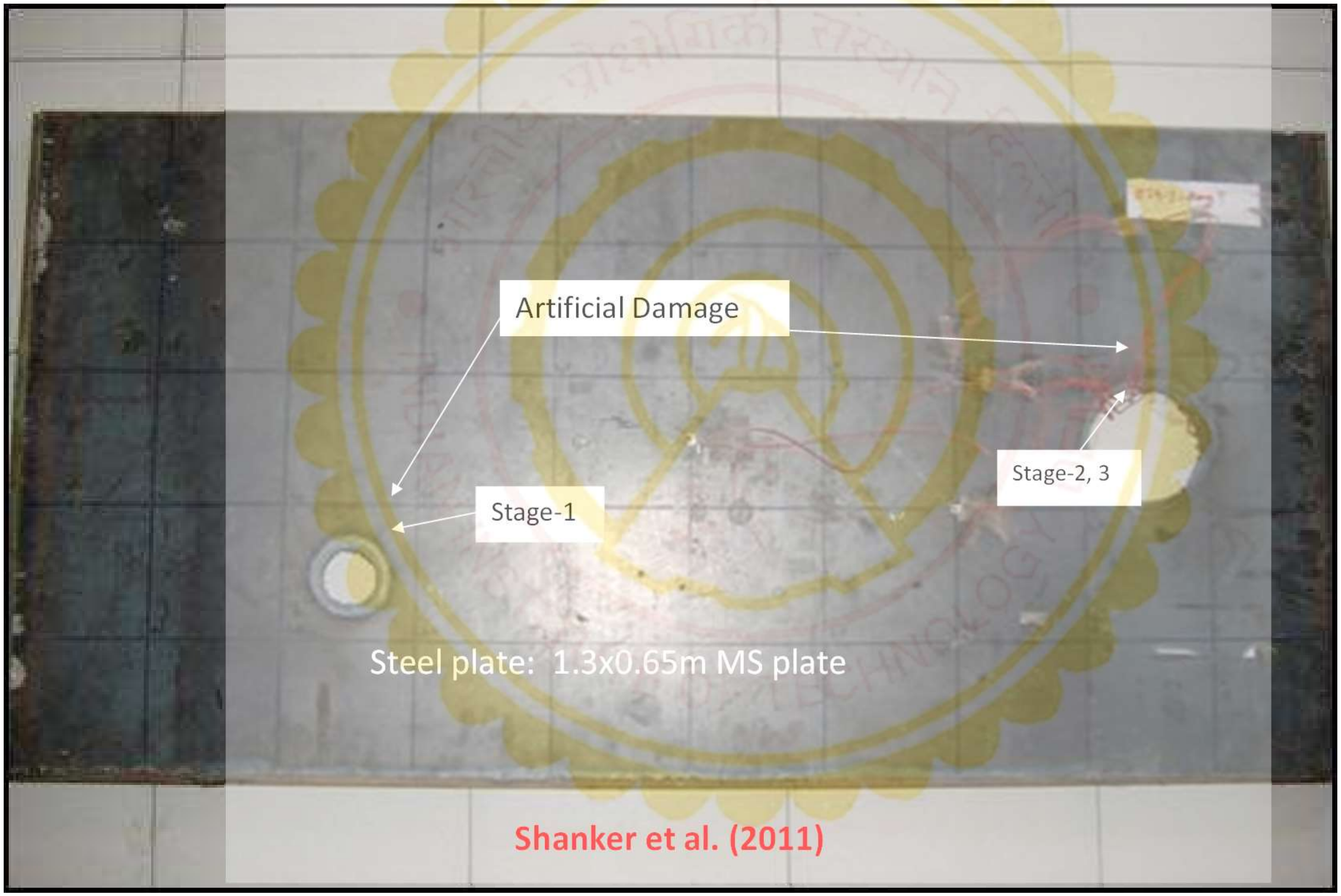


SAMPLE OUTPUTS



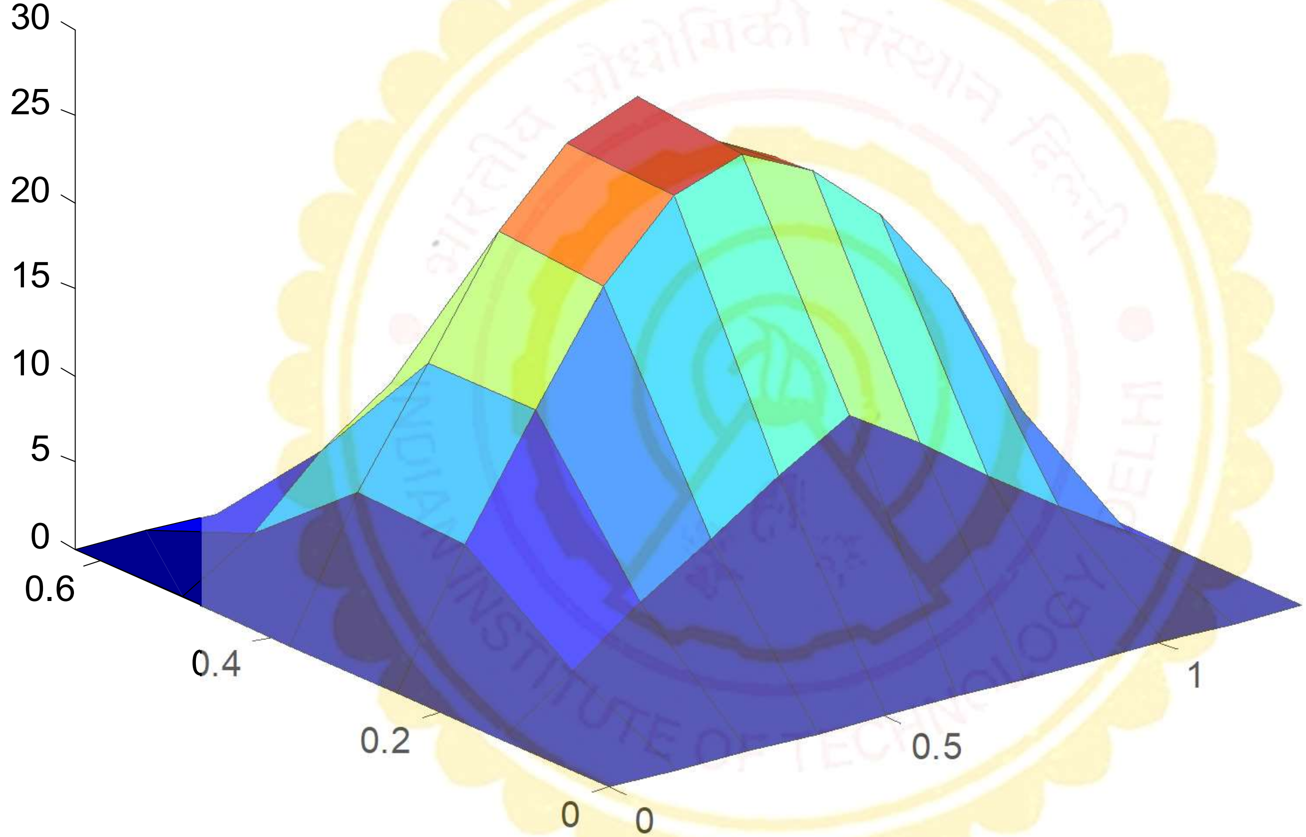


EXTENSION TO 2D STRUCTURES

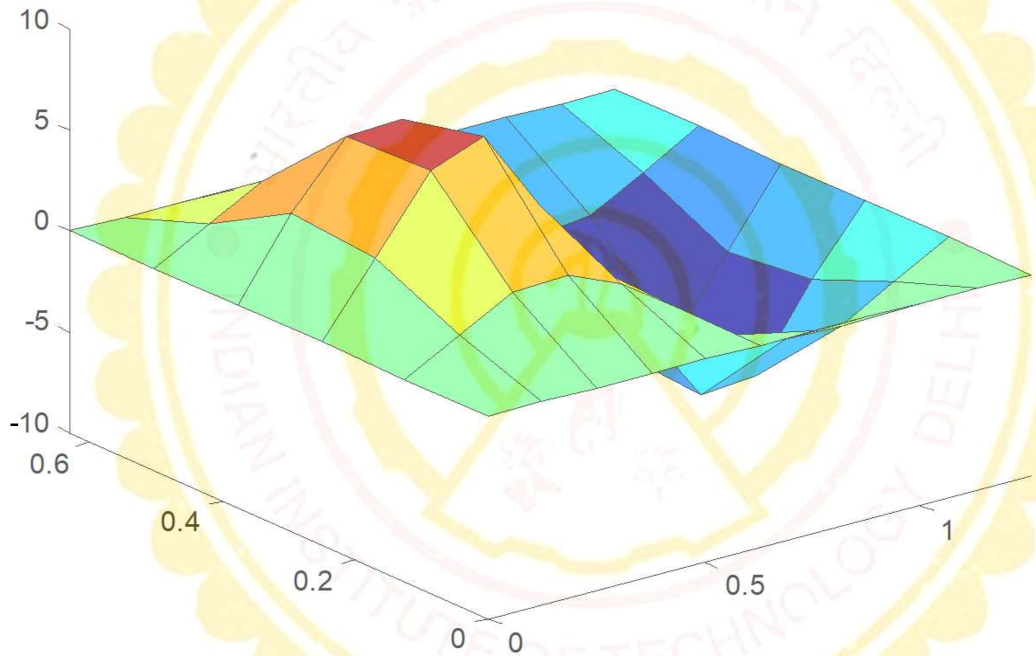


Shanker et al. (2011)

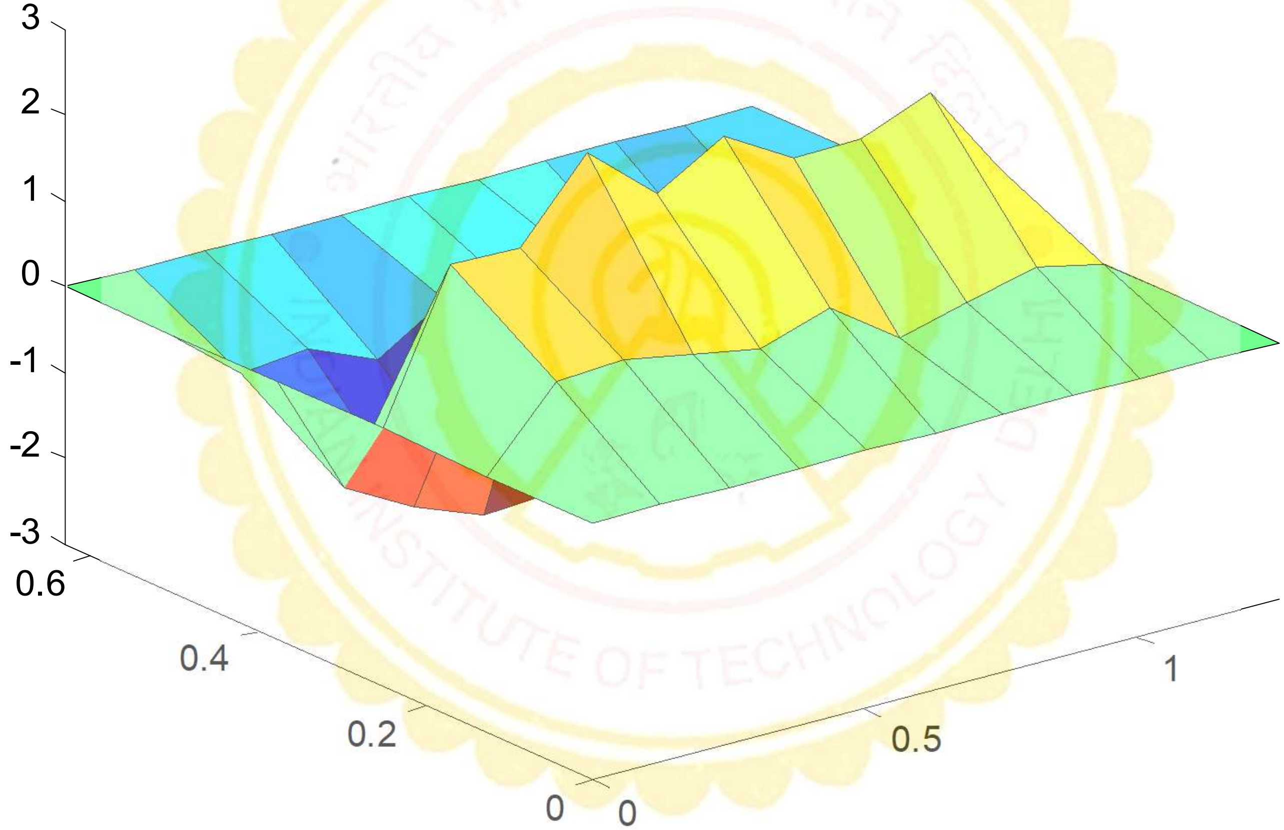
FIRST CURVATURE MODE SHAPE



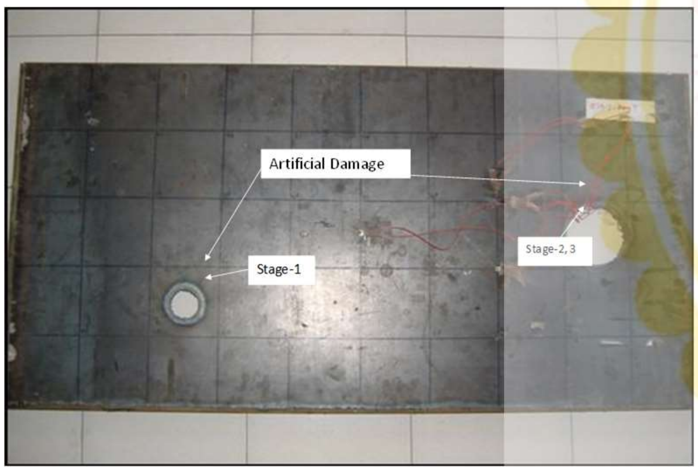
SECOND CURVATURE MODE SHAPE

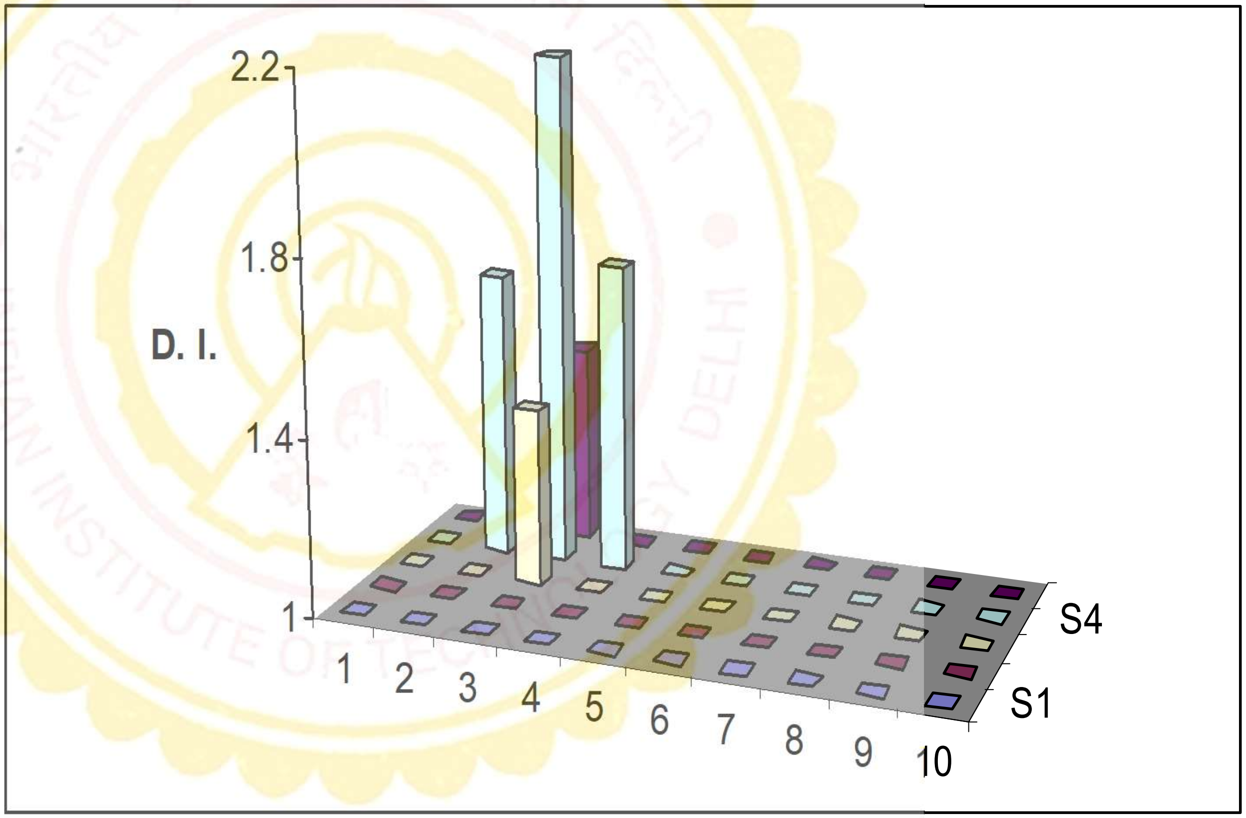


THIRD CURVATURE MODE SHAPE

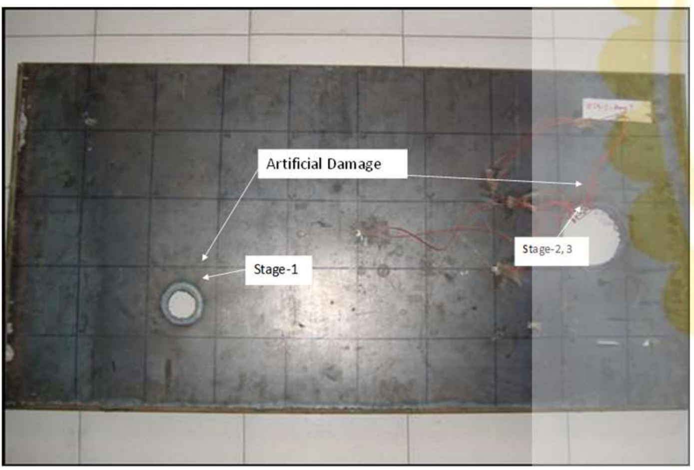


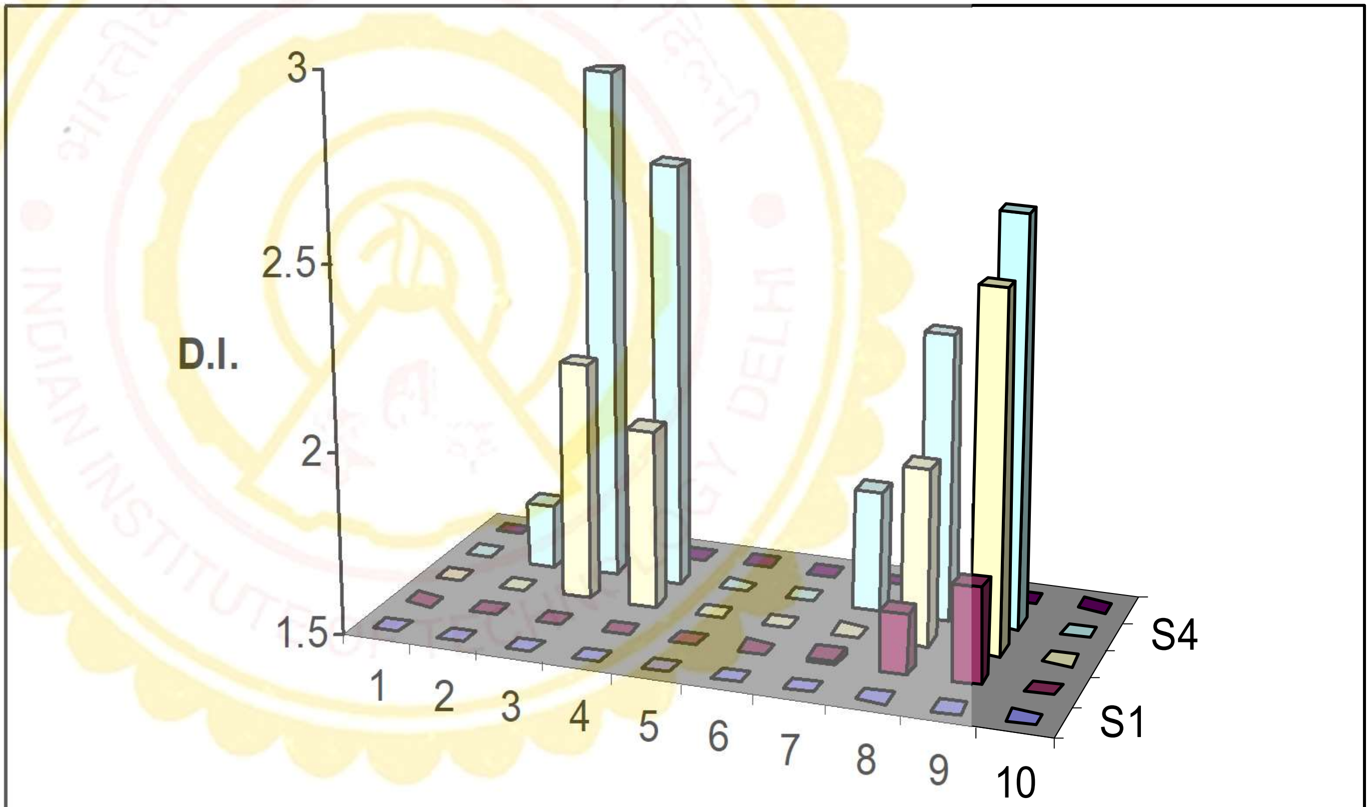
DAMAGE INDEX OF PLATE FOR SINGLE DAMAGE LOCATION





DAMAGE INDEX OF PLATE FOR MULTIPLE LOCATION DAMAGES





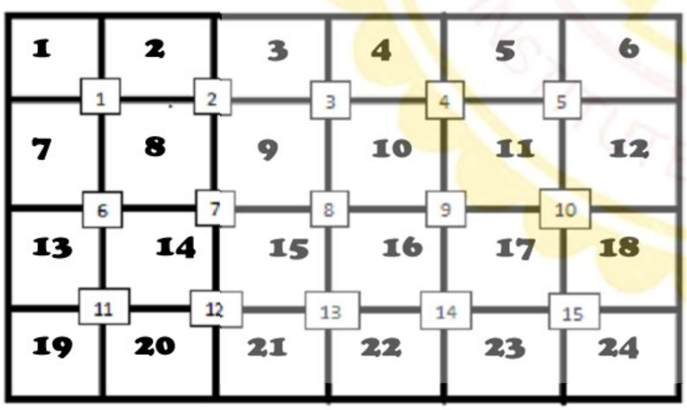
DAMAGE LOCATION ALGORITHM (Talwar, 2011)

ϕ_i = Curvature of 1st mode shape at node i before damage

ϕ_i^* = Curvature of 1st mode shape at node i after damage

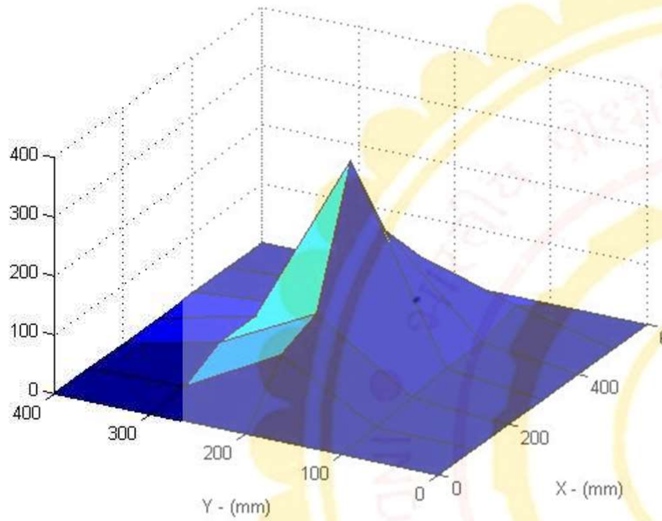
$$D_i = abs(1 - \frac{\phi_i^*}{\phi_i})$$

D_i is the Damage index for i th node

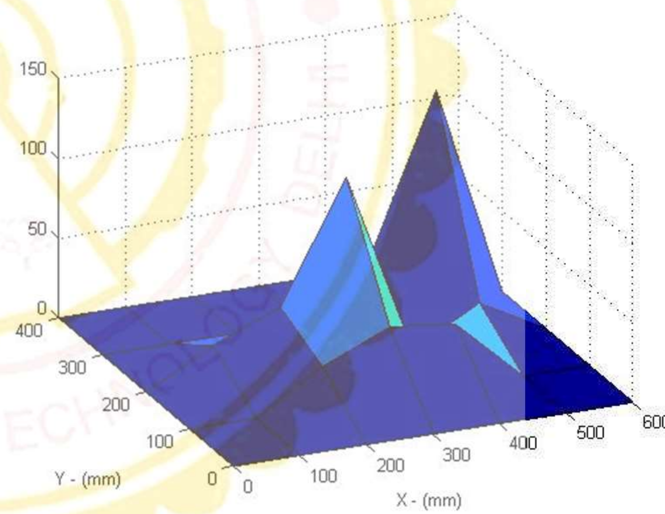


Damage index of an element
=
Sum of D_i of four surrounding nodes

EFFECT OF DAMAGE

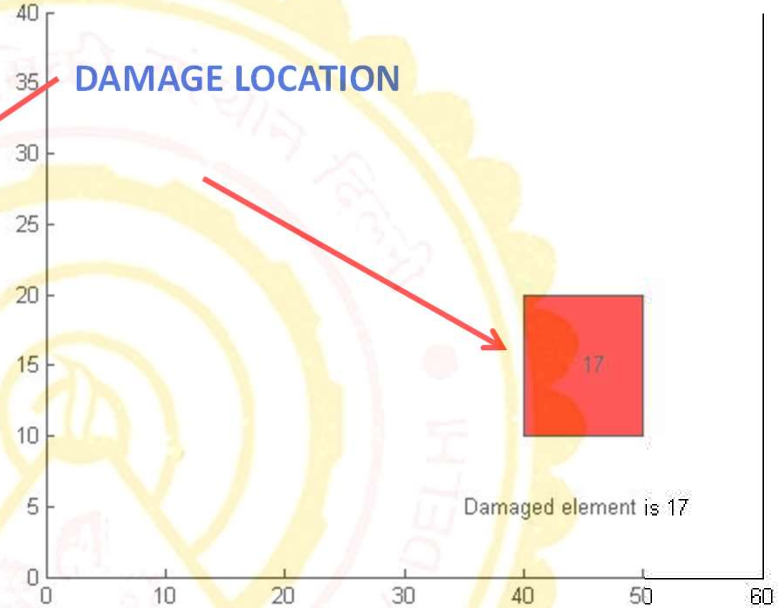


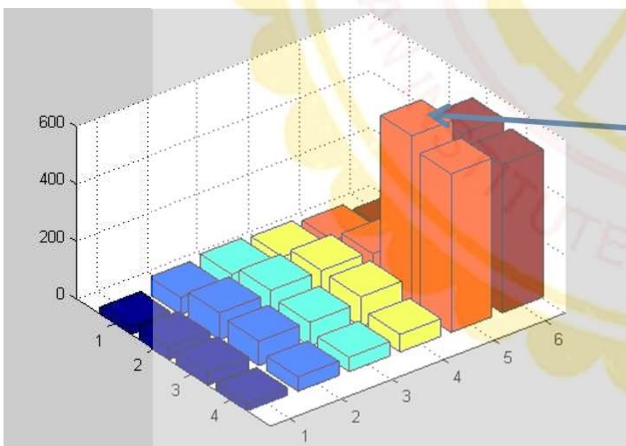
Pre-damage



Post-damage

DAMAGE INDEX FOR ELEMENTS





DAMAGE LOCATION CLEARLY IDENTIFIED BY MAXIMUM **DAMAGE INDEX FOR ELEMENT**

IN CONCLUSION.....

- **Surface bonded PZT patches can be employed to obtain curvature mode shapes directly. Expensive accelerometers substituted by low-cost PZT patches.**
- **The experimental undamaged and damaged mode can be directly utilized, circumventing any numerical modelling of structure.**
- **Damage detection, location and severity covered.**

UNANSWERED.....

- How to identify incipient damage????

THANK YOU

Suggested reading: **Farrar and Jauregui (1998)**
Supplementary reading: **Pandey and Biswas (1994),**
Pandey et al. (1991)
Shanker et al. (2011)