DEPARTMENT OF CIVIL ENGINEERING

IIT DELHI

Lec 9

ELECTRO-MECHANICAL IMPEDANCE (EMI) TECHNIQUE:

1D IMPEDANCE FORMULATIONS

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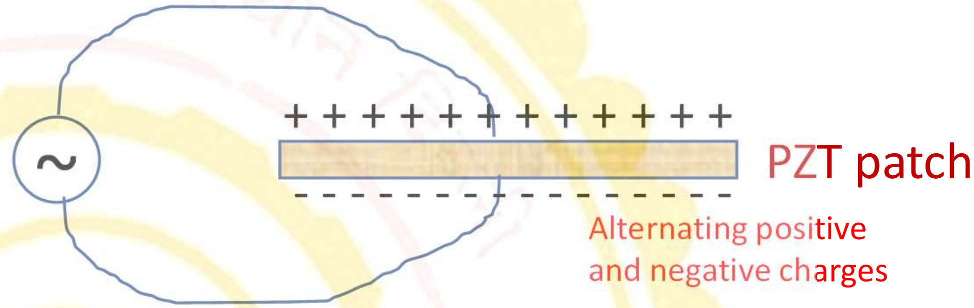
BACKGROUND

$$\overline{E_3} = E_{3o}e^{j\omega t}$$



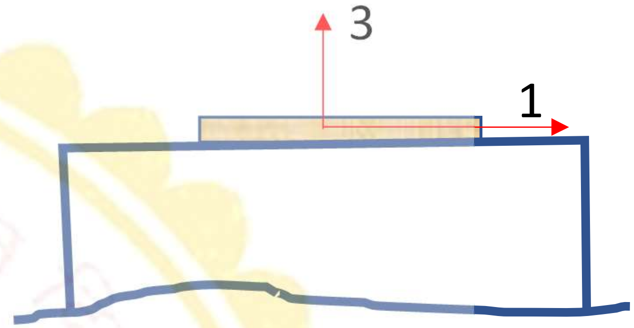
Parallel plate capacitors $\quad Q = CV$

This can be reduced to $\quad D_3 = \overline{\varepsilon_{33}^T} E_3$

$$\overline{\varepsilon_{33}^T} = \varepsilon_{33}^T(1 - \delta j)$$

δ Is the dielectric loss tangent and it signifies "electrical damping". As previously explained, the negative sign indicates that the charges lag behind the applied electric field.

ASSUMPTIONS

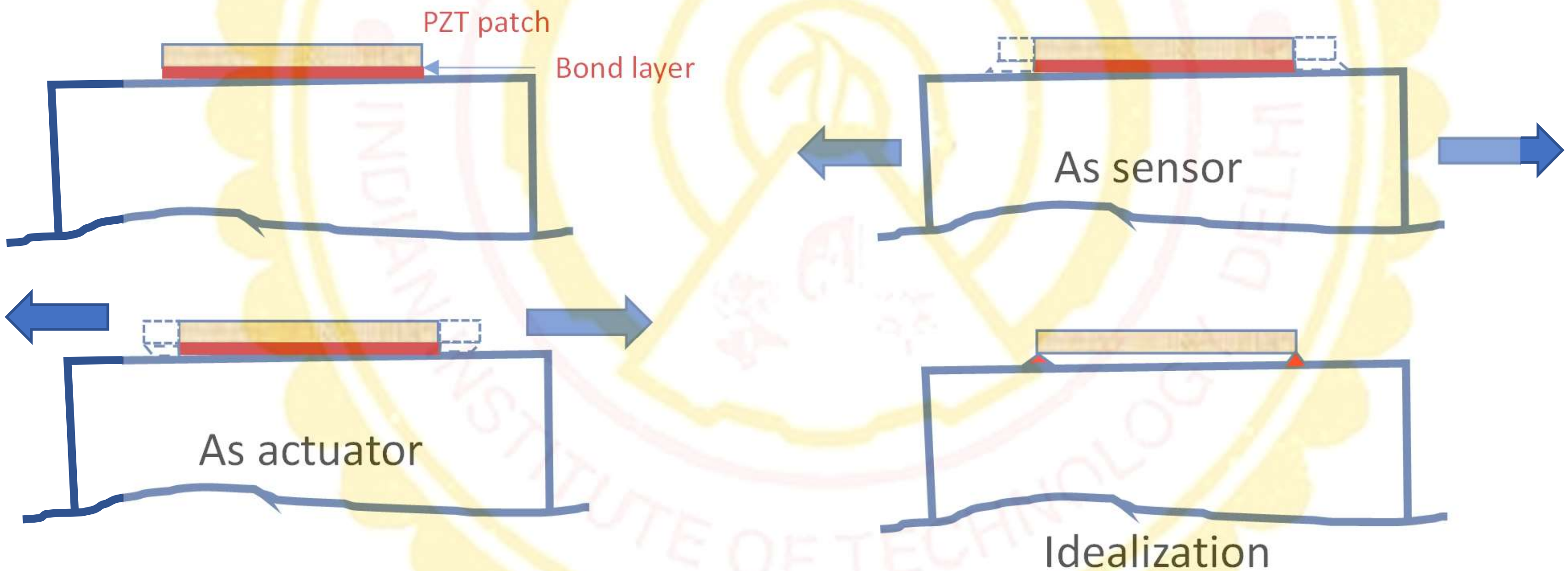


1. We consider only 1D (unidirectional) interaction

   Axis 1: Direction of strain

   Axis 3: Direction of electric field

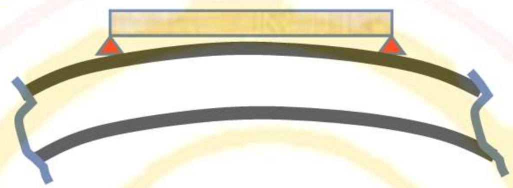
2. Force transfer occurs at the ends of the PZT patch only.



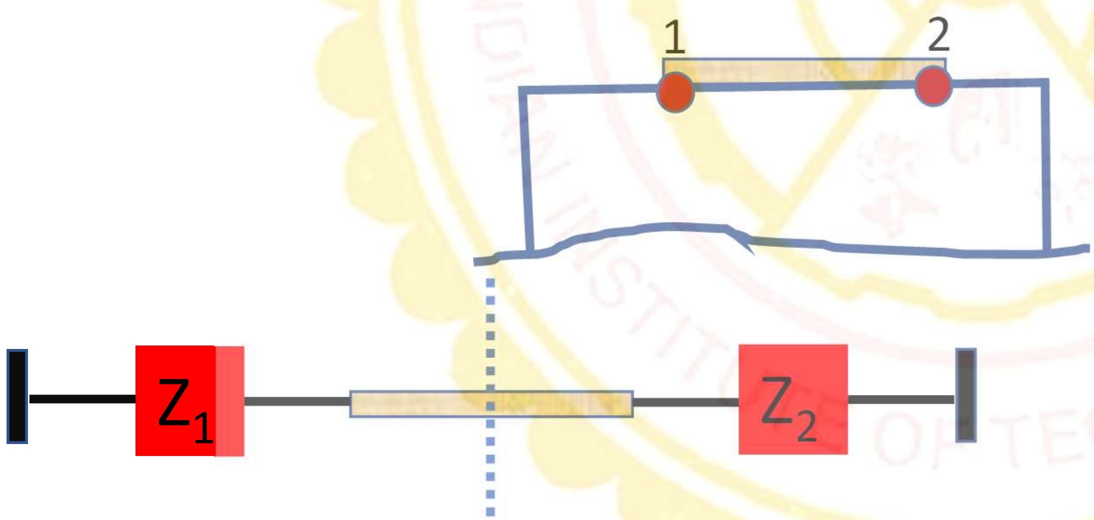
Hence, it is idealized that a PZT patch acts like a bar undergoing pure axial vibrations with pinned ends. This assumption is realistic in view of the fact that force transfer between the PZT patch and the structure takes place through shear mode of the bond layer, shear stress being dominant at the ends.

ASSUMPTIONS

3. Bending of the PZT patch, if any, is ignored even when the host **structure** undergoes bending.



4. PZT patch infinitesimally small as compared to the host structure. **Hence, dr**ive point impedances at points of connection (1 and 2) equal.



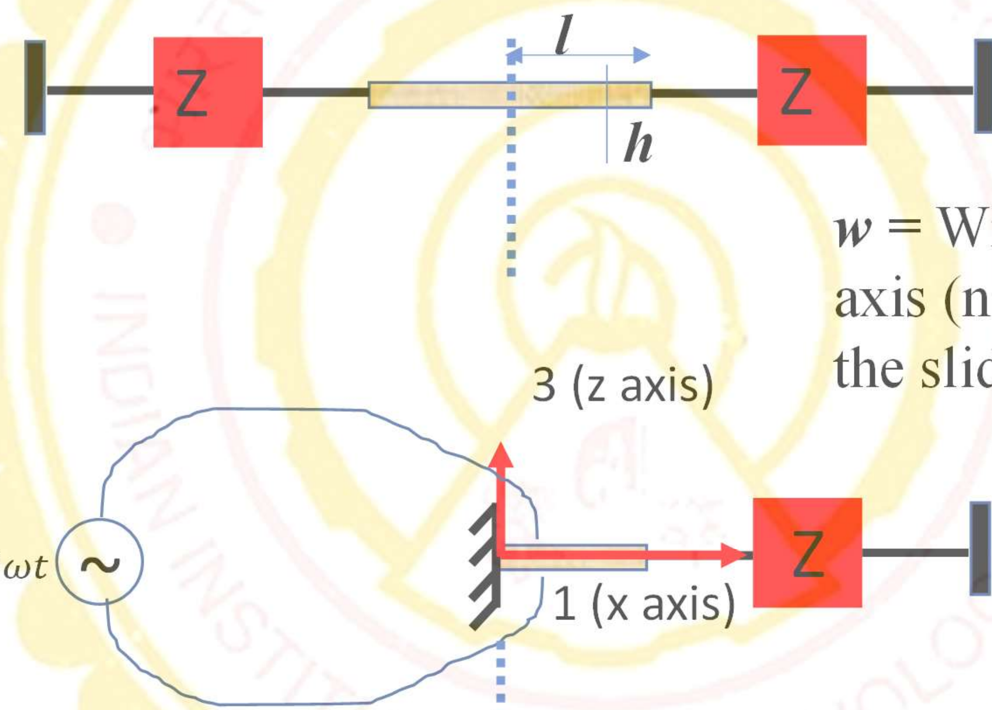
$$Z_1 = Z_2 = Z$$

$Z_1 = Z_2 = Z$ implies that the nodal lines pass through **the** mid point of the PZT pat**ch.**

Nodal line will always pass through the mid point irrespective of the poin**t of attachmen**t of the PZT patch on the structure (whether symmetrical or no**t)**

MODEL

Taking advantage of the symmetry, only one half of the PZT-structure system shall be modelled.

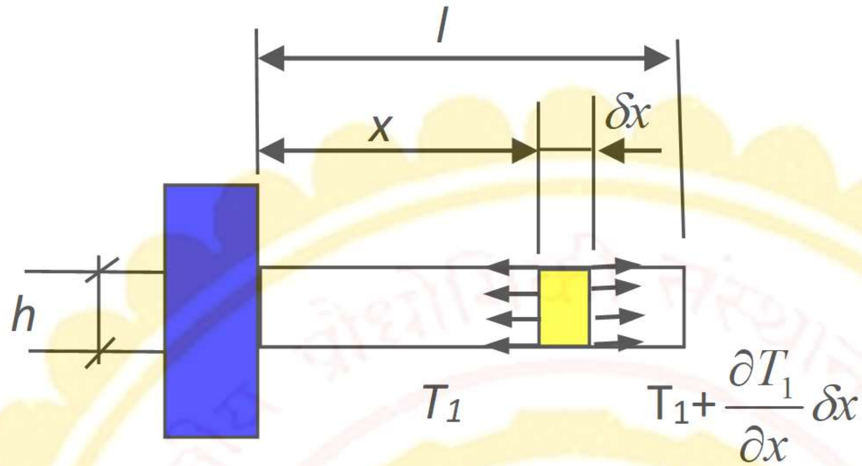


$\overline{E_3} = E_{3o} e^{j\omega t}$

w = Width of patch along y axis (normal to the plane of the slide)

$$\overline{E_3} = \frac{\bar{V}}{h} = \frac{V_o}{h} e^{j\omega t}$$

At x =0,
u (disp) = 0



Let us consider a small element of the PZT patch of length *δx*, situated at a distance *x* from the centre of the patch under dynamic equilibrium. The infinitesimal element has a mass of

$$dm = \rho w h \delta x$$

where ρ is the density of the patch, *w* its width and *h* its thickness.

Let *u*(*x*) be the displacement at location *x* in the patch. Application of D'Alembert's principle on the infinitesimal element yields

$$\left[\left(T_1 + \frac{\partial T_1}{\partial x}\delta x\right) - T_1\right] wh = dm\frac{\partial^2 u}{\partial t^2}$$

Q: Where is damping force in the above equilibrium equation?

$$\frac{\partial T_1}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}$$

Substitute $$T_1 = \overline{Y^E} \frac{\partial u}{\partial x}$$

$$\overline{Y^E} \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \kappa^2 \frac{\partial^2 u}{\partial t^2}$$

$$\kappa^2 = \frac{\rho}{\overline{Y^E}} \omega^2$$

Solution of the above differential equation (variable separable type), by method of separation of variables, yields

$$u = (A \sin \kappa x + B \cos \kappa x) e^{j\omega t}$$ Eq. (1)

where κ, the wave number, is related to the angular frequency of excitation ω by

$$\kappa = \omega \sqrt{\frac{\rho}{\overline{Y^E}}}$$

$$u = (A\sin \kappa x + B\cos \kappa x)e^{j\omega t}$$ Eq. (1)

Boundary condition 1: At x = 0 (nodal point), u = 0

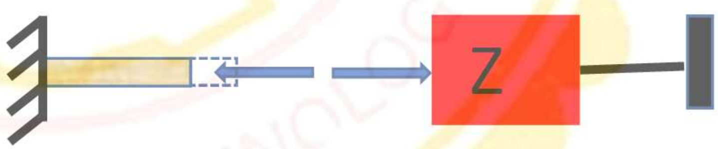
From Eq (1), B = 0

Strain: $S_1 = \frac{\partial u}{\partial x} = A\kappa(\cos \kappa x)e^{j\omega t}$ Eq. (2a)

Velocity: $\dot{u} = \frac{\partial u}{\partial t} = Aj\omega(\sin \kappa\, x)e^{j\omega t}$ Eq. (2b)

Boundary condition 2: $F_{(x=l)} = -Z\dot{u}_{(x=l)}$

The negative sign here simply implies that a positive *u* gives rise to a force in compressive force in the PZT patch.



$$F_{(x=l)} = -Aj\omega Z\sin(\kappa l)e^{j\omega t}$$ Eq. (3)

Consider piezoelectric converse effect

$$S_1 = \frac{T_1}{\overline{Y^E}} + d_{31}\overline{E}_3$$

$$S_1 = \frac{F_1}{\overline{Y^E}wh} + d_{31}\frac{\overline{V}}{h}$$

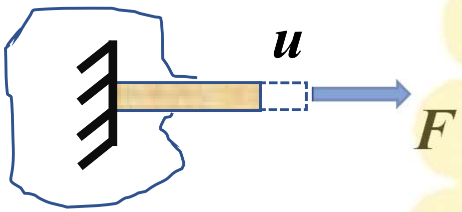
From Eq 2(a) and (3), at x = l

$$A\kappa\cos(\kappa l)e^{j\omega t} = \frac{-Aj\omega Z\sin(\kappa l)e^{j\omega t}}{wh\overline{Y^E}} + \frac{d_{31}}{h}\overline{V} \qquad \text{Eq. (4)}$$

At this juncture, the mechanical impedance of the PZT patch, similar to that of the structure is introduced.

As a general practice, the mechanical impedance of the PZT patch is determined in short circuited condition, so as to eliminate the piezoelectric effect and only invoke pure mechanical response.

The short-circuited mechanical impedance of the patch, Z_a, can be determined as



$$Z_a = \frac{F_{(x=l)}}{\dot{u}_{(x=l)}} = \frac{whT_{1(x=l)}}{\dot{u}_{(x=l)}} = \frac{wh\overline{Y^E}S_{1(x=l)}}{j\omega u_{(x=l)}}$$

No minus sign here since both force and displacement in same direction

Substituting from Eqs (2a) and (2b) and solving:

$$Z_a = \frac{\kappa wh\overline{Y^E}}{(j\omega)\tan(\kappa l)}$$

Substituting in Eq (4) and solving:

$$A = \frac{Z_a V_o d_{31}}{h\kappa\cos(\kappa l)(Z + Z_a)}$$

$$S_1 = \frac{T_1}{Y^E} + d_{31}\overline{E}_3$$

$$S_1 = \frac{T_1}{Y^E} + d_{31}\frac{\overline{V}}{h}$$

Substituting from Eq (2a) and solving:

$$T_1 = \left[\frac{Z_a \cos \kappa x}{\cos(kl\ )(Z + Z_a)} - 1\right]\frac{V_o \overline{Y^E} d_{31} e^{j\omega t}}{h}$$

Eq. (5)

$$D_3 = \overline{\varepsilon_{33}^T} E_3 + d_{31} T_1$$

$$\overline{E_3} = \frac{\bar{V}}{h} = \frac{V_o}{h} e^{j\omega t}$$

Substituting from Eq (5) and simplifying:

$$D_3 = \left[\frac{Z_a d_{31}^2 \overline{Y^E} \cos \kappa x}{(Z + Z_a) \cos(\kappa l)} + (\overline{\varepsilon_{33}^T} - d_{31}^2 \overline{Y^E}) \right] \frac{V_o}{h} e^{j\omega t}$$

The electric current can be obtained by integrating the rate of change of the electric charge density over the surface of the PZT patch, that is,

$$\bar{I} = \int_{y=0}^{y=w} \int_{x=-l}^{x=l} \frac{dD_3}{dt} dx dy = \int_{y=0}^{y=w} \int_{x=-l}^{x=l} D_3 j\omega \, dx dy$$

$$\bar{I} = \frac{2V_o e^{j\omega t} wl\omega j}{h}\left[\frac{Z_a d_{31}^2 \overline{Y^E}}{(Z+Z_a)}\frac{\tan\kappa\, l}{\kappa l} + (\overline{\varepsilon_{33}^T} - d_{31}^2 \overline{Y^E})\right]$$

$$\bar{V} = V_o \;\; e^{j\omega t}$$

$$\overline{Y} = 2\omega j\frac{wl}{h}\left[(\overline{\varepsilon_{33}^T} - d_{31}^2\overline{Y^E}) + \left(\frac{Z_a}{Z+Z_a}\right)d_{31}^2\overline{Y^E}\left(\frac{\tan\kappa l}{\kappa l}\right)\right]$$

Capacitive part

This equation is the same as that derived by Liang et al. (1994), except with a multiplication factor of two, which comes into picture since the limits of integration are from -*l* to +*l*, contrary to Liang et al. (1994), who had considered only one half (the right one) of the patch.

Homework: Separate the equation into real and imaginary parts G + Bj

Question: Which part shall be larger? Which part shall be more capacitive? WHY

QUASISTATIC SENSOR APPROXIMATION

It may be noted that no assumption about the operating frequency range is made in the derivation of the above equation. For the particular case of

$$\omega << \omega_{res}$$

$$\left[\frac{\tan \kappa l}{\kappa l}\right] \approx 1$$

$$\overline{Y} = 2\omega j \frac{wl}{h}\left[(\overline{\varepsilon_{33}^T} - \left(\frac{Z}{Z + Z_a}\right) d_{31}^2 \overline{Y^E}\right]$$

RESONANCE FREQEUNCY OF PZT PATCH

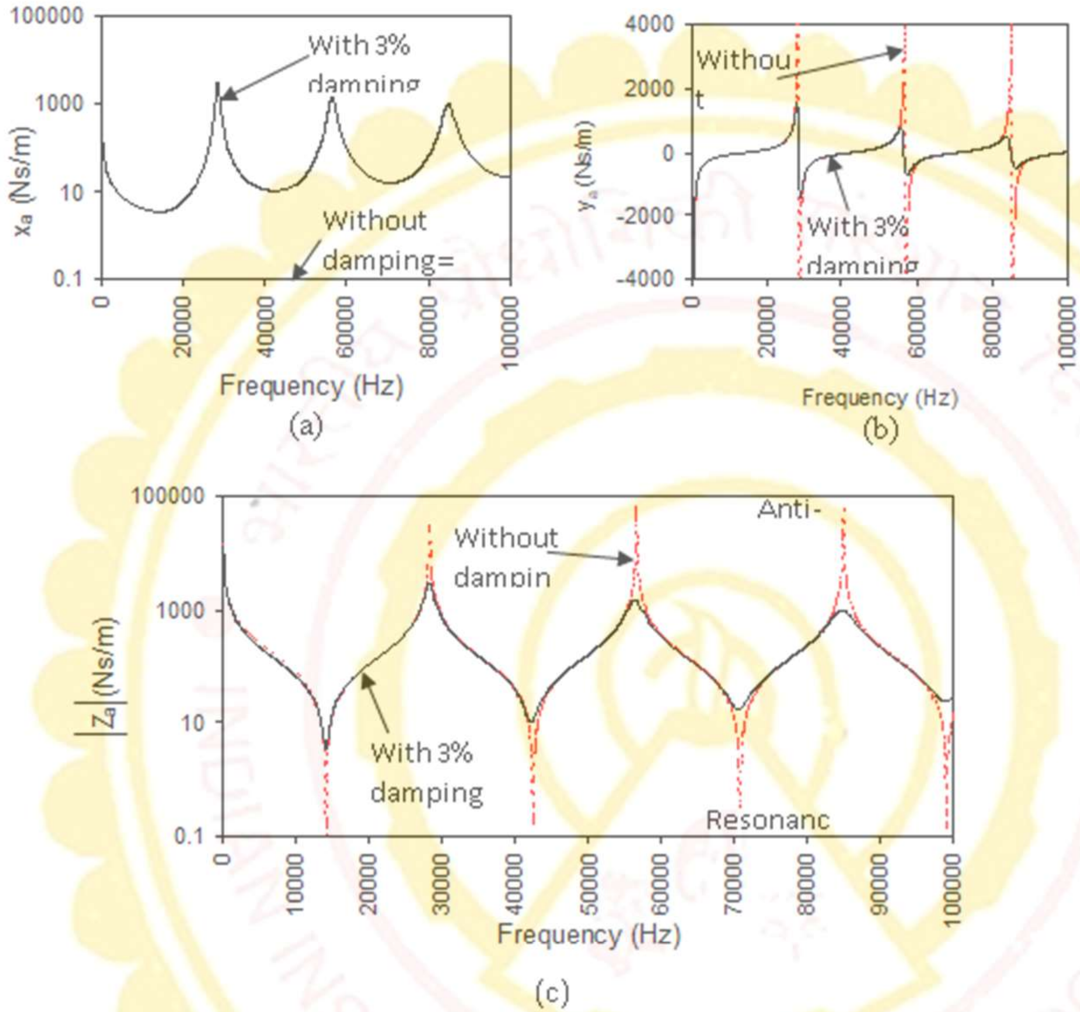
$$Z_a = \frac{\kappa w h \overline{Y^E}}{(j\omega)\tan(\kappa l)}$$

Resonance frequency is that frequency at which this is minimum

This would be minimum at $$\kappa l = (2n-1)\frac{\pi}{2}$$

Homework 1: Determine the resonance frequency of PZT patch of grade PIC 151 for sizes 10x10x0.3 mm and 5x5x0.3 mm

Homework 2: Draw plot of |Z| using MATLAB for above two sizes in frequency range 0-1000 kHz. How can you get resonance frequency of patch from this plot?



Variation of impedance components with frequency for a 1x10x0.3 mm PZT patch (a) Real part (b) Imaginary part (c) Absolute value

THANK

YOU