CVL 756

PLASTIC ANALYSIS

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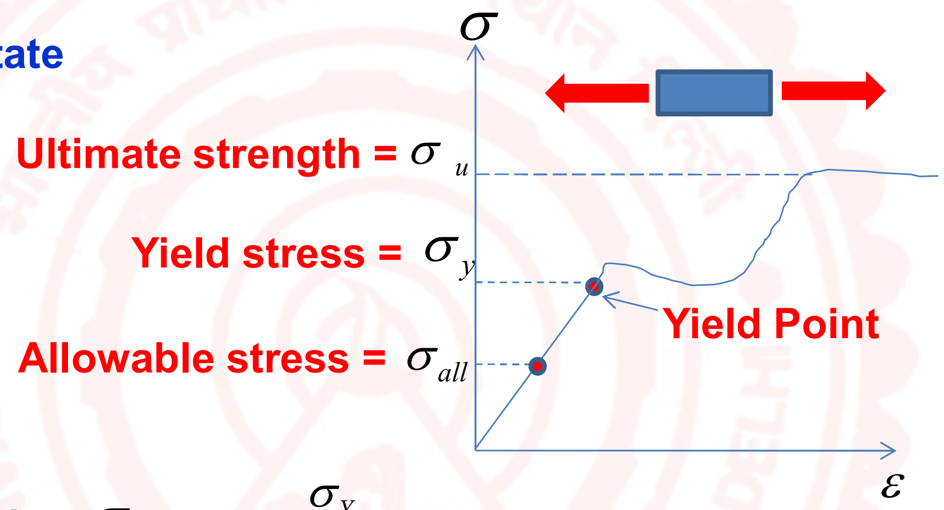
ENGINEERING DESIGN APPROACH

Design Approach
- Working stress
- Limit State



Working stress method: $$\sigma_{all} = \frac{\sigma_Y}{FOS}$$

-Stress is restricted to σ_{all} under working loads. No load factor.

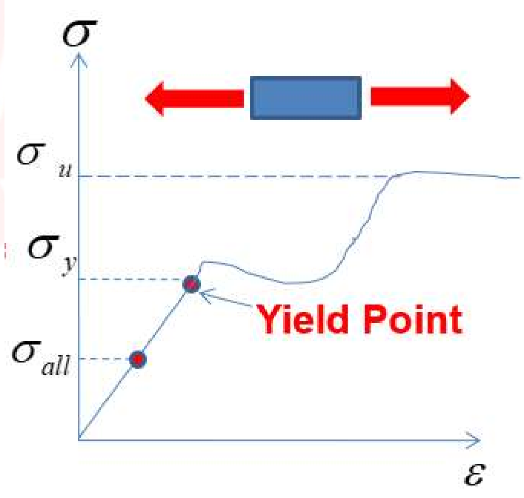
-Elastic analysis of structure is considered adequate

LIMIT STATE METHOD

1. Design the structure for limit state of <u>collapse</u>
2. Check for limit state of <u>serviceability</u>.
3. <u>Partial factors</u> of safety for both loads as well as stresses

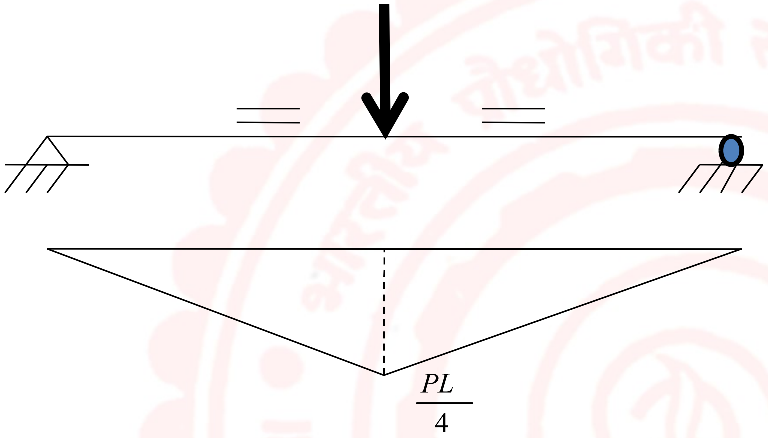
Conventional analysis approach for limit state design

1. Carry out linear elastic analysis (unfactored load values)
2. For limit state values of axial force, shear force and bending moment we <u>simply multiply by load factor</u>.
3. Section design, we follow <u>rigorous non-linear computations</u>.

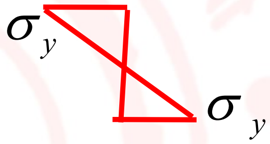


This makes the process somewhat contradictory in nature.. HOW?

OTHER FLAWS WITH CONVENTIONAL ANALYSIS APPROACH

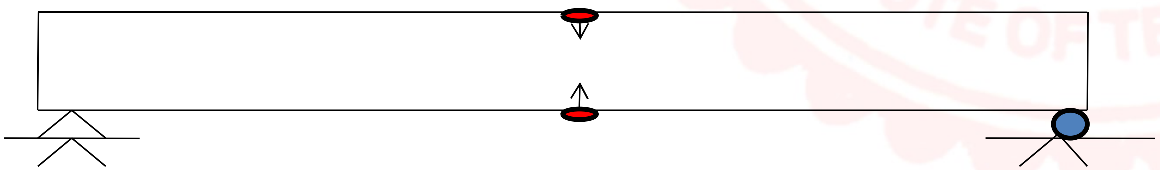


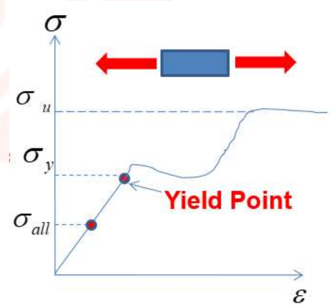
Only 2 points reach yield stress
(symmetrical section, else only one)



Corresponding moment is called as Yield Moment M_y

But the structure has not yet reached collapse state, can still carry further load





WHY PLASTIC ANALYSIS?

The structure deemed failed at Yield Moment still has capacity to sustain higher loads and bending moment.

Actual failure values P_{ult}, M_{ult} are much higher than M_y P_y

Correspond to yielding at two points only.

By plastic analysis, we can get

Actual Load Factor $$\lambda = \frac{P_{ult}}{P_y}$$

Plastic Analysis basically extends the Limit state approach (so far restricted to design aspect only) to load analysis

In Plastic Analysis, we take into account the actual behaviour of structure beyond the yield point.

Actual behavior of structure @ collapse (rather than yield point) is realistically taken into account.

PLASTIC ANALYSIS: ASSUMPTIONS

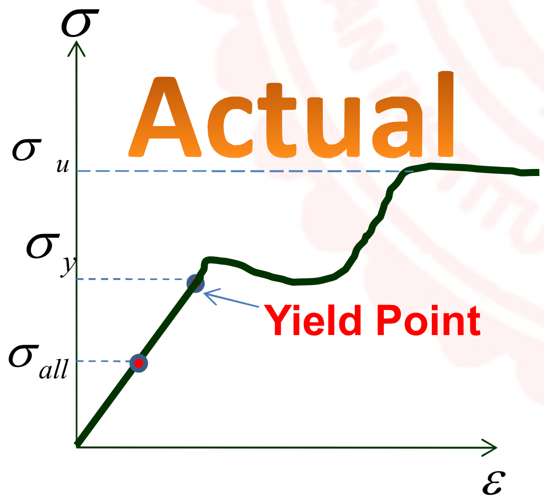
1) Hookes's law of elaticity holds until yield point

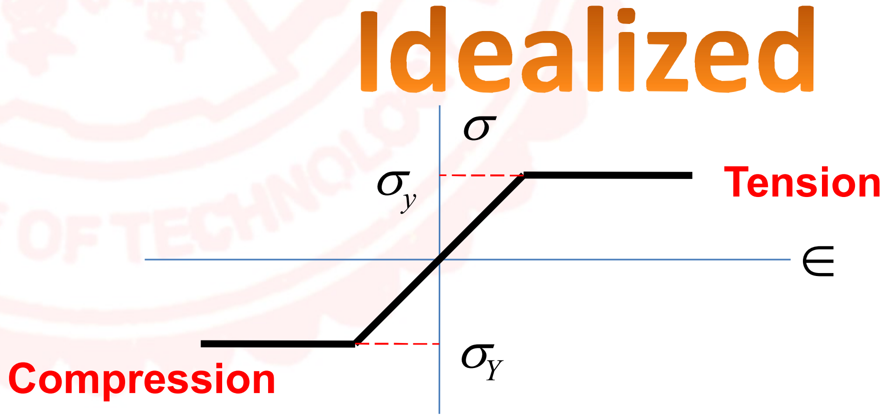
   Stress – strain curve is linear till yield point.

2) Yield stress and Young's modulus have same values for tension and compression.

3) Stress-strain curve is same in tension and compression.

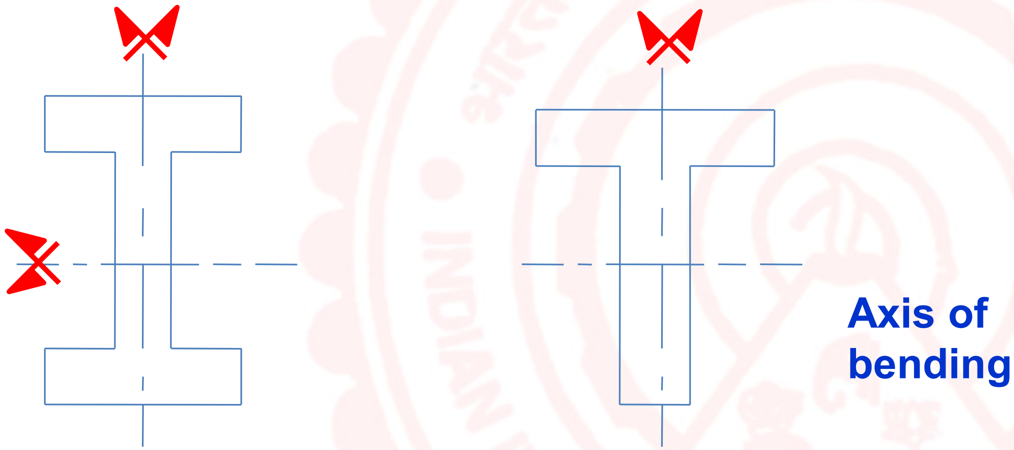
4) Strain hardening is ignored





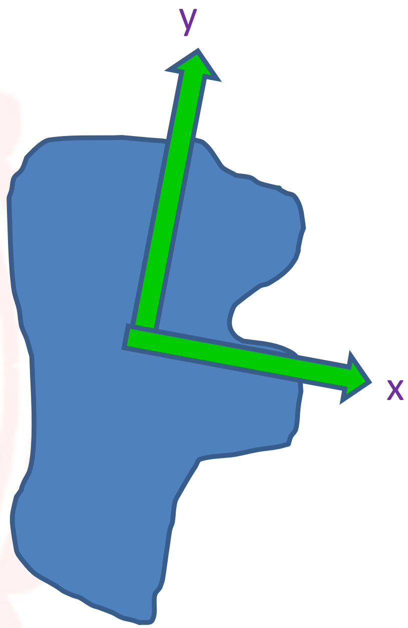
ASSUMPTIONS (Contd.)…..

5) Section has at least one line of symmetry.



BI-SYMMETRIC

MONO-SYMMERTIC

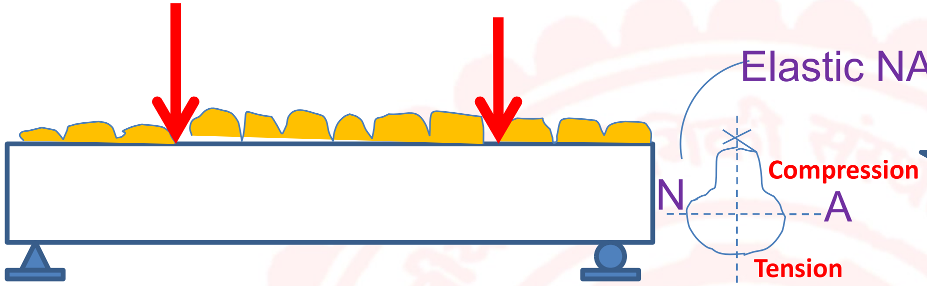


Not considered

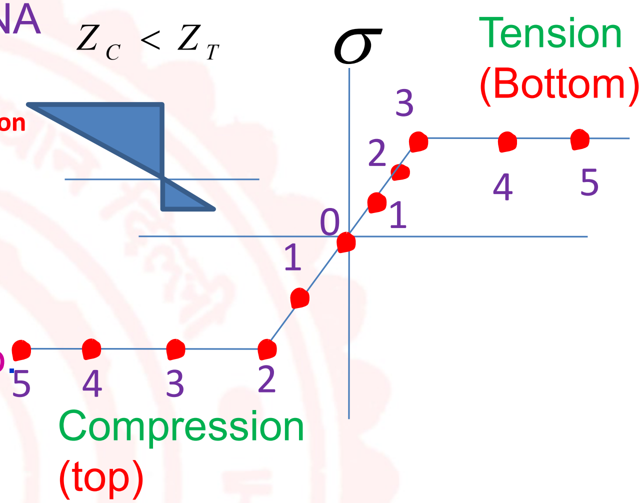
Why…..???

So that the plane of loading and the plane of bending coincide with the plane of symmetry….else formulations won't apply

STEP-BY-STEP ANALYIS TILL COLLAPSE



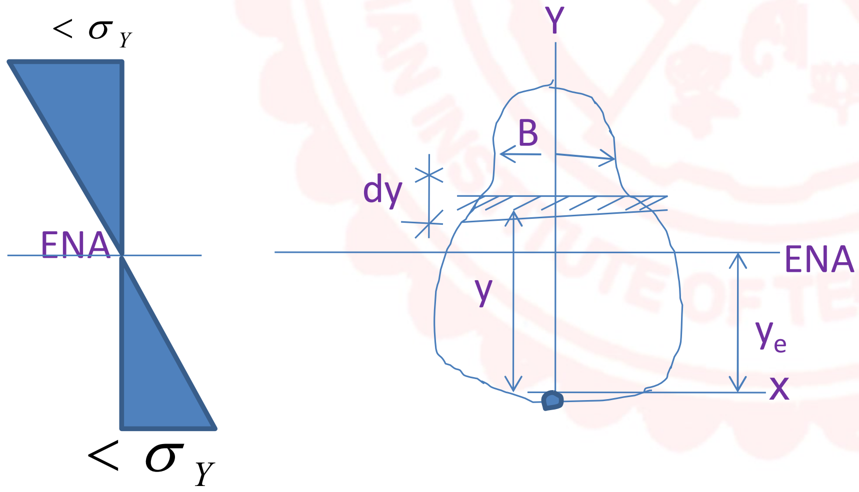
$Z_C < Z_T$



-Let plane section remain plane.

-Let loads be gradually increased starting from zero.

At point 1: Elastic behaviour, both tension and compression zone.



C-T = No net axial force.

$$\int \sigma b dy = o$$

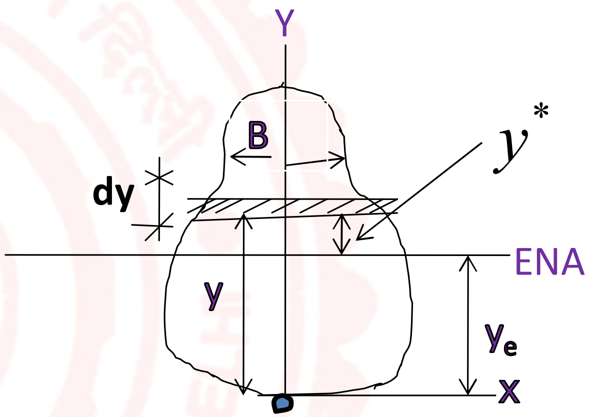
$$\sigma = \frac{M}{I} y^{*} = \frac{M}{I}(y - y_e)$$

$$\int \sigma\, b dy = 0$$

$$\int_A \frac{M}{I}(y - y_e)\, b dy = 0$$

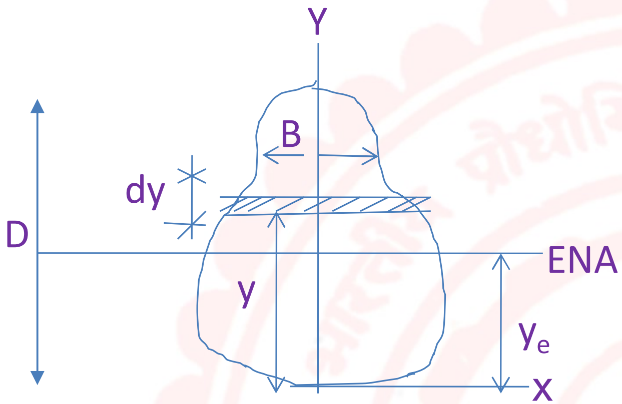
$$\int y \; bdy - y_e \int bdy = 0$$

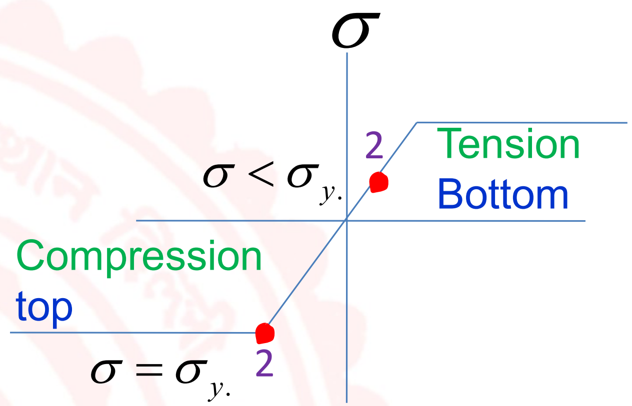
$$y_e = \frac{1}{A} \int_A y \; dA$$



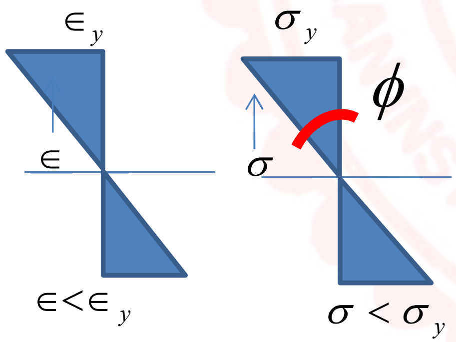
A= Total are of section.

INCREASE LOADS TO REACH POINT<2>





Yield point is reached for compression.



Yield condition arrived !
(**one point only, @ compression**)

Derive ??

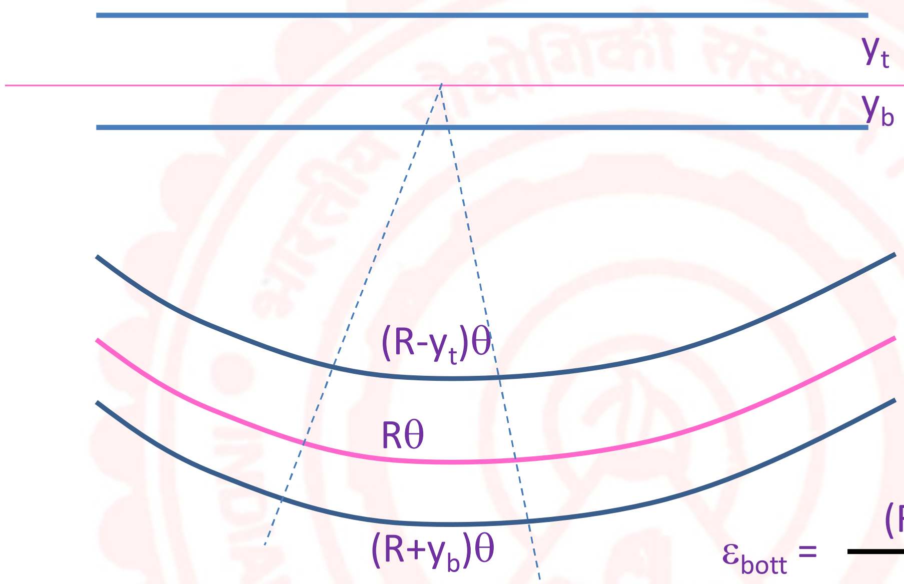
Comp. Tension

$$\text{Curvature} = \tan\phi = \frac{\varepsilon_{top} + \varepsilon_{bott}}{D}$$ **Absolute values**

Depth of section

$$\phi = \left(\frac{\in_b - \in_t}{D} \right)$$ **With sign convention**

+ive value implies concave upwards



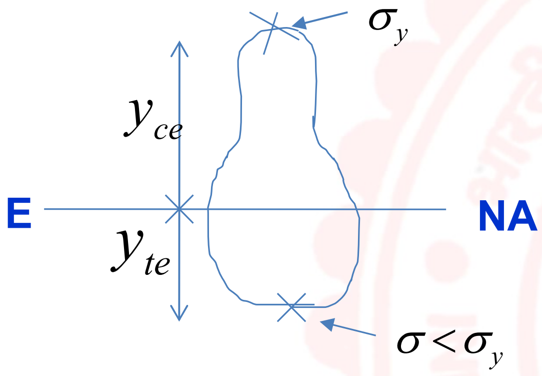
$$\varepsilon_{top} = \frac{R\,\theta-(R-y_t)\theta}{R\,\theta}$$

$$\varepsilon_{bott} = \frac{(R+y_b)\theta - R\,\theta}{R\,\theta}$$

$$\phi = \frac{\varepsilon_{top} + \varepsilon_{bott}}{D}$$

POINT<2> contd…….

Hence moment acting an the section = YIELD MOMENT = M_y



$$\sigma = \frac{My}{I}$$

Yield moment = $M_y = \dfrac{\sigma_y I}{y_{ce}} = \sigma_y Z_e$

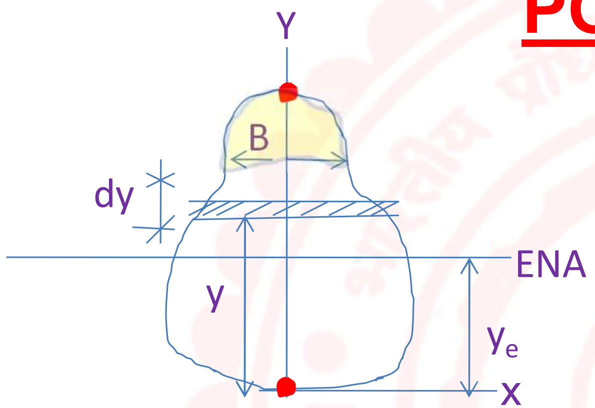
Elastic section modulus

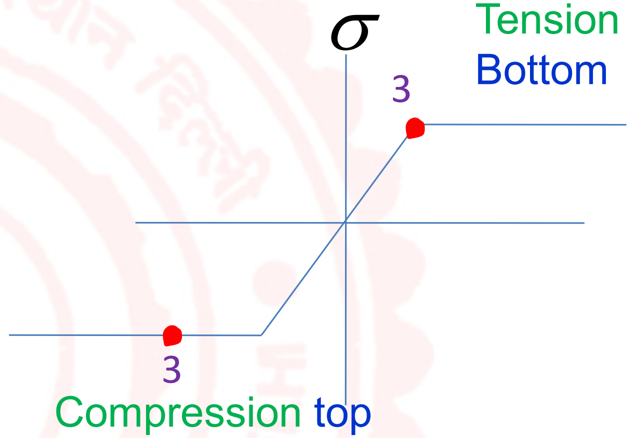
$$Z_e = \frac{I}{y_{e,\max}}$$

Larger of the two extreme fibre distances from ENA

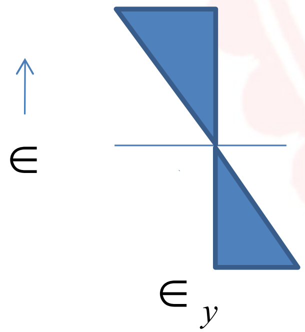
Distance from ENA, so here equal to y_{ce}

INCREASE LOAD FURTHER YIELD POINT ARRIVES AT BOTTOM (TENSION) POINT <3>

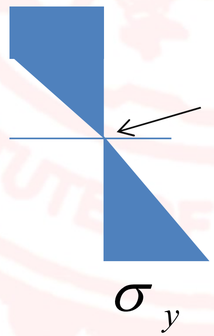




$\varepsilon > \varepsilon_y$



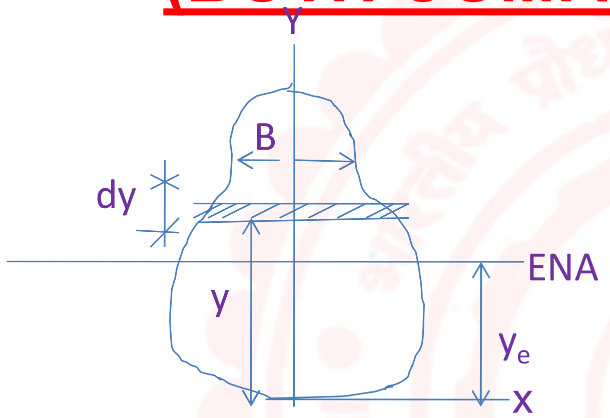
$\sigma = \sigma_y$

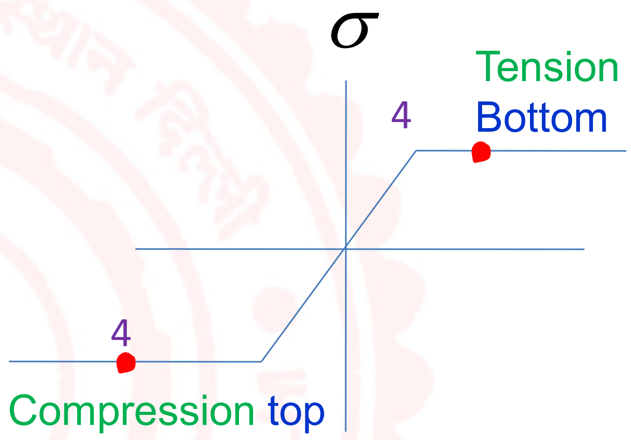


No Longer elastic neutral axis (ENA)
Nor fully plastic neutral axis (PNA)

Stress no longer proportional to y

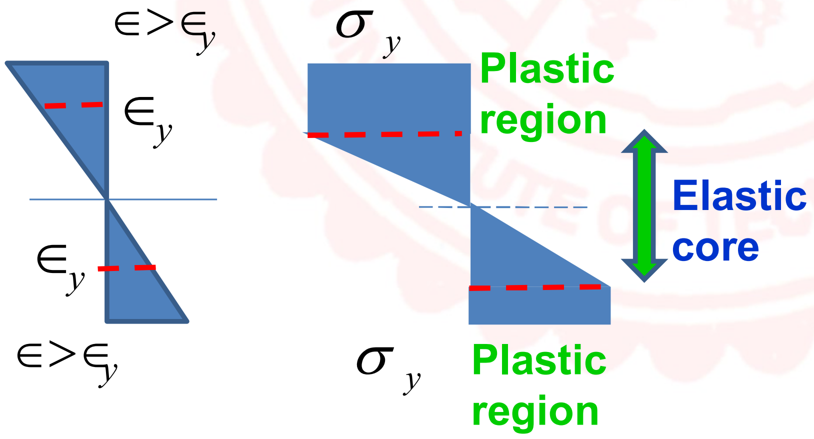
INCREASE LOAD FURTHER SO AS TO REACH BEYOND YIELD POINT (BOTH COMP. AND TENSION) <4>





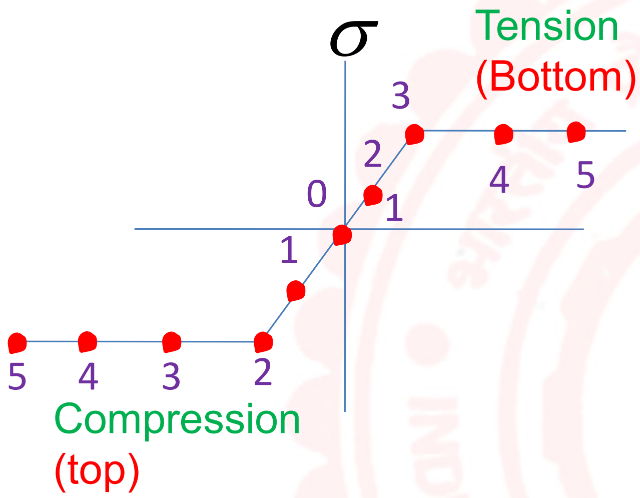
$$\phi = \frac{\varepsilon_{top} + \varepsilon_{bott}}{D}$$

$$= \frac{2\varepsilon_y}{D_{core}}$$

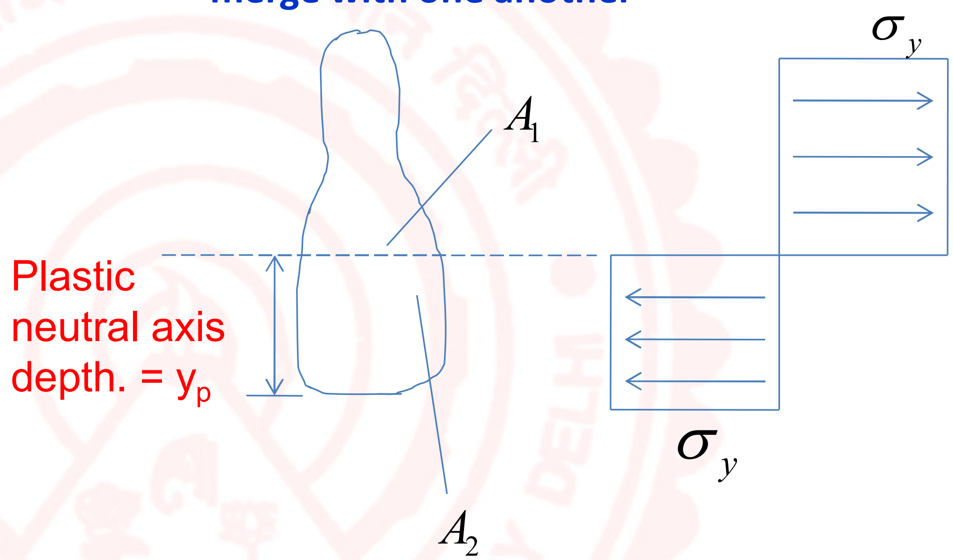


Provides a measure of curvature

LOAD THE STRUCTURE SUCH THAT ELASTIC CORE VANISHES POINT <5>



Top and bottom plastic zones merge with one another



‘NA’ defined by **C = T**

New neutral axis is called as Plastic neutral axis (PNA)

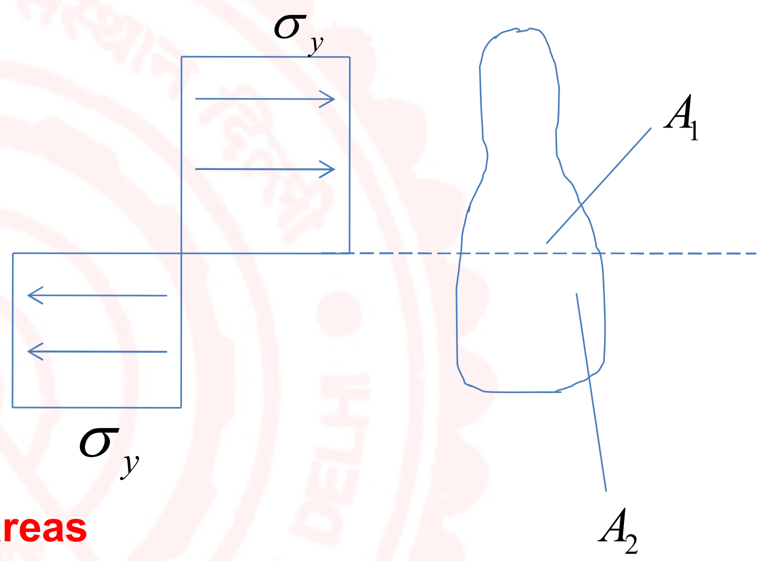
FULLY PLASTIC SECTION.

Curvature $\longrightarrow \infty$

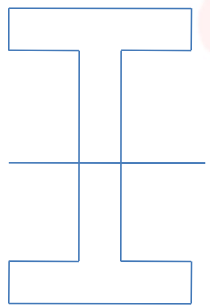
$$C = T$$

$$\sigma_y A_1 = \sigma_y A_2$$

$$A_1 = A_2$$



PNA bifurcates the section into 2 equal areas



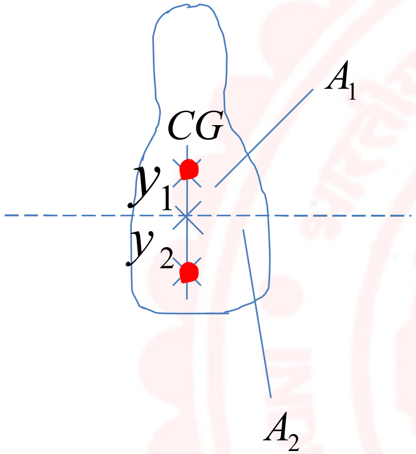
ENA = PNA for Doubly symmetric sections

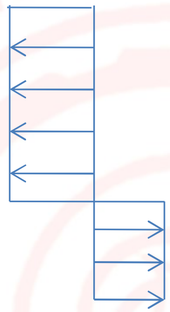
ENA ≠ PNA for mono symmetric sections
(symmetric about y axis only)

PLASTIC MOMENT CAPACITY AND SECTION MODULUS

$A_1 = A_2$

Can be obtained by taking the moment of C & T about the PNA





$$M_P = \sigma_y A_1 y_1 + \sigma_y A_2 y_2$$

$$A_1 = A_2 = \frac{A}{2}$$

$$M_P = \sigma_y \frac{A}{2}(y_1 + y_2)$$

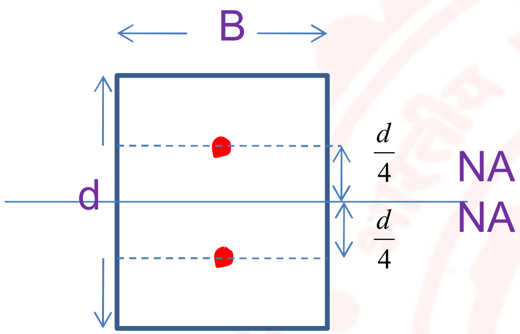
Distance between the CGs of two areas into which PNA divides the cross section.

$$M_P = \sigma_y Z_P$$

$Z_P =$ Plastic section modulus $= \dfrac{A}{2}(y_1 + y_2)$

Shape factor = $\dfrac{M_p}{M_y} = f = \dfrac{Z_p}{Z_e}$

EXAMPLE (1)



$$Z_e = \frac{1}{6} bd^2$$

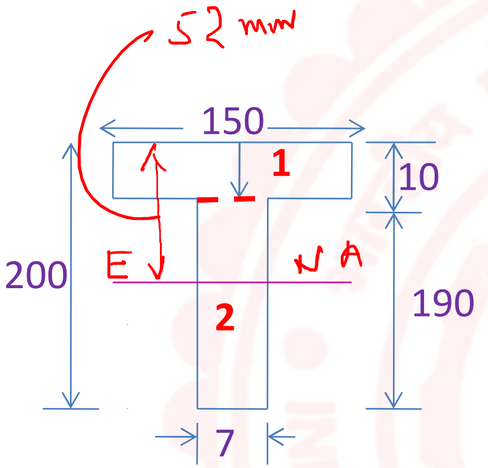
$$Z_p = \frac{A}{2}(y_1 + y_2)$$

$$= \frac{bd^2}{4}$$

$$\boxed{f = \frac{Z_p}{Z_e} = 1.5}$$

Plastic collapse moment: 50% higher than yield moment.

EXAMPLE (2)



ALL DIMENDIONS IN MM

Yield stress= 250MPa

Z_e ? Z_p ?

f ?

$10 + \frac{190}{2}$

$= 105$

$$Y_e = \frac{1}{A}\int ydA$$

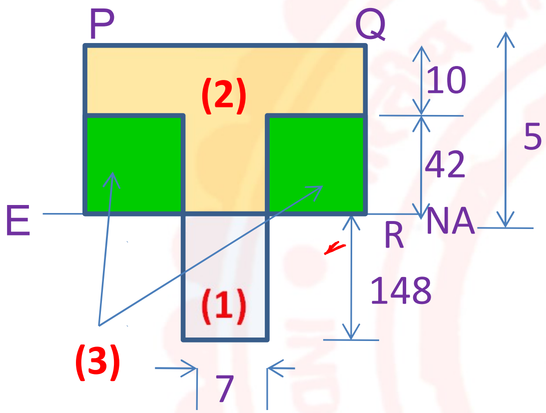
5 150×10 190×7

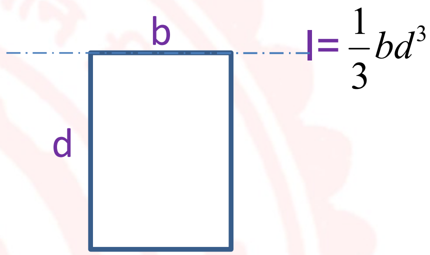
$$= \frac{Y_1 A_1 + Y_2 A_2}{A_1 + A_2}$$

$$= \frac{5\times(150\times 10) + (10+\frac{190}{2})(190\times 7)}{150\times 10 + 190\times 7}$$

$$= 52 \quad mm$$

EXAMPLE (2) contd…





$$I = \underbrace{\frac{1}{3}(7)(148)^3}_{(1)} \underbrace{+\frac{1}{3}(150)(52)^3}_{(2)} \underbrace{-\frac{1}{3}(150-7)(42)^3}_{(3)}$$

$$= 11.06 \times 10^6 mm^4$$

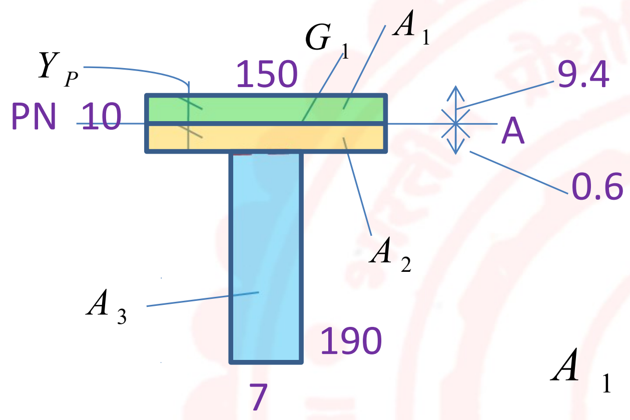
$$Z_e = \frac{I}{y_{max}} = \frac{11.06 \times 10^6}{148} = 74750 mm^3$$

Tension

$$M_y = \sigma_y Z_e = 18.66 \times 10^6 Nmm = 18.66 kNm$$

250

PLASTIC MOMENT



$$A_{web} = 190 \times 7 = 1330 \; mm^2$$

$$A_f = 1500 \; mm^2$$

$\therefore$ NA must lie in flange.

$$A_1 = A_2 + A_3$$

$$150 \; Y_p = 150 \; (10 - y_p) + 190 \times 7$$

$$Y_p = 9.4 \, mm \, .$$

Z_p = First moment of A_1, A_2, A_3 About PNA

$$= \underbrace{(150 \times 9.4)(\tfrac{9.4}{2})}_{A_1} + \underbrace{(150 \times 0.6) \times (\tfrac{0.6}{2})}_{A_2} + \underbrace{(190 \times 7)(\tfrac{190}{2} + 0.6)}_{A_3}$$

$$= 133801 \quad mm^3$$

M_p = Plastic moment capacity

$$= \sigma_y Z_p$$

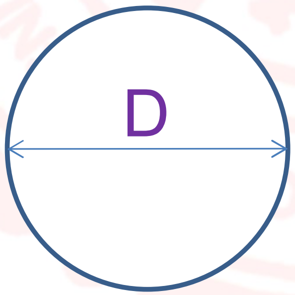
$$= 33.45 \times 10^6 \, Nmm$$

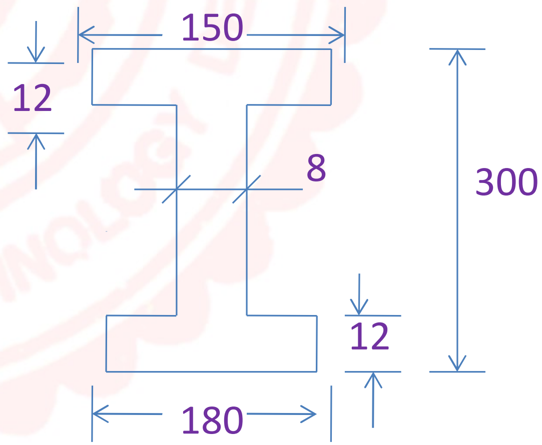
$$= 33.45 \, kNm$$

$$f = \frac{Mp}{My} = \frac{Zp}{Ze} = \frac{33.45}{18.66} = 1.79$$

<u>H.W:</u>

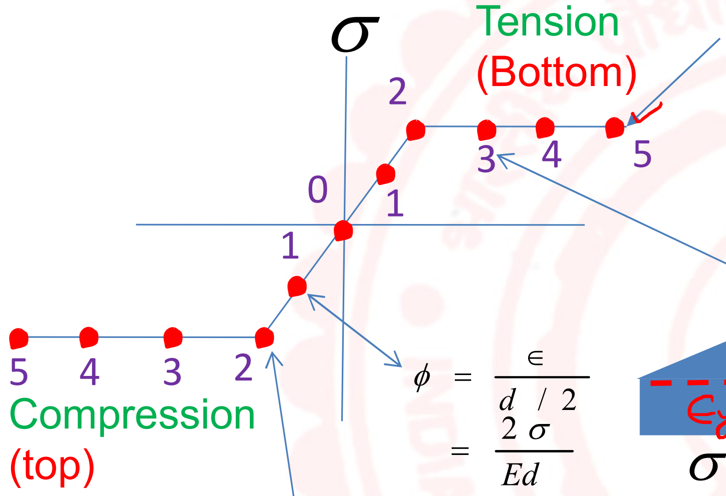
Determine f





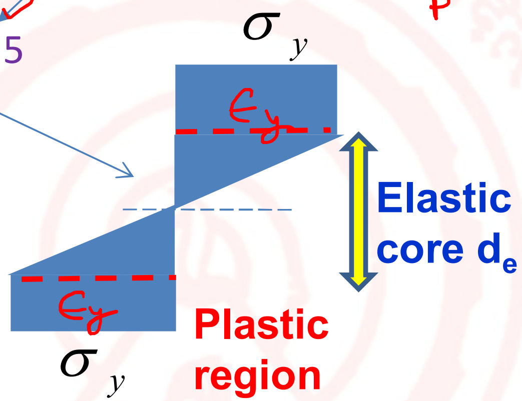
MOMENT CURVATURE RELATION

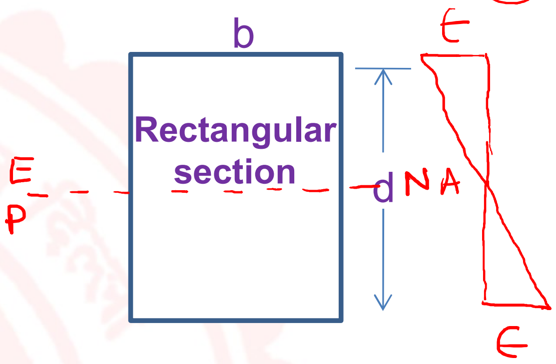
①



$$\phi = \frac{\in}{d/2} = \frac{2\sigma}{Ed}$$

$$M_y = \sigma_y Z_e = \frac{1}{6}\sigma_y bd^2$$





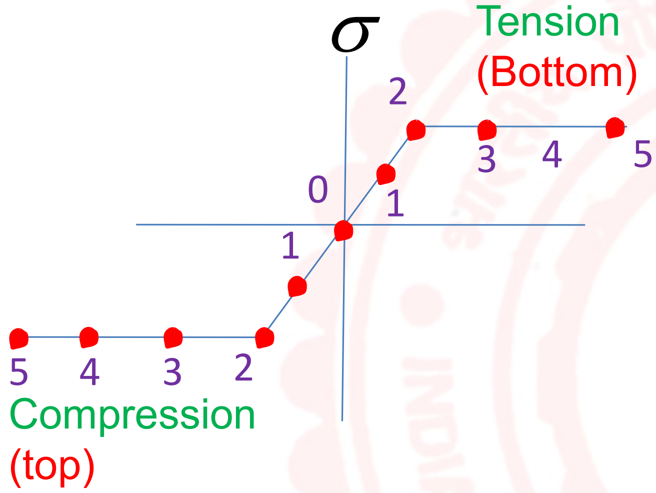
$$\phi = \frac{\varepsilon_y}{d_e/2} = \frac{2\sigma_y}{Ed_e}$$

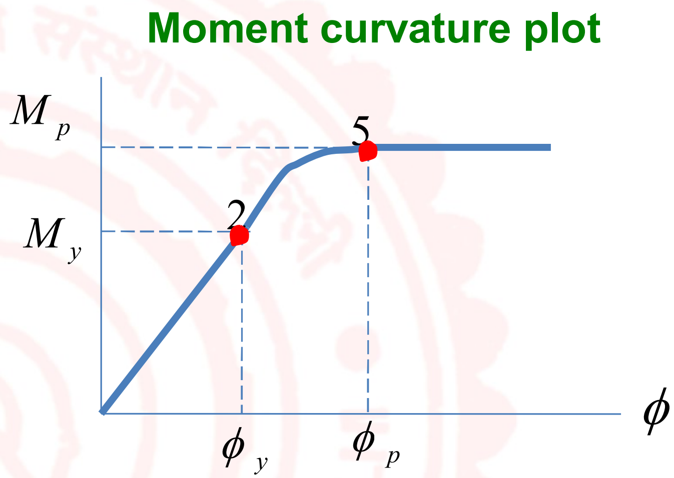
-Plastic (end) regions – will undergo strain without any increase in stress.

-Any moment beyond M_y will be resisted by shrinking of the elastic core .

Fully plastic condition $\quad d_e \rightarrow 0 \qquad \phi \rightarrow \infty$

MOMENT CURVATURE RELATION



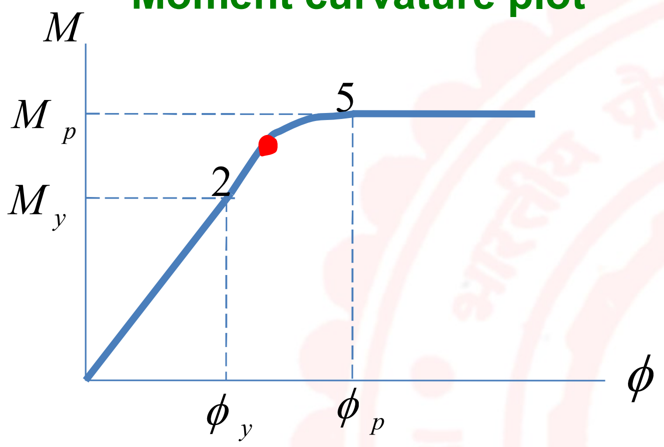


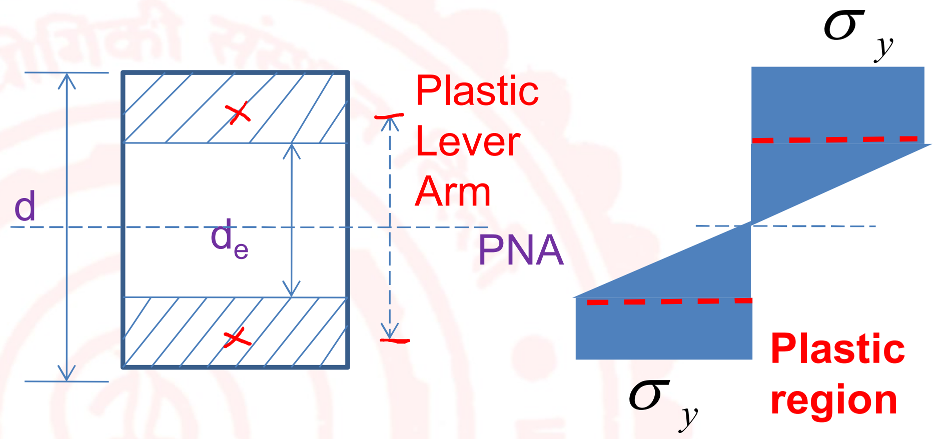
For linear portion of curve $\quad \in < \in_y \; or \; \sigma < \sigma_y$

$$\phi_y = \frac{2\sigma_y}{Ed} \qquad \phi_y = \frac{2M_y}{EdZ_e}$$

MOMENT CURVATURE RELATION

Moment curvature plot





$$M = M_{core} + M_{plastic}$$

$$M = \sigma_y Z_{ec} + \sigma_y b\left(\frac{d-de}{2}\right)\left(\frac{d+de}{2}\right)$$

$$Z_{ec} = \tfrac{1}{6} b d_e^{\,2}$$

$$M = \tfrac{1}{2} M_y \left[3 - \left(\tfrac{de}{d}\right)^2\right] \qquad M_y < M < M_p$$

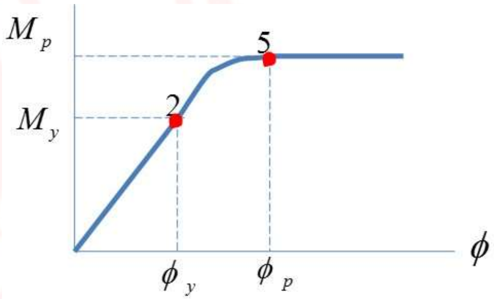
MOMENT CURVATURE RELATION

$$\underline{M_y < M < M_p} \qquad or \qquad \underline{\phi_y < \phi < \phi_p}$$

$$M = \tfrac{1}{2} M_y \left[3 - \left(\tfrac{d_e}{d}\right)^2\right]$$

$$\phi = \frac{2\sigma_y}{E d_e} \qquad \frac{\phi}{\phi_y} = \frac{d}{d_e}$$

$$\phi_y = \frac{2\sigma_y}{E d}$$



$$M = \tfrac{3}{2} M_y - \tfrac{1}{2} M_y \left(\tfrac{\phi_y}{\phi}\right)^2$$

$$\frac{\phi}{\phi_y} = \frac{1}{\sqrt{3 - 2\left(\frac{M}{M_y}\right)}}$$

$$M_y = \frac{1}{6}\sigma_y b d^2$$

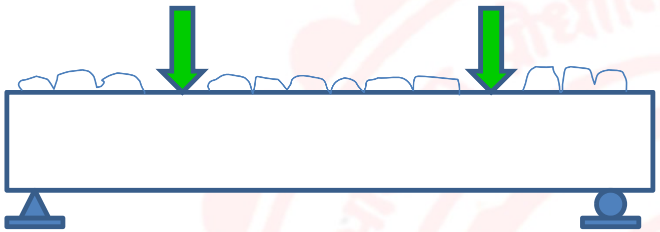
$$M_P = \frac{1}{4}\sigma_y b d^2$$

Condition $M = M_p$

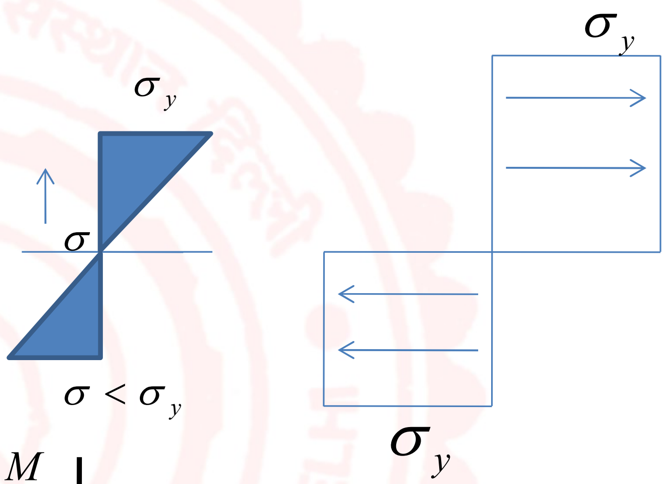
$$\frac{M_p}{M_y} = \frac{3}{2} \qquad \phi \to \infty$$

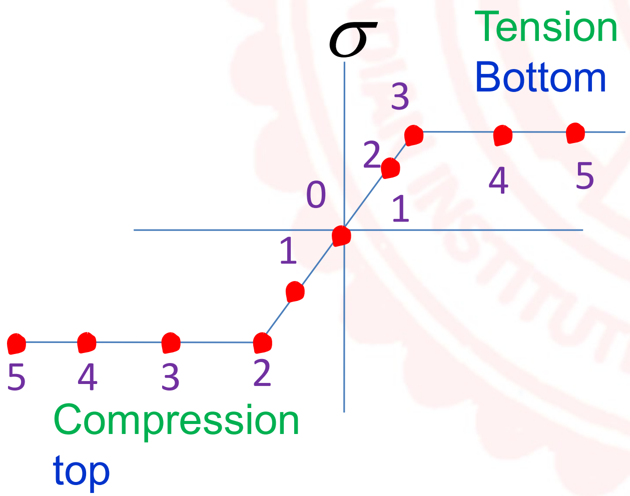
H.W. ϕ_p ??

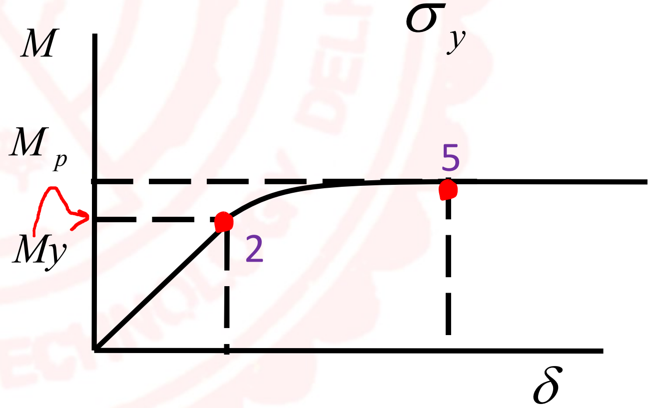
MOMENT DEFLECTION RELATION



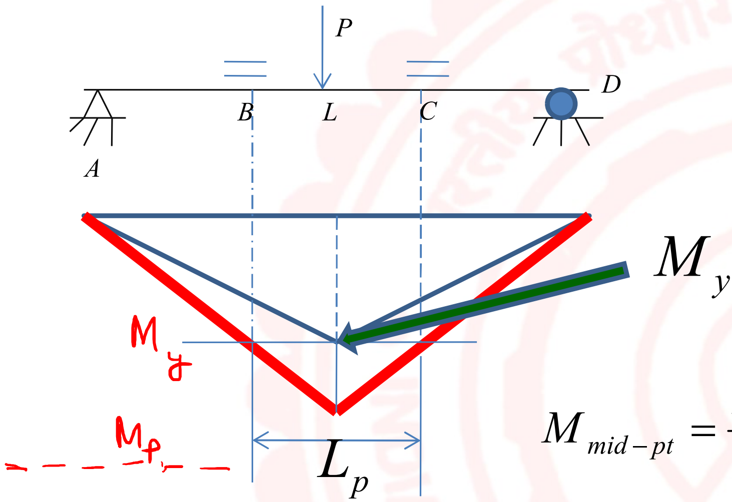
Assume that structure is determinate





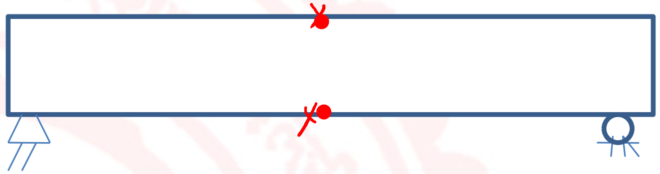


SEQUENCE OF PLASTIFICATION



Region under plastification

$$M_{mid-pt} = \frac{PL}{4}$$



Increase load gradually.

$$P = P_y \qquad M = M_y$$

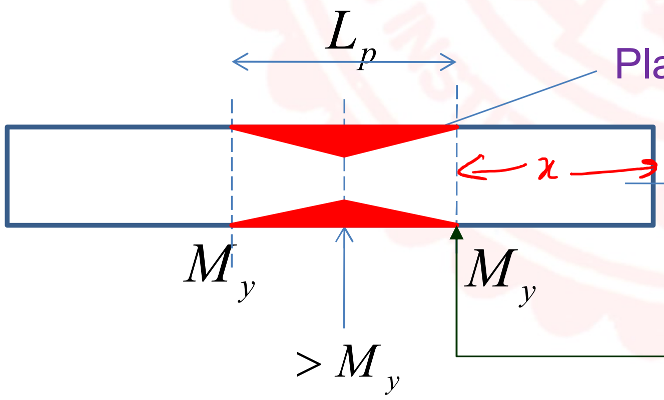
Increase load beyond P_y

$$M_y < M < M_P$$

$M = 1.2 M_y$

$\frac{P}{2} * x$

$= L - 4^{.8}\left(\frac{M_y}{P}\right)$



Plastic zone

Rest of the beam is elastic.

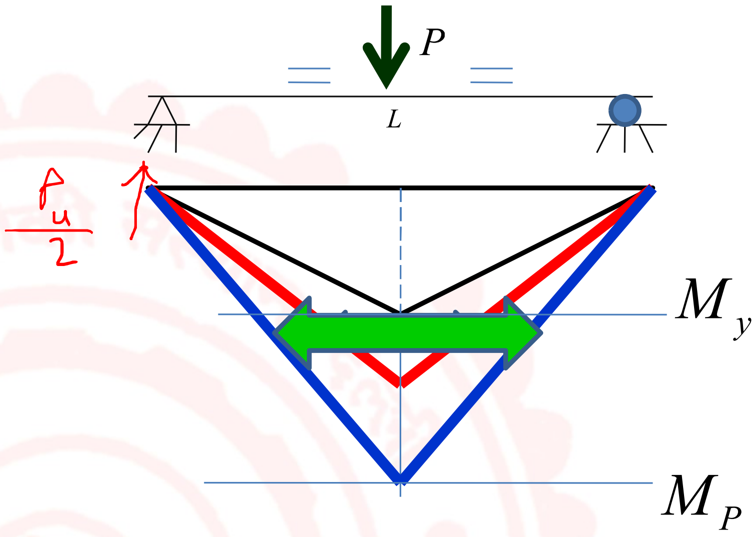
$$M = M_y = \frac{P}{2}\left(\frac{L - L_p}{2}\right)$$

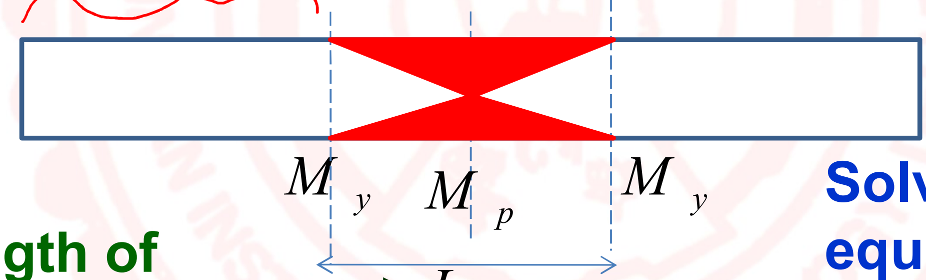
Increase P such that

$$P = P_u \qquad M = M_P$$

$$M_P = \frac{P_u L}{4}$$

$$\Rightarrow P_u = \frac{4 M_P}{L}$$





Limiting length of plastic zone → L_{po}

Solving the two equations

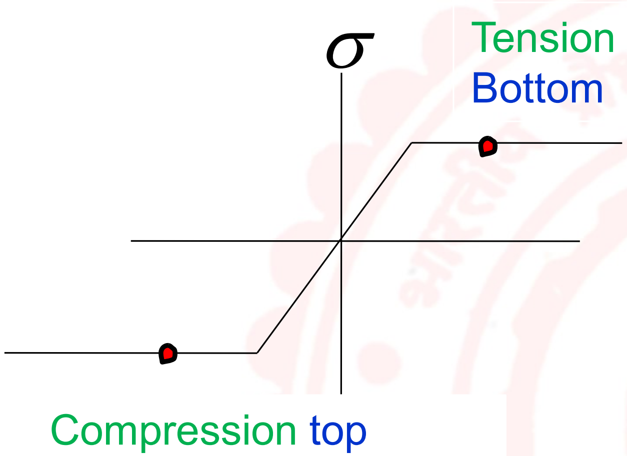
$$\frac{P_u}{2}\left(\frac{L - L_{po}}{2}\right) = M_y \qquad M_P = \frac{P_u L}{4}$$

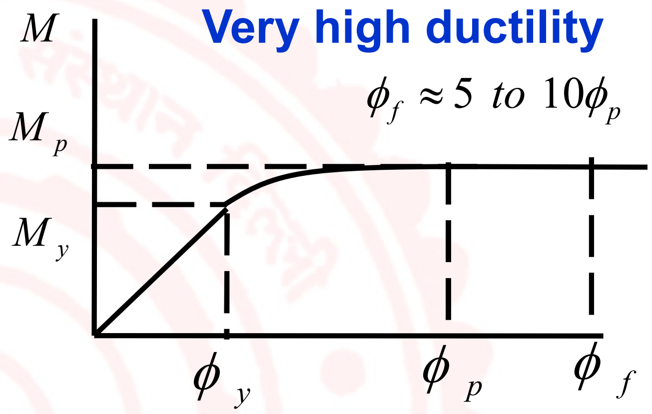
$$\frac{L_{po}}{L} = 1 - f^{-1}$$

Rectangular section: 0.33

H.W.: Shape of plastic region?

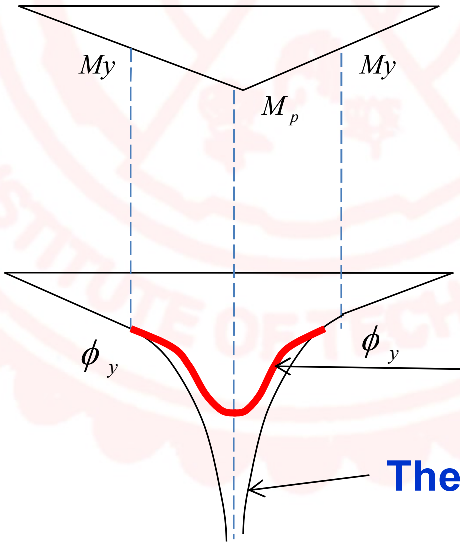
MOMENT CURVATURE RELATION

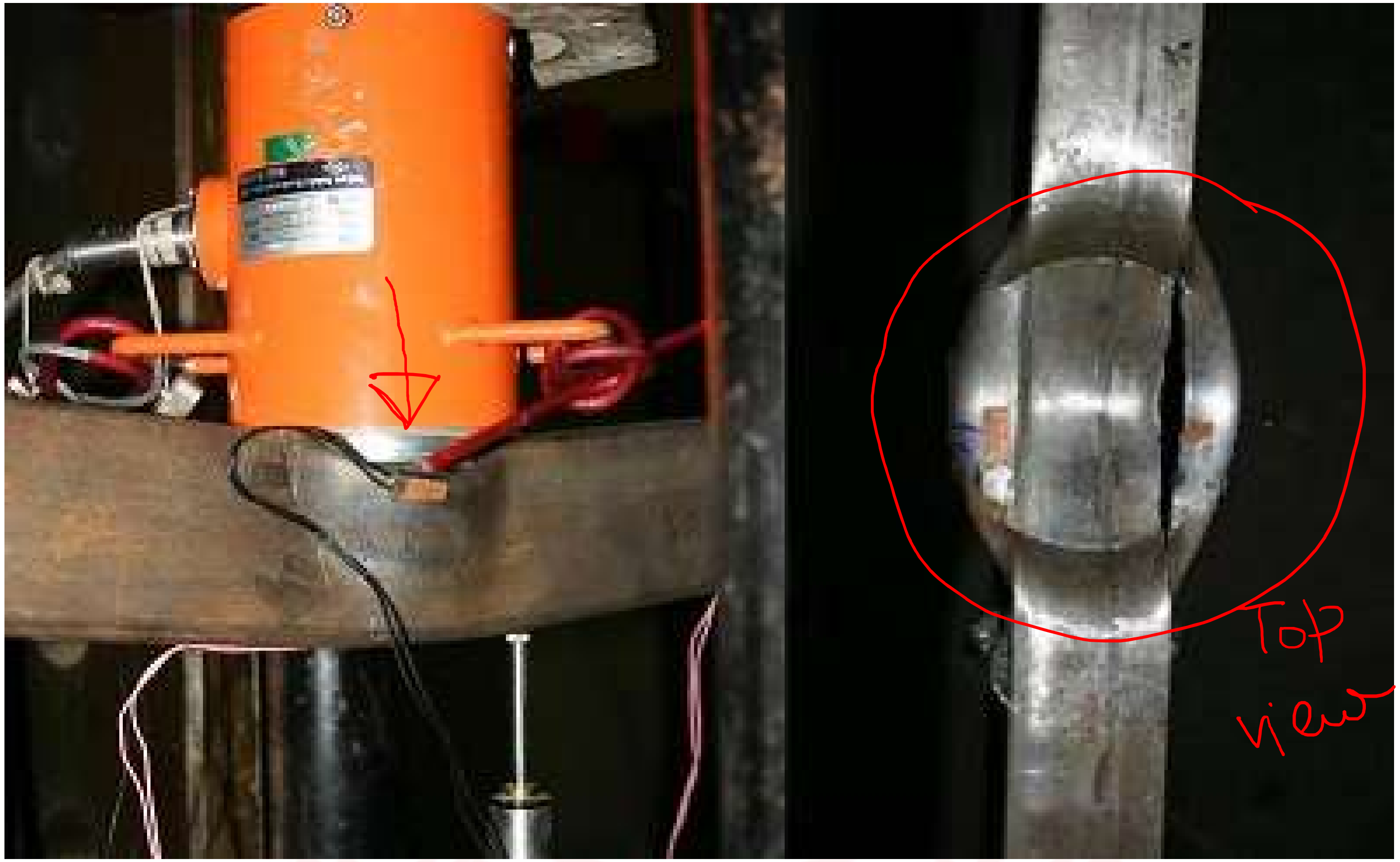




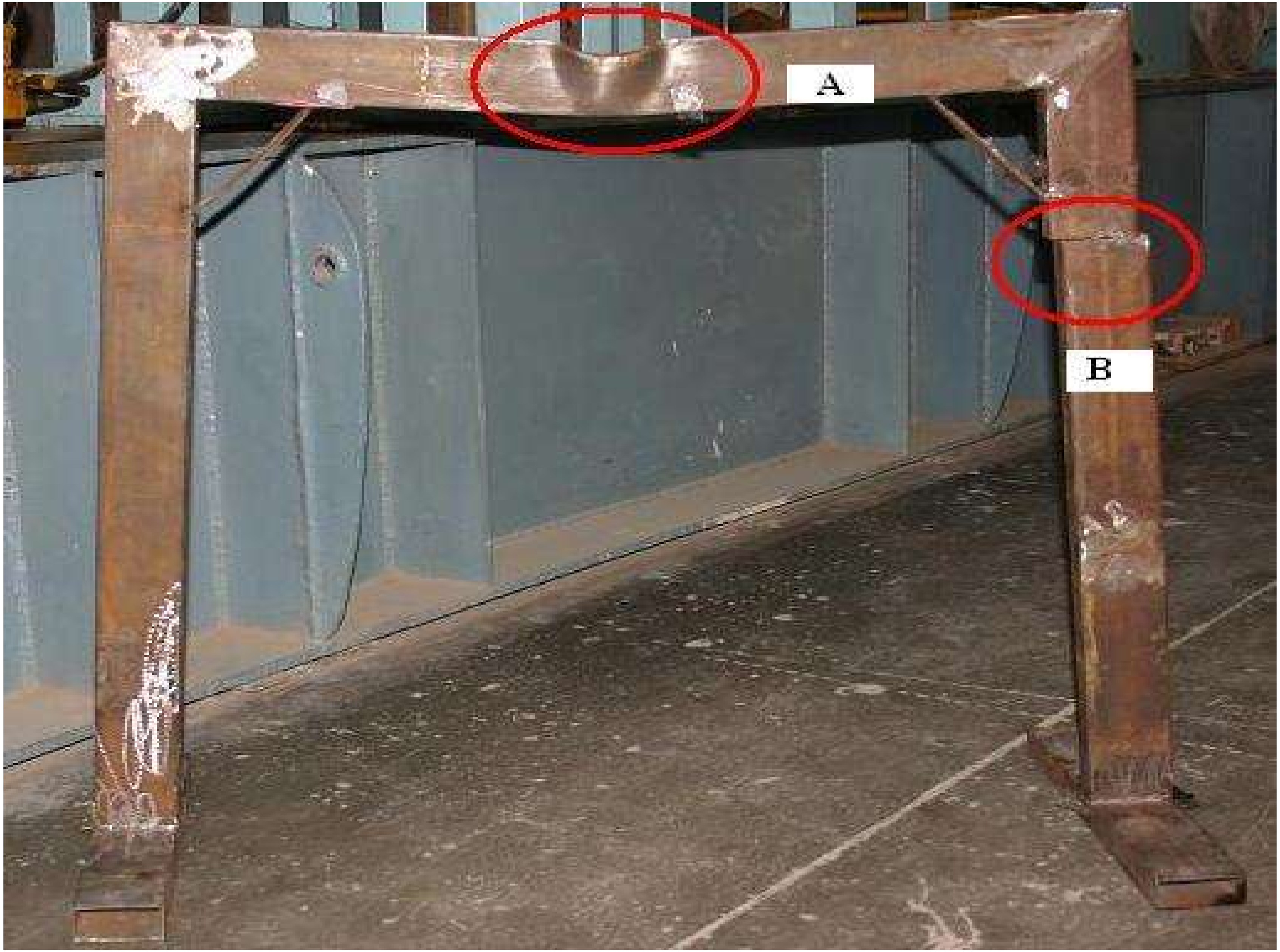
The phenomenon of large increase in curvature when load reaches P_u is called as "unrestricted plastic flow"

Variation of curvature





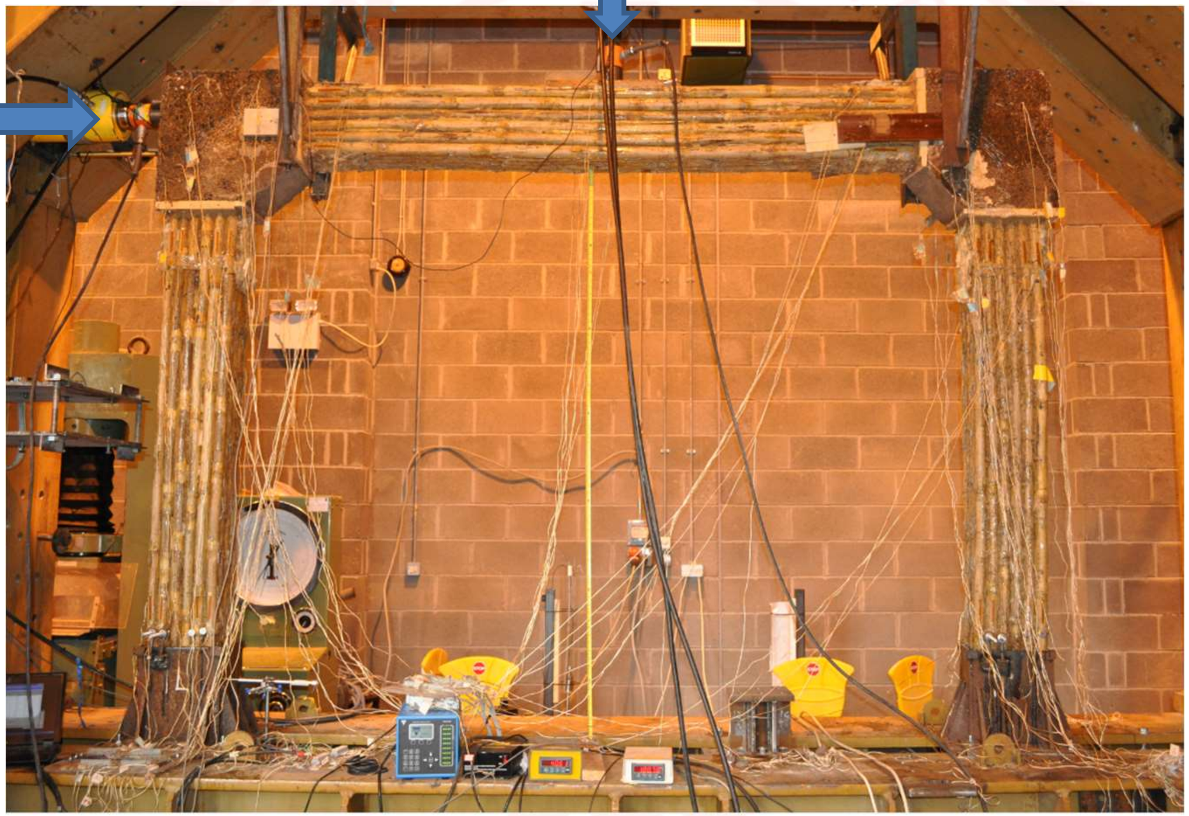
TYPICAL PLASTIC HINGE

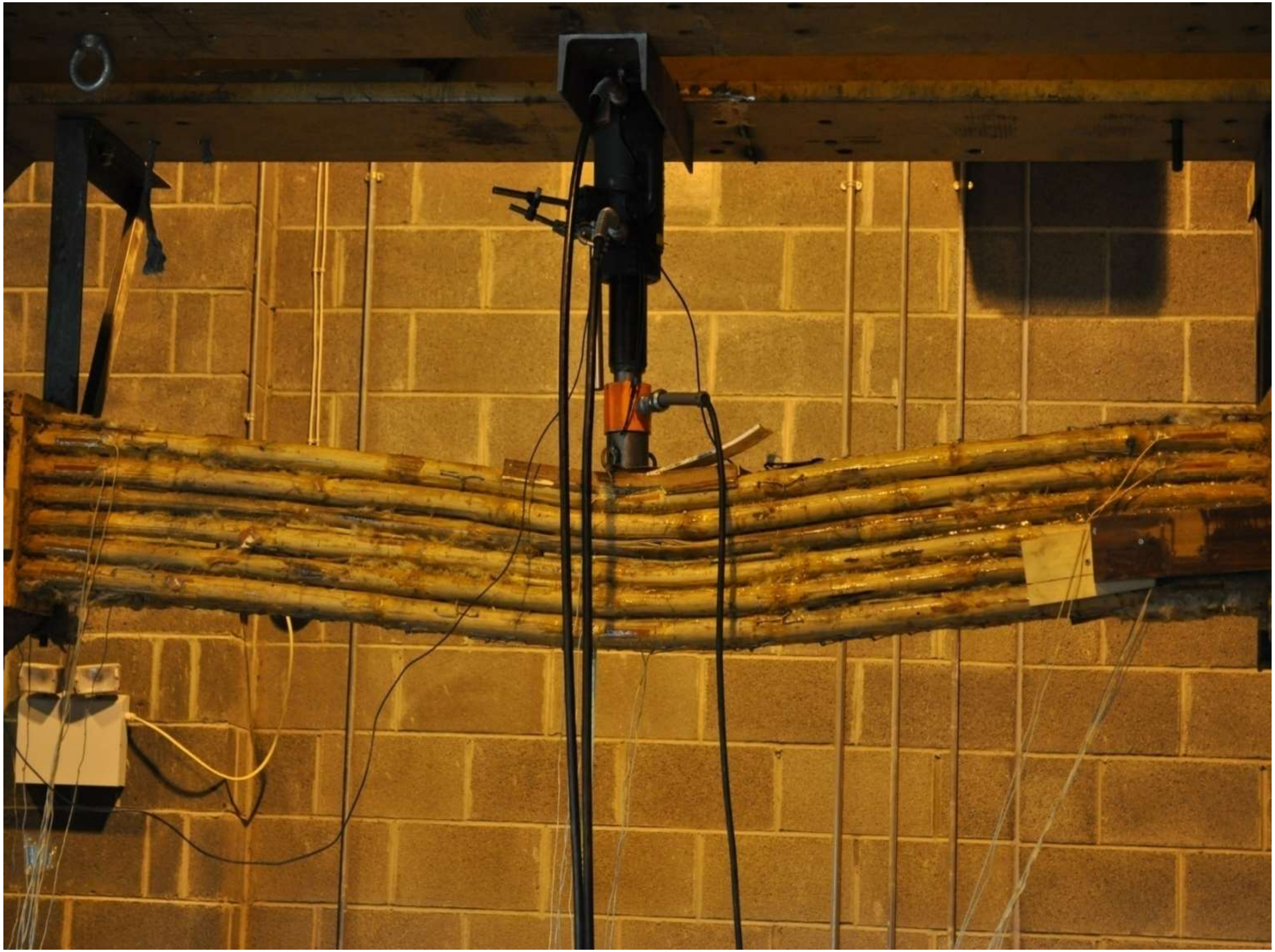

A
B

LOAD APPLICATION

Vertical Load

Horizontal
Load





PROVISIONS OF IS 800 (2007)

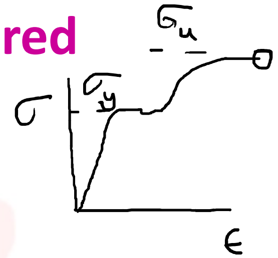
SEC 3.7 CLASSIFICATION OF SECTIONS

Plate elements of a cross-section may buckle locally due to compressive stresses. The local buckling can be avoided before the limit state is achieved by limiting the width to thickness ratio of each element of a cross-section subjected to compression due to axial force, moment or shear.

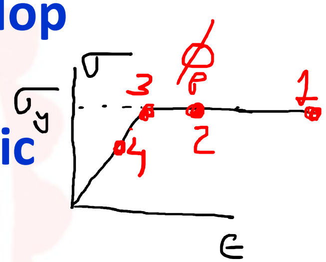
When plastic analysis is used, the members shall be capable of forming plastic hinges with sufficient rotation capacity (ductility) without local buckling, to enable the redistribution of bending moment required before formation of the failure mechanism.

When elastic analysis is used, the member shall be capable of developing the yield stress under compression without local buckling.

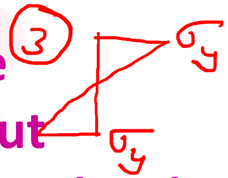
Class 1 (Plastic) — Cross-sections, which can develop plastic hinges and have the rotation capacity required for failure of the structure by formation of plastic mechanism.



***Class* 2 (Compact) — Cross-sections, which can develop plastic moment of resistance, but have inadequate plastic hinge rotation capacity for formation of plastic mechanism, due to local buckling.**



Class 3 (Semi-compact) — Cross-sections, in which the extreme fiber in compression can reach yield stress, but cannot develop the plastic moment of resistance, due to local buckling.



Class 4 (Slender) Cross-sections in which the elements buckle locally even before reaching yield stress.

Table 2 Limiting Width to Thickness Ratio

(*Clauses* 3.7.2 *and* 3.7.4)

Compression Element			Ratio	Class of Section		
				Class 1 Plastic	Class 2 Compact	Class 3 Semi-compact
(1)			(2)	(3)	(4)	(5)
Outstanding element of compression flange	Rolled section		b/t_f	9.4ε	10.5ε	15.7ε
	Welded section		b/t_f	8.4ε	9.4ε	13.6ε
Internal element of compression flange	Compression due to bending		b/t_f	29.3ε	$33.5\,\varepsilon$	42ε
	Axial compression		b/t_f	Not applicable		
Web of an I, H or box section	Neutral axis at mid-depth		d/t_w	84ε	105ε	126ε
	Generally	If r_1 is negative:	d/t_w	$\frac{84\varepsilon}{1+r_1}$	$\frac{105.0\,\varepsilon}{1+r_1}$	$\frac{126.0\,\varepsilon}{1+2r_1}$
					$105.0\,\varepsilon$	

4.5.2 *Requirements*

When a plastic method of analysis is used, all of the following conditions shall be satisfied, unless adequate ductility of the structure and plastic rotation capacity of its members and connections are established for the design loading conditions by other means of evaluation:

a) The yield stress of the grade of the steel used shall not exceed 450 MPa. Ductility

b) The stress-strain characteristics of the steel shall not be significantly different from those obtained for steels complying with IS 2062 or equivalent and shall be such as to ensure complete plastic moment redistribution. The stress-strain diagram shall have a plateau at the yield stress, extending for at least six times the yield strain. The ratio of the tensile strength to the yield stress specified for the grade of the steel shall not be less than 1.2. The elongation on a gauge length complying with IS 2062 shall not be than 15 percent, and the steel shall exhibit strain-hardening capability. Steels conforming to IS 2062 shall be deemed to satisfy the above requirements.

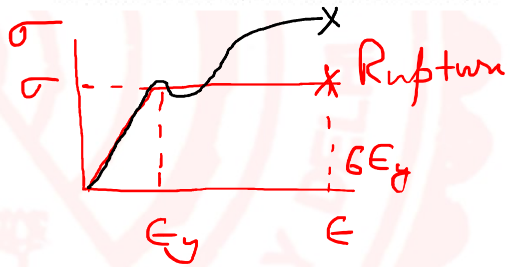
c) The members used shall be hot-rolled or fabricated using hot-rolled plates and sections.

d) The cross-section of members not containing plastic hinges should be at least that of compact section (*see* **3.7.2**), unless the members meet the strength requirements from elastic analysis.

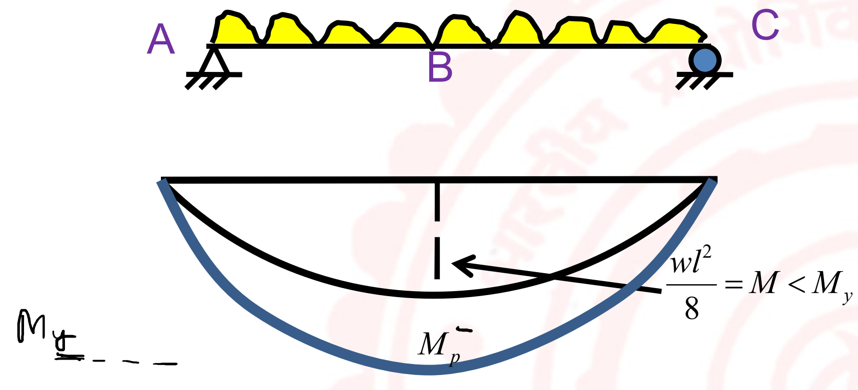
e) Where plastic hinges occur in a member, the proportions of its cross-section should not exceed the limiting values for plastic section given in **3.7.2**.

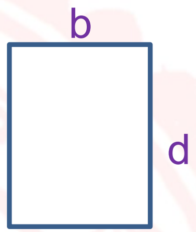
f) The cross-section should be symmetrical about its axis perpendicular to the axis of the plastic hinge rotation.

g) The members shall not be subject to impact loading, requiring fracture assessment or fluctuating loading, requiring a fatigue assessment (*see* Section 13).

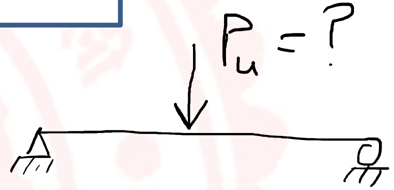


EX 1: SIMPLY SUPPORTED BEAM UNDER U.D.L





Repeat



$When \quad M > M_y \quad but \quad M < M_p$

M_p

No change in shape of BMD continue till M_p

$$But\ when\ M = M_p$$

Plastic hinge @ B
Mechanism formation

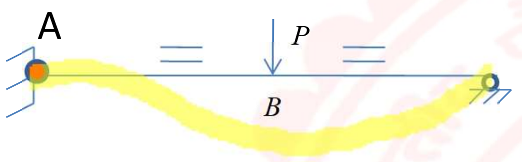
$$w = w_u$$

$$\frac{w_u l^2}{8} = M_p = Z_p \sigma_y = \frac{1}{4} bd^2 \sigma_y$$

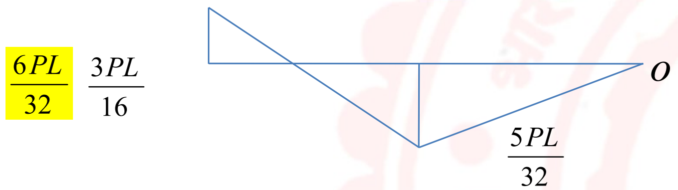
$$w_u = \frac{2\sigma_y bd^2}{L^2}$$ = Ultimate collapse load.

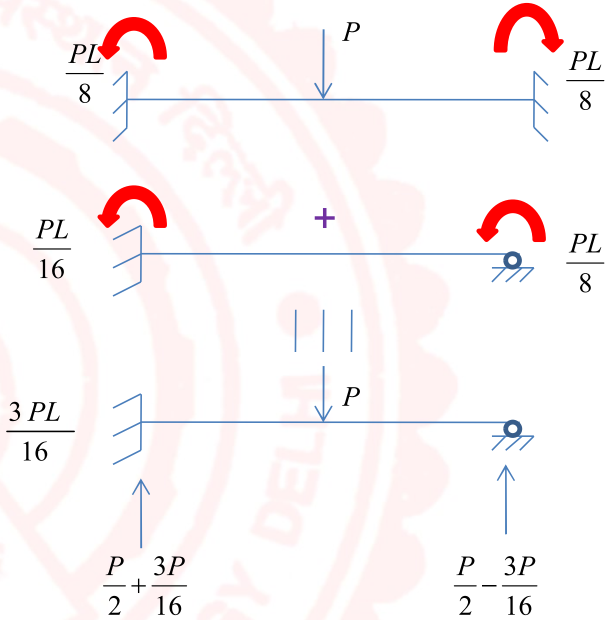
Compare with yield capacity

EX 2: PROPPED CANTILEVER



$P < P_u$

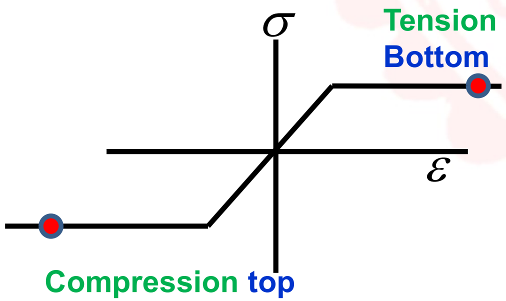




1st Plastic hinge shall form at 'A'

Collapse will not occur

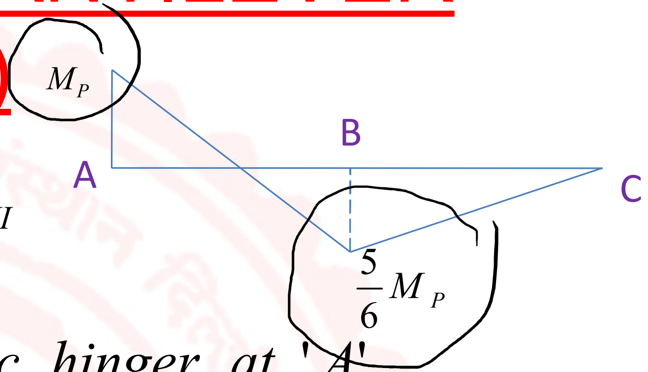
$$Let \quad P = P_{H1} \quad when \quad BM \ @ \ A = M_p$$



$$\frac{3P_{H1}\, l}{16} = M_p \qquad P_{H1} = \frac{16\, M_p}{3L}$$

Structure can still carry further loads

EX 2: PROPPED CANTILEVER (Contd.)



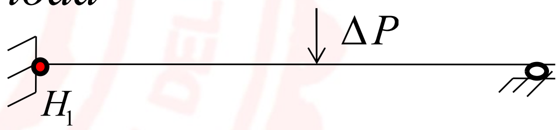
$$When \;\; P = P_{HI}$$

$$Increase \;\; "P" \, after \;\; formation \;\; of \;\; Plastic \;\; hinger \;\; at \;\; 'A'$$

$$For \;\; load \;\; P > P_{HI} \rightarrow B.M \;\; cannot \;\; increase \;\; @ \; A$$

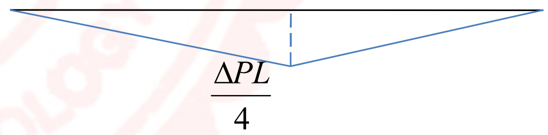
$\Delta P = P - P_{H1}$

$$When \; P > P_{HI} : for \; incremental \; load$$

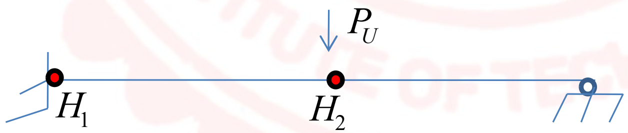


$$\Delta P = P - P_{HI}$$

$$When \;\; P = P_u , \Delta P = \Delta P_u$$



$$Net \;\; BM \;\; @ \; B = \frac{5}{6} M_P + \frac{\Delta PL}{4}$$



Mechanism

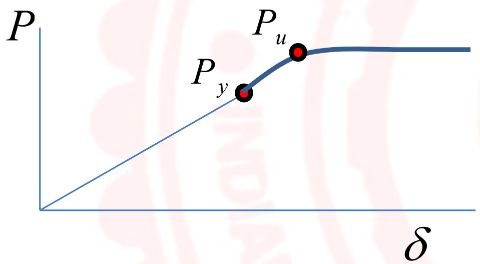
$$M_p = \frac{5}{6} M_p + \frac{\Delta P_U L}{4}$$

$$\Delta P_U = \frac{2}{3} \left(\frac{M_p}{L} \right)$$

$$P_u = P_{H1} + \Delta P_u \qquad\qquad P_u = \frac{6M_P}{L}$$

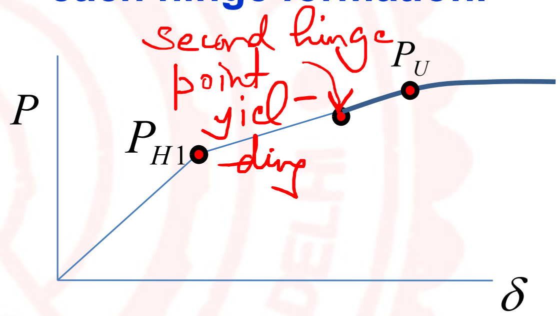
Determinate structures

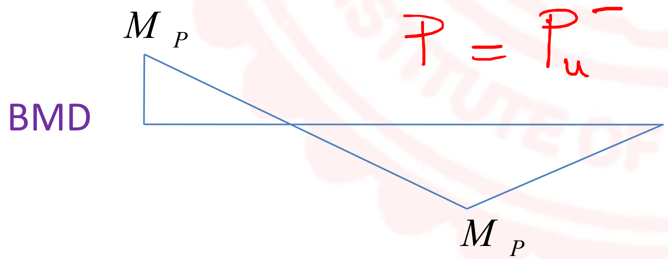
Shape of BMD does not change till failure



Indeterminate structures

Shape of BMD changes after each hinge formation.

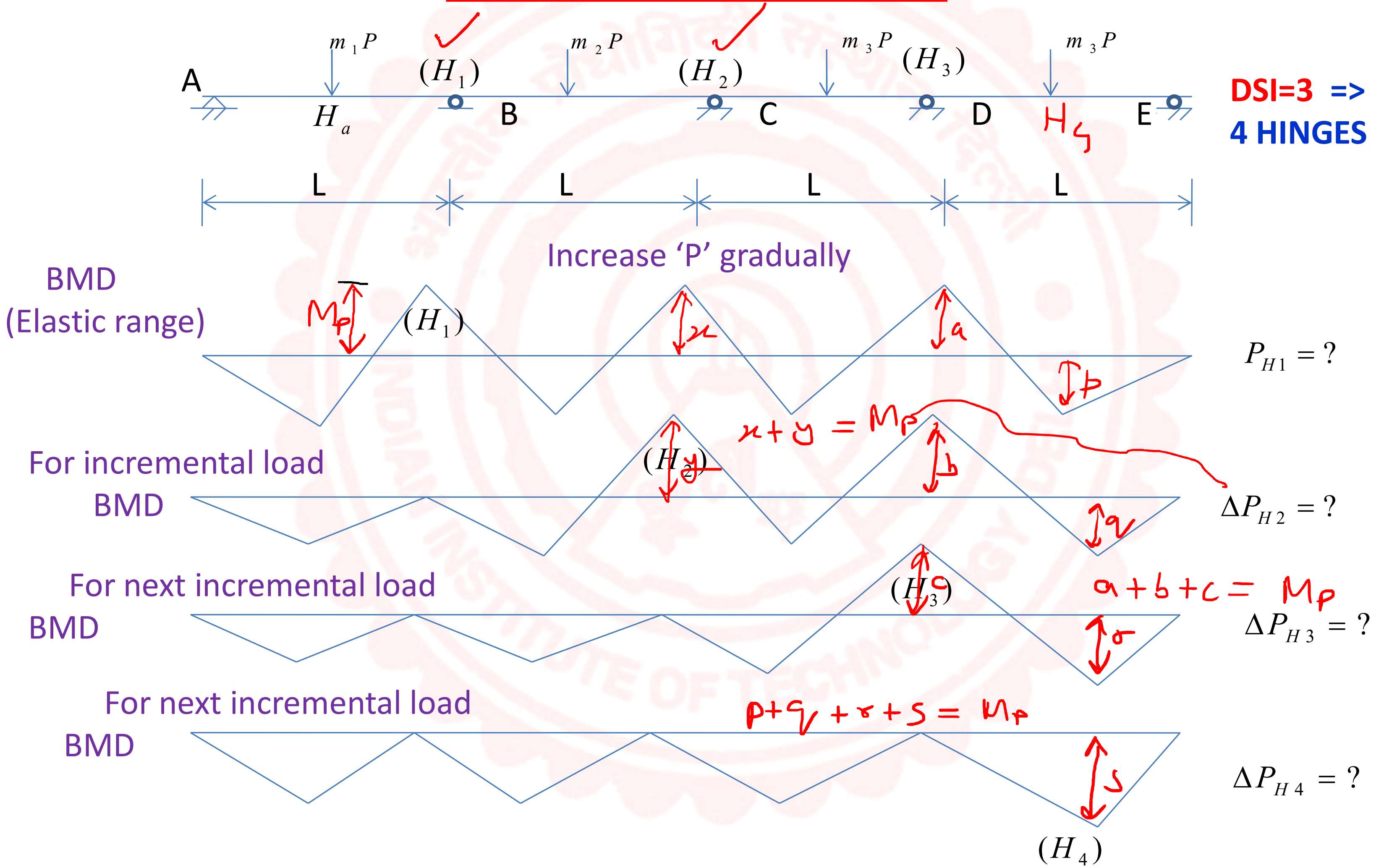




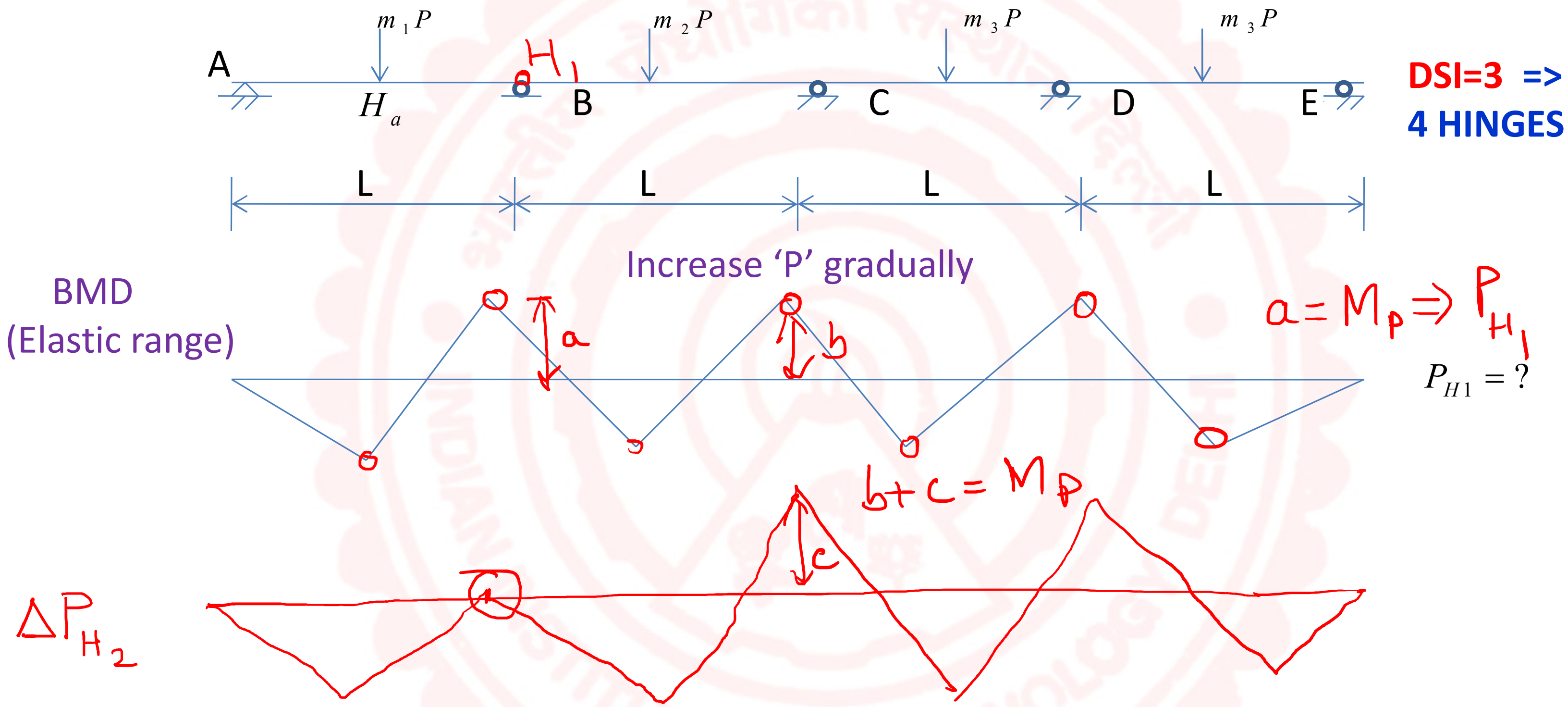
$P = P_u^-$

Final reactions @collapse ???

MULTISPAN INDERTMINATE BEAM UNDER U.D.L



MULTISPAN INDERTMINATE BEAM UNDER U.D.L



INDETERMINATE STRUCTURES

The solution process very tedious

$$P_u = P_{HI} + \Delta P_{H_2} + \Delta P_{H3} + \Delta P_{HN}$$

3 2 1

Need to analyse indeterminate structure

We also get to know….. ….the sequence of hinge formation.

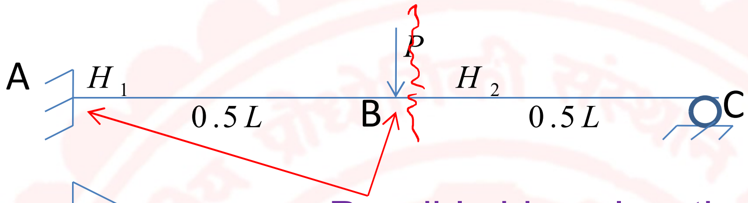
Often, we are not interested in sequence.

Methods for simplified analysis

EQULIBRIUM

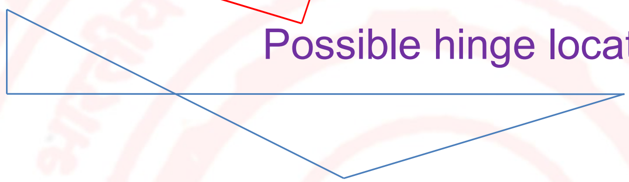
VIRTUAL WORK

EQUILIBRIUM APPROACH



Shape of BMD

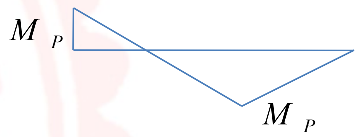
(actual value not needed)



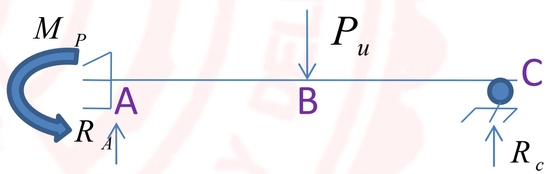
Possible hinge locations.

Consider equilibrium when structure is at verge of collapse

$$P = P^{-}_{u}$$



Consider overall equilibrium

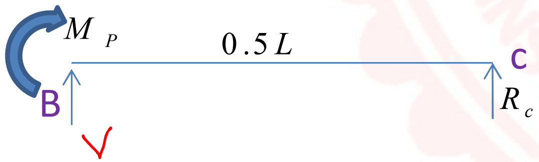


Moment about A

$$M_P + \left(\frac{2Mp}{L}\right)L = P_u \times 0.5L$$

Solving $P_u = \dfrac{6M_p}{L}$

Consider equilibrium of BC

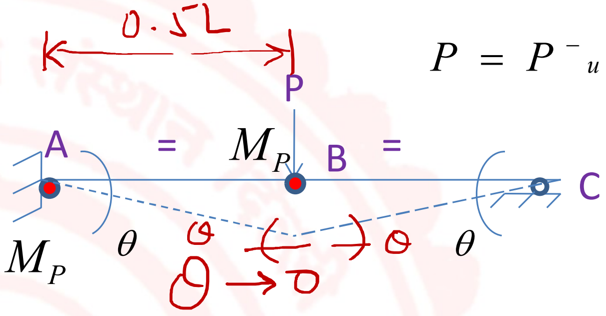


Take moment about B $R_c = \dfrac{2Mp}{L}$

Advantage – Skip solving an indeterminate structure, which is otherwise tedious….

VIRTUAL WORK APPROACH

ASSUMPTIONS:



1. Structure @ verge of collapse

2. Geometry of the structure not change as a result of plastification.

Identify hinge locations and apply small virtual (RIGID BODY) rotation θ.

Ext. Virtual work = Internal Virtual work

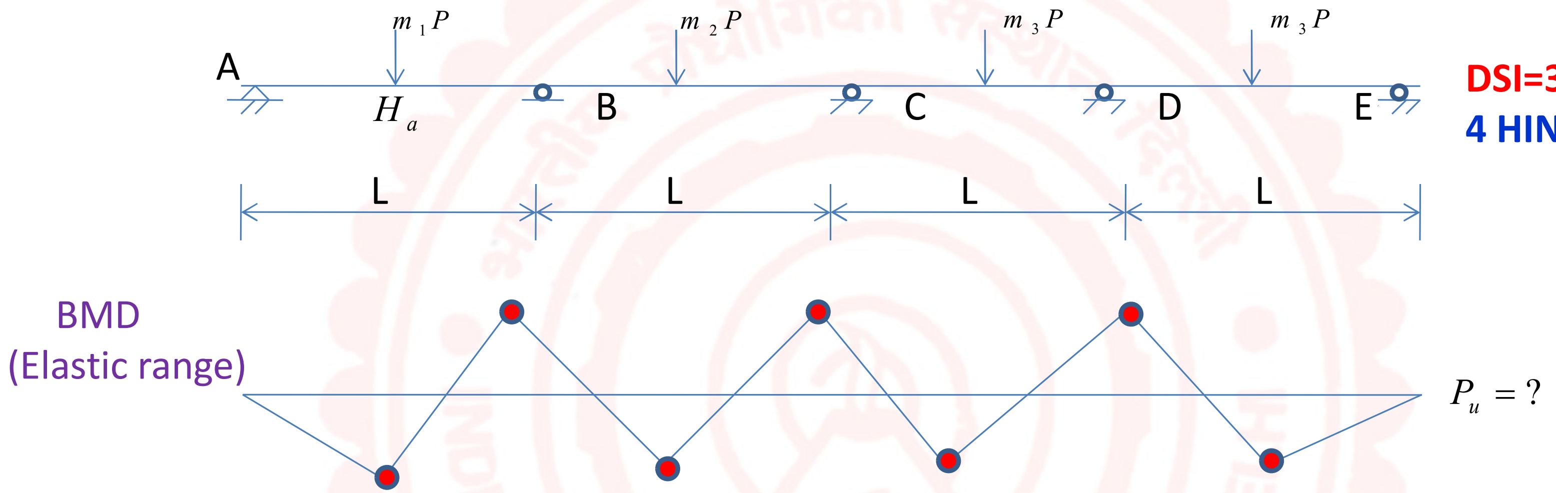
$$P_U \frac{L}{2}\theta = \overbrace{M_P\theta}^{A} + \underbrace{M_P(2\theta)}_{B}$$

$$\Rightarrow P_U = \frac{6M_P}{L}$$

Only M_p does the internal work, WHY?

Which method would you judge best out of the three methods??

APPLICATION OF SHORT-CUT APPROACH ON COMPLICATED STRUCTURE



DSI=3 => 4 HINGES

Problem: Four hinges till collapse, but seven possible locations

Which four of the seven possible hinges will be the governing mechanism….???

This won't be problem in the long-hand method of sequential formation of hinges…

In equilibrium or virtual work method, we rely on some important theorems..

THEOREMS FOR PLASTIC ANALYSIS

ASSUMPTIONS: At the point of collapse
(1) Loading system not affected
(2) Geometry of structure not affected.

UNIQUENESS THEOREM (UT)

Following conditions must be satisfied simultaneously at point of collapse of structure.

LBT

1) EQUILIBRIUM CONDITION
All bending moments in equilibrium with the applied forces and reactions

2) YIELD CONDITION
At all points of the structure, the bending moment $\leq M_P$

3) MECHANISM CONDITION - Sufficient number of plastic hinges are formed so that mechanism condition results….. UBT

UPPER BOUND THEOREM (UBT)

If a loading can be found such that a mechanism is formed, then

$$P \geq P_U$$

LOWER BOUND THEOREM (LBT)

If a distribution of moments can be found such that equilibrium and yield conditions are satisfied…..

…This would imply that the structure is either safe or just at the verge of collapse

$$P \leq P_U$$

If LBT and UBT are satisfied simultaneously

$$P = P_U$$

ANAYLYSIS PROCEDURE

(1) Formulate possible collapse mechanisms, based on indeterminacy. (Guesswork involved!)

(2) According to UBT $P_U \leq (P_{1u}, P_{2u}, P_{3u} ......)$

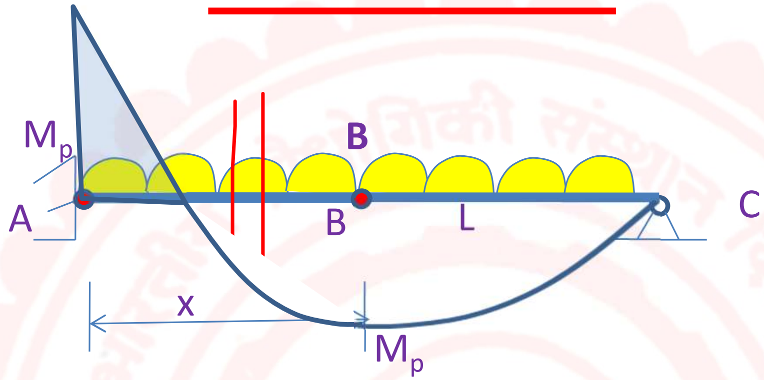
(3) Critical value of P_u : Lowest among these values ………..provided all possible mechanisms included.

Possibility of missing the critical mechanism exists.

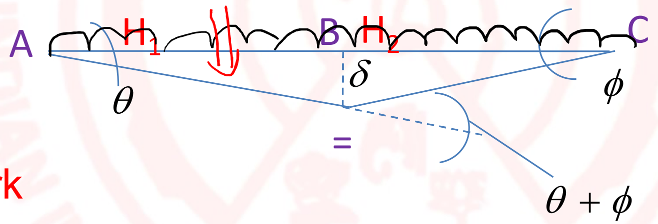
(4) To rule out missing of critical mechanism, apply LBT (Yield and equilibrium criteria) $P_u \geq P_{crit}$

(5) If both LBT & UBT satisfied simultaneously imply correct collapse load $P = P_u$

EXAMPLE



Let structure be @ verge of collapse. Apply virtual displacement & @ B



External virtual work

Internal virtual work

$$(wx)\frac{\delta}{2} + w(l - x)\frac{\delta}{2} = \underbrace{M_P\theta}_{@A} + \underbrace{M_P(\theta + \phi)}_{@B}$$

Disp. of CG

$$\theta = \frac{\delta}{x} \quad \phi = \frac{\delta}{L - x}$$

Solving $w = \left(\frac{2M_p}{L}\right)\left[\frac{2L - x}{(L - x)x}\right]$ Infinite mechanisms depending an value of x

EXAMPLE (CONTD..)

$$To \quad get \quad lowest \quad \frac{dw}{dx} = 0 \Rightarrow x = 0.586 L$$

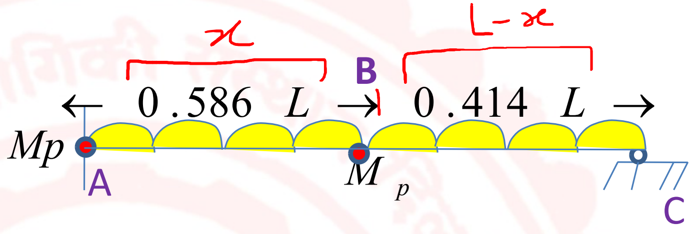
$$w_u = w_{(x=0.586 L)} = 11.65 \left(\frac{M_p}{L^2}\right)$$

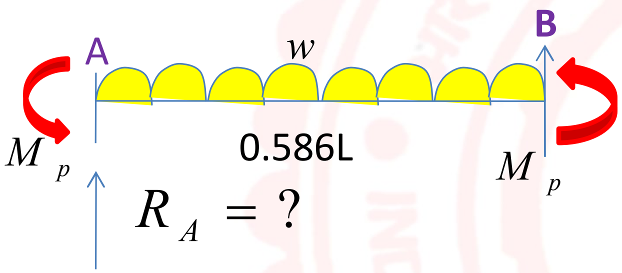
According to UBT $\quad W_u \geq W_{uc}$

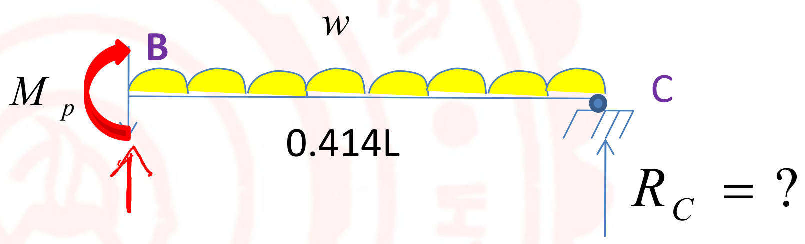
To ensure correct mechanism, use LBT & check yield & equilibrium condition.

CHECK BY LBT

Equilibrium Check :







Moment about 'B'

$$R_A \times 0.586\ L = w\frac{x^2}{2} + 2M_p$$

$$R_A = 0.586\ wL$$

Moment about 'B'

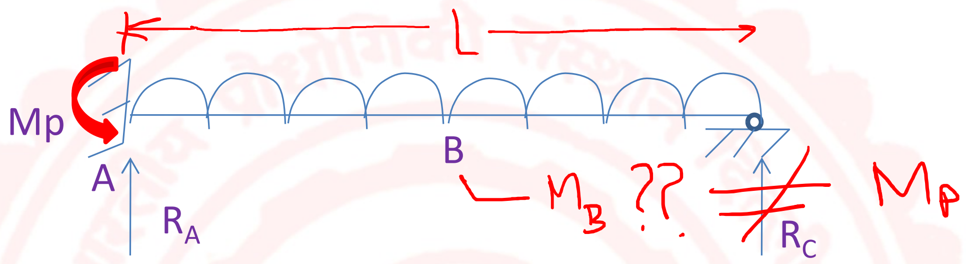
$$R_C(L-x) = M_p + w\frac{(L-x)^2}{2}$$

$$R_C = 0.4140\ wL$$

$$Check\ \ R_A + R_C = wL$$

.. Equilibrium condition satisfied

YIELD CHECK



REACTIONS ON THE BASIS OF THE ABOVE BOUNDARY CONDITIONS ONLY

$$R_A = 6.83 \frac{Mp}{L} \qquad Rc = 4.83 \frac{Mp}{L}$$

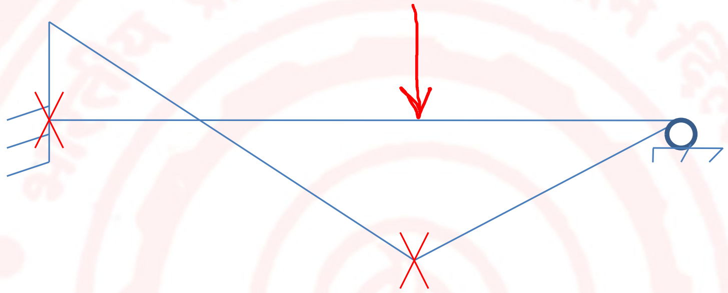
0.586 L $\left(11.65 \frac{M_p}{L^2}\right)$

$$Compute \quad M_B = R_A x - \frac{wx^2}{2} - M_P$$

$$= ?? \; M_P$$

Alternately, we can keep M_A as unknown fix M_B to a value equal to M_P

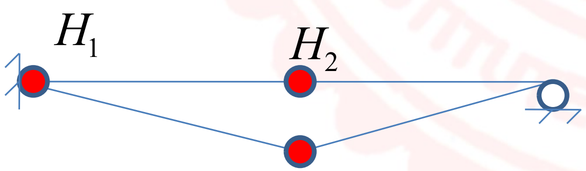
NUMBER OF INDEPENDENT MECHANISMS



Possible hinge locations? n 2

Degree of indeterminacy r 1

Possible **independent** mechanisms $m = n - r$



$$= 2 - 1$$

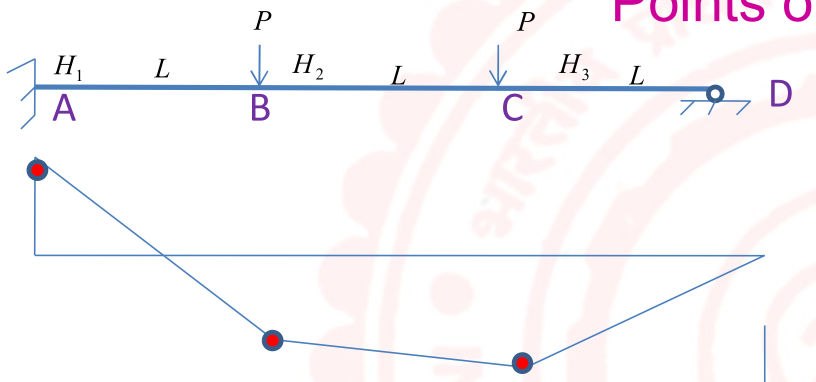
$$= 1$$

CAUTION: THIS NUMBER IS ONLY SUGGESTIVE

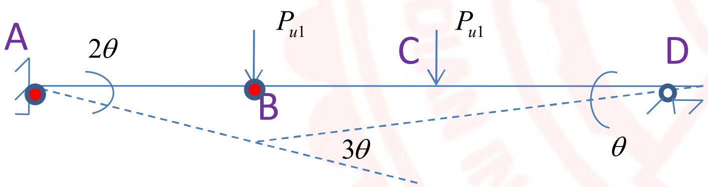
EXAMPLE

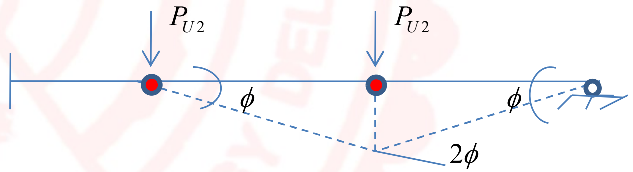
How to identify possible hinge locations ???

Points of discontinuity/ maxima of BMD



$$m = n - r$$
$$= 3 - 1$$
$$= 2$$





Ext virtual work = Int. Virtual work

$$P_{u1}(2\theta L) + P_{u1}(L\theta) = M_P(2\theta) + M_p(3\theta)$$

$$P_{u1} = \frac{5}{3}\left(\frac{Mp}{L}\right)$$

Ext virtual work = Int. Virtual work

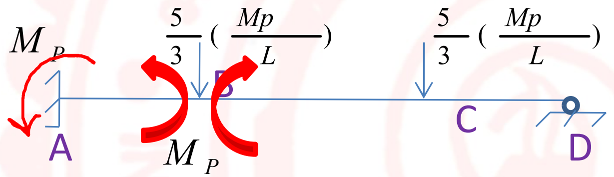
$$P_{U2}(\phi l) = M_P\phi + Mp\,(2\phi)$$

$$P_{U2} = 3\left(\frac{Mp}{L}\right)$$

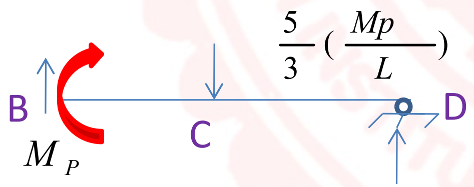
MECH 1: EQUILIBRIUM CHECK

$$UBT \Rightarrow P_{U1},\ P_{U2},\ > P_U$$

LBT – To rule out missing any mechanism. Check for equilibrium & yield.

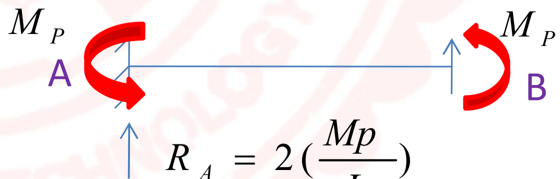


Consider equilibrium of BCD



$$R_D = \frac{4}{3}(\frac{Mp}{L})$$

Consider equilibrium of AB

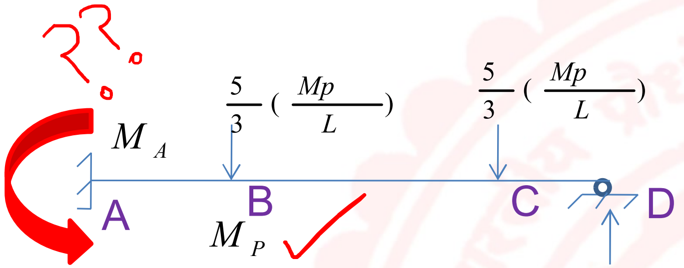


$$R_A = 2(\frac{Mp}{L})$$

$$Check \quad : R_A + R_D = \frac{10}{3}\frac{Mp}{L} = 2P_{U1}$$

$$\therefore \quad Satified \quad .$$

MECH 1: YIELD CHECK



Use BCD- $R_D = \frac{4}{3}(\frac{Mp}{L})$

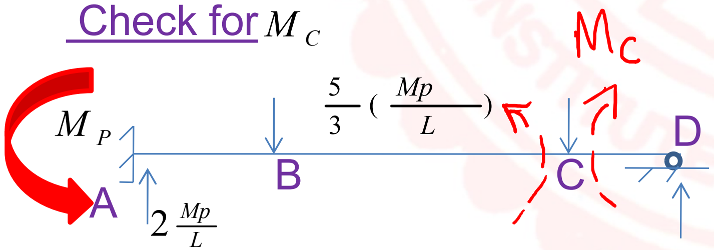
Vertical equilibrium

$$R_A = 2(\frac{Mp}{L})$$

Overall equilibrium - $M_A + \frac{4}{3}(\frac{Mp}{L}) \times 3L = \frac{5}{3}(\frac{Mp}{L})[L + 2L]$

$$M_A = M_P$$

Check for M_C



Using CD

$$M_C = \frac{4}{3}(M_P)$$

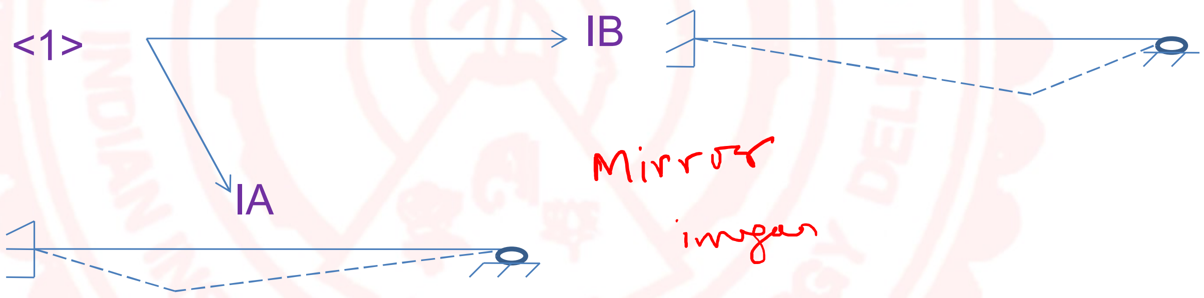
Yield criteria not satisfied

=> WE MISSED OUT A CRITICAL MECHANISM

NUMBER OF INDEPENDENT MECHANISMS

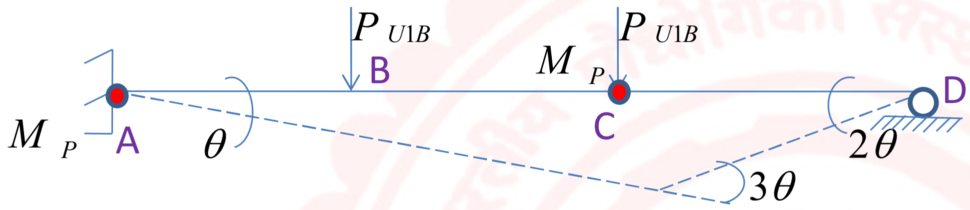
LBT- Some mechanism missed out……………

Mechanism <1>



Check I (B)

MECHANISM I(B): MISSED OUT



External virtual work = Internal virtual work

$$P_{U1B}(L\theta) + P_{1B}(2L\theta) = \underset{\overline{@A}}{M_P\theta} + \underset{\overline{@C}}{M_P(3\theta)}$$

$$\boxed{P_{U1B} = \frac{4}{3}\left(\frac{M_p}{L}\right)}$$

This is lower than P'_{UI}
Check for equilibrium & yield condition.

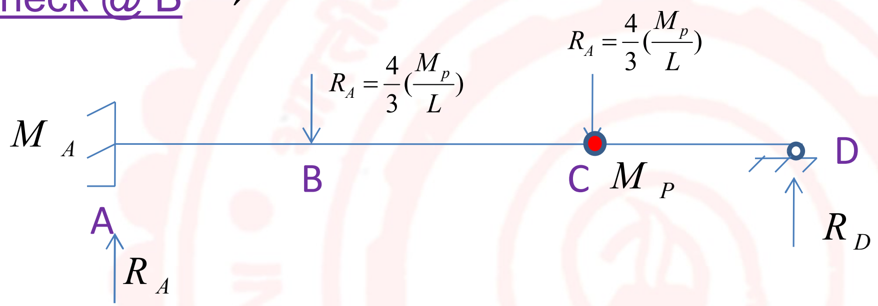
<1> ABC $\rightarrow R_A$ CD $\rightarrow R_D$ check vertical equilibrium.

<2> <u>Yield check</u>

- <2> Point A(assume $M_A \neq M_P$)
- <2> Point B

YIELD CHECK @ B

Yield Check @ B $\rightarrow$



Equilibrium of CD and then overall vertical equilibrium

$$R_D = \frac{M_p}{L} \qquad R_A = \frac{5}{3}(\frac{M_p}{L})$$

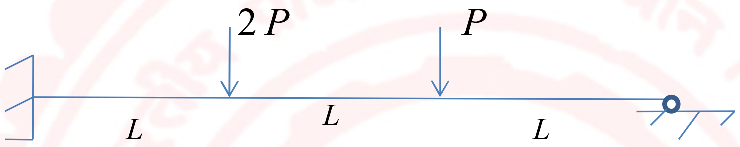
Overall moment equilibrium about A $\quad M_A = M_p$

$$M_B = R_A L - M_A$$
$$= \tfrac{5}{3} M_p - M_p$$
$$M_B = \tfrac{2}{3} M_p$$

Yield check satisfied.

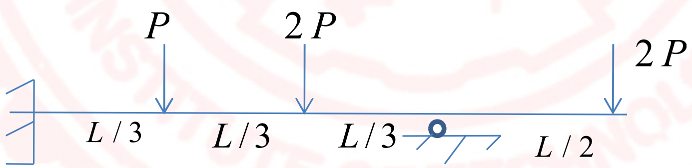
PRACTICE PROBLEMS

1.

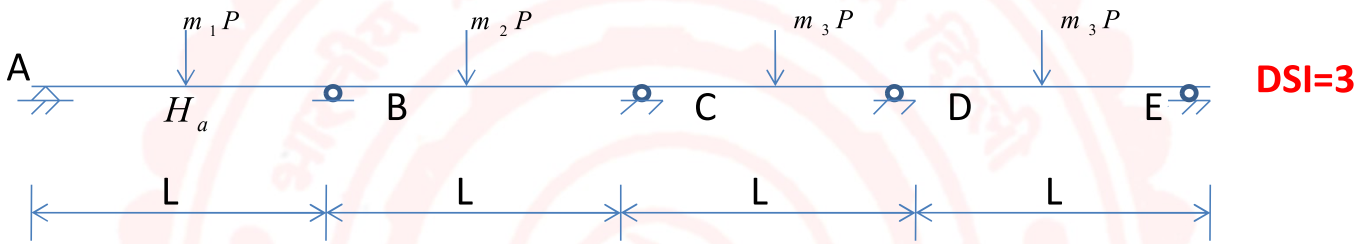


First solve by first principles

2.

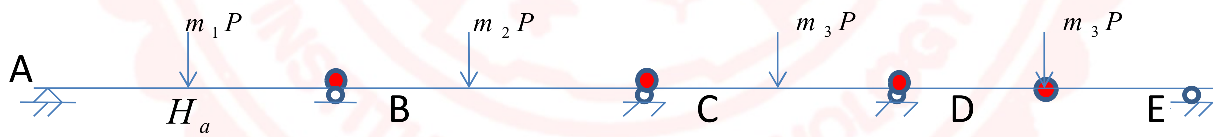


APPLICATION OF SHORT-CUT APPROACH ON MULTISPAN STRUCTURE



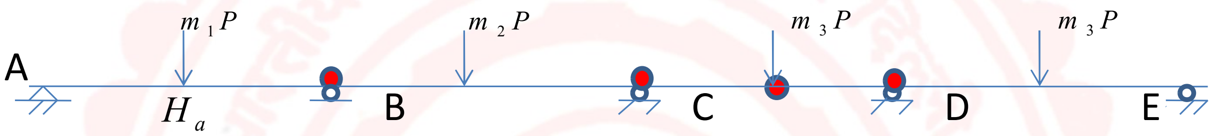
Possible independent mechanisms = 7 – 3 = 4

Mechanism 1

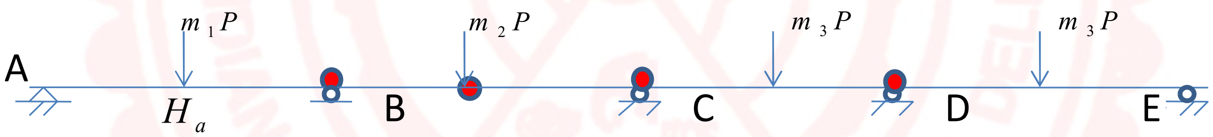


APPLICATION OF SHORT-CUT APPROACH ON MULTISPAN STRUCTURE

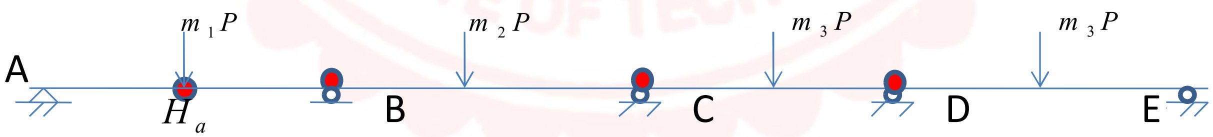
Mechanism 2



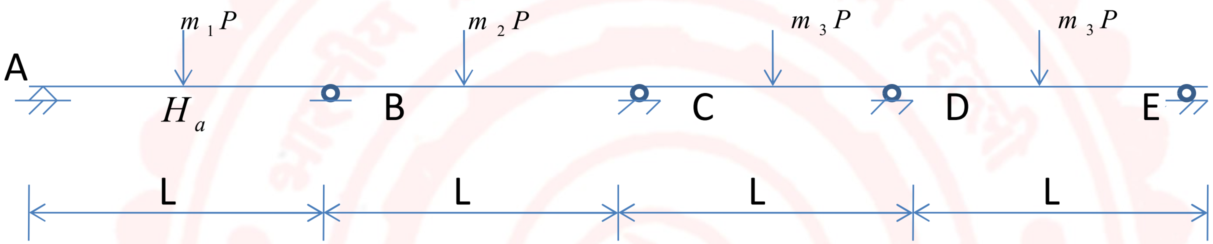
Mechanism 3



Mechanism 4



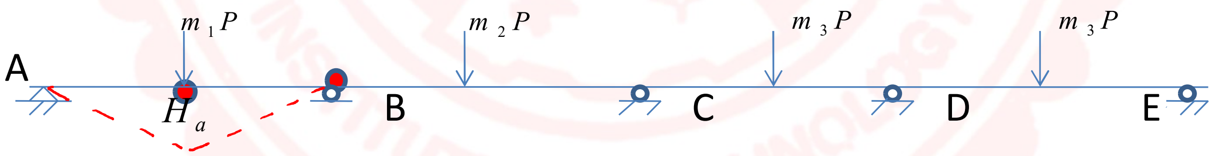
ALTERNATE FOUR MECHANISMS



DSI=3

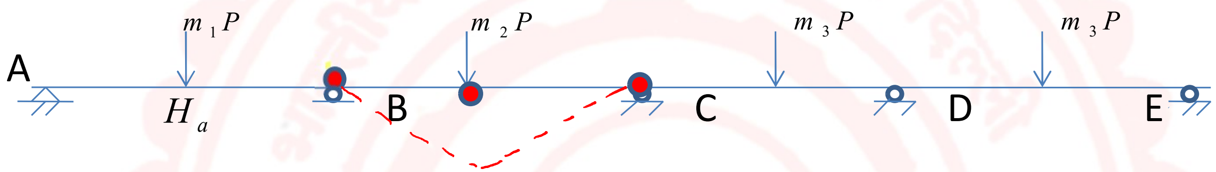
Possible independent mechanisms = 7 – 3 = 4

Mechanism 1

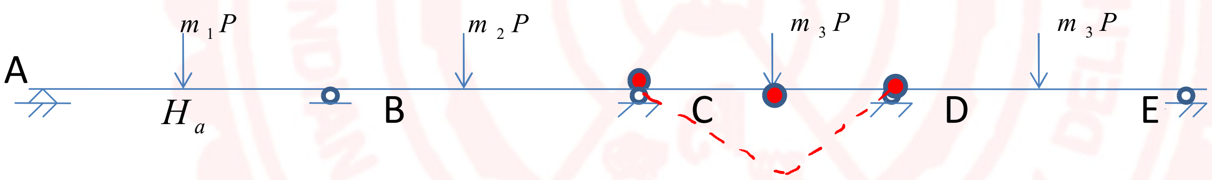


APPLICATION OF SHORT-CUT APPROACH ON COMPLICATED STRUCTURE

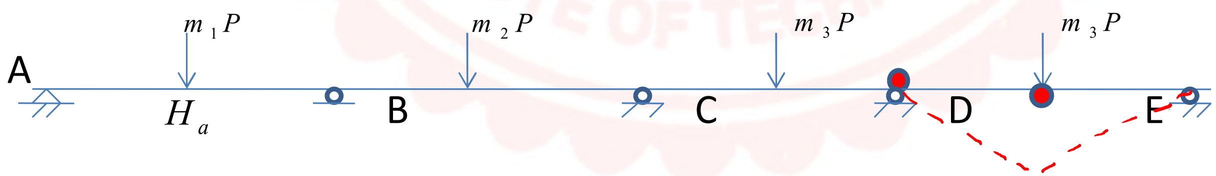
Mechanism 2



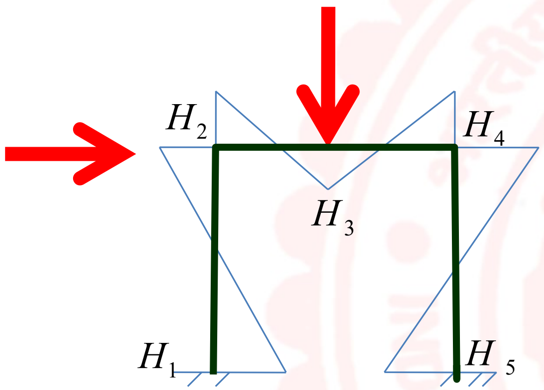
Mechanism 3

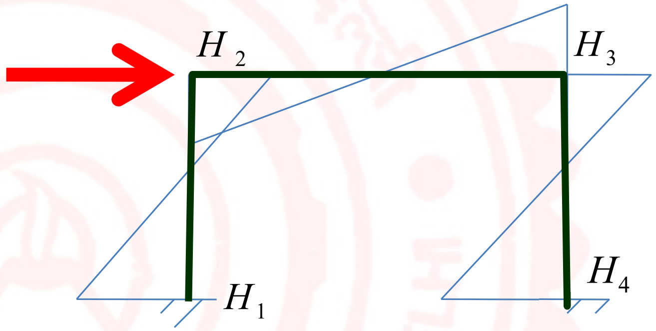


Mechanism 4



2 D FRAMES





$$m = \underset{5}{n} - \underset{3}{r} = 2$$

Beam

Sway

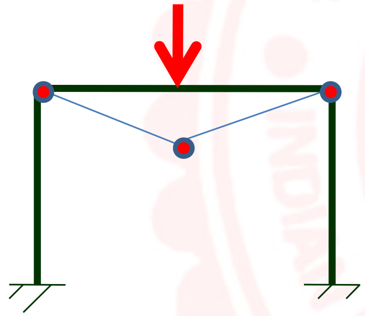
$$m = \underset{4}{n} - \underset{3}{r} = 1$$

2 D FRAMES

Symmetric

Under symmetrical vertical loads

No sway frame



Beam mechanism
(local collapse)

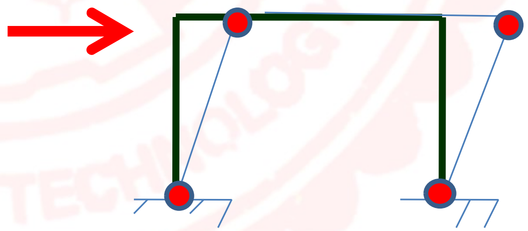
Under horizontal loads or unsymmetrical vertical loading or unsymmetrical structure under vertical loading

Sway frame

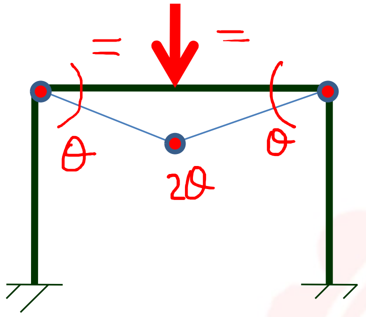
Beam mechanism

Sway mechanism.

Beam + Sway – Combined Mechanism



Sway mechanism
(complete collapse)

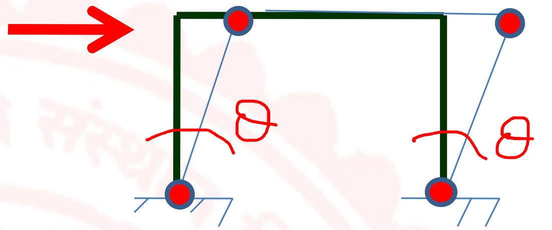


Beam mechanism

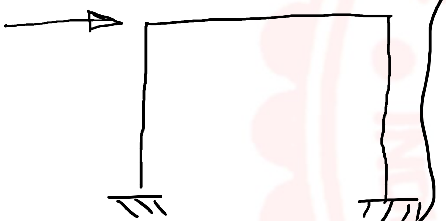
$n = 6$

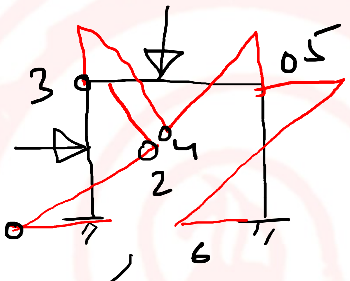
$r = 3$

$m = 3$

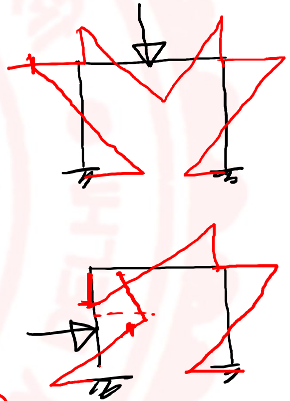


Sway mechanism

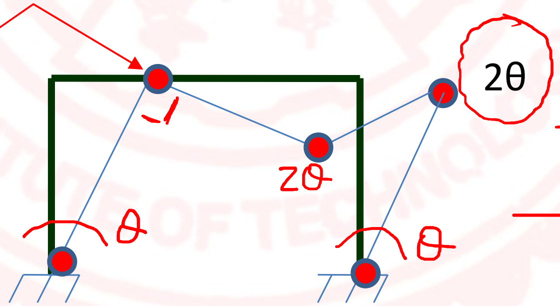


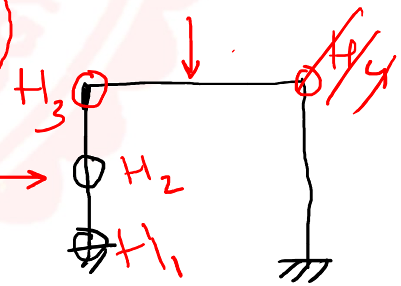


Combined mechanism



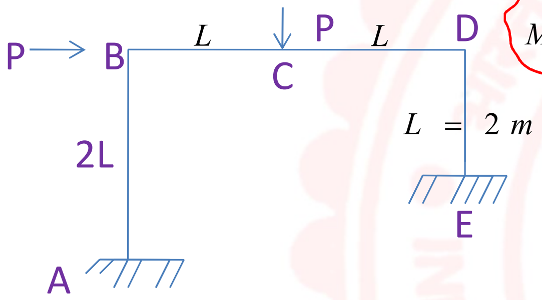
Hinge cancellation





- Beam
- Sway
- Col. hinge mech

EXAMPLE



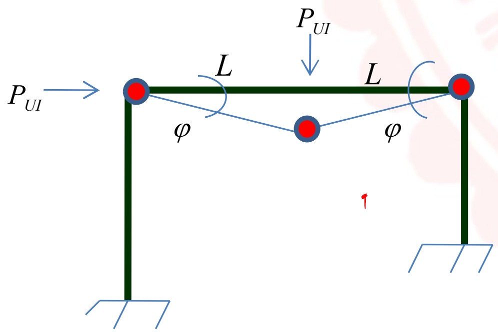
$M_P = 300\ kNm$

$$m = n - r = 2$$

1 Beam
2 Sway
3 Combined.

Independent

<1> BEAM MECHANISM



$$EVW = IVW$$

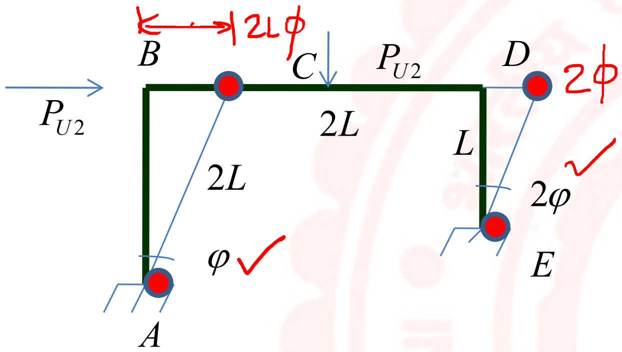
$$P_{UI}\, L\varphi = \underset{B}{\underline{Mp\ \varphi}} + \underset{C}{\underline{Mp\ (2\varphi)}} + \underset{D}{\underline{Mp\ \varphi}}$$

2m

$$P_{UI} = 600\ kN$$

2 D FRAMES

<2> SWAY MECHANISM



$EVW = IVW$

2m 300

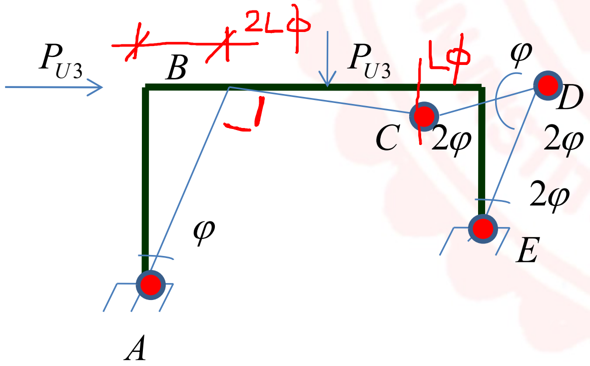
$$P_{U2}(2L\varphi) = (Mp\ \varphi) + Mp\ \varphi + Mp\ (2\varphi) + Mp\ (2\varphi)$$

A B D E

$$2P_U L = 6Mp$$

$$P_{U2} = 450\ kN$$

<3> COMBINED MECHANISM



$EVW = IVW$

Hor Vert.

$$P_{U3}(2L\varphi) + P_{U3}(L\varphi) =$$

$$Mp\ \varphi + Mp\ (2\varphi) + Mp\ (3\varphi) + Mp\ (2\varphi)$$

A C D E

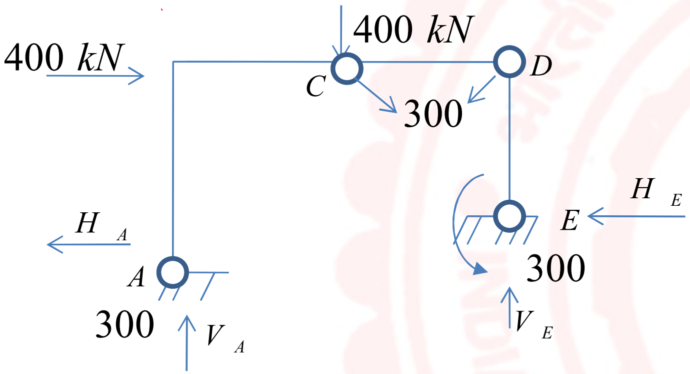
$$3P_{U3}L\varphi = 8Mp\ \varphi \Rightarrow P_{U3} = 400\ kN$$

A C D E

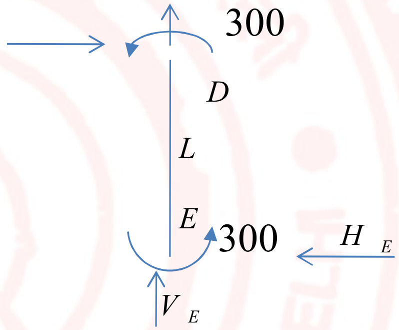
No work done at "B" due to mutual cancellation….

2 D FRAMES

Combined mechanism is the critical (true) mechanism provided equilibrium & yield conditions are satisfied.



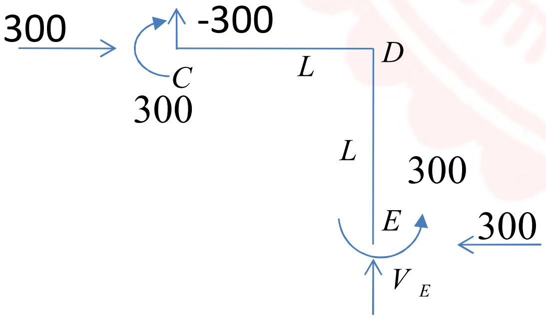
Equilibrium of DE -



$$H_E L = 300 + 300$$

$$H_E = 300 \quad kN$$

Equilibrium of CDE -



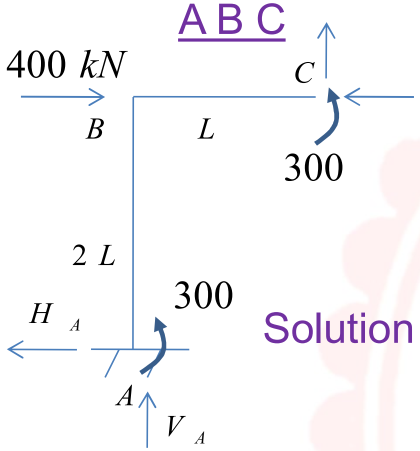
Moment about C –

$$300 L = V_E L$$

$$V_E = 300 \quad kN$$

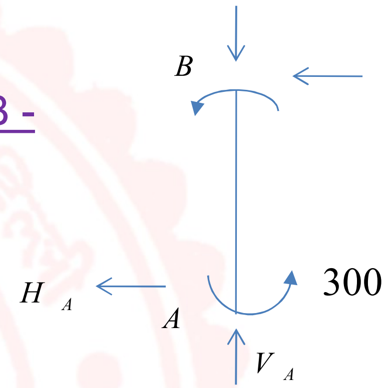
2 D FRAMES

A B C



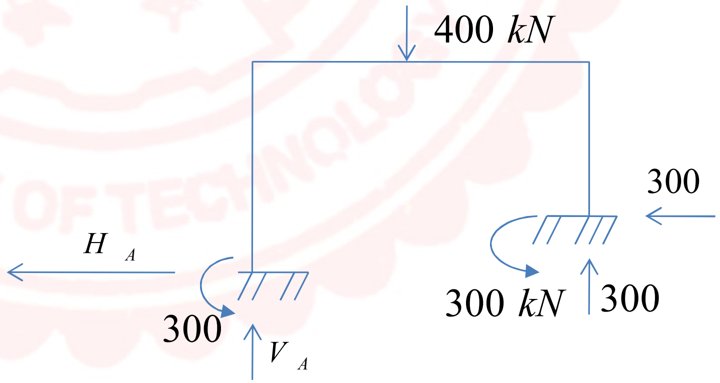
Solution Not possible!

A B -



Cannot find H_A, V_A

Use equilibrium of over all structure

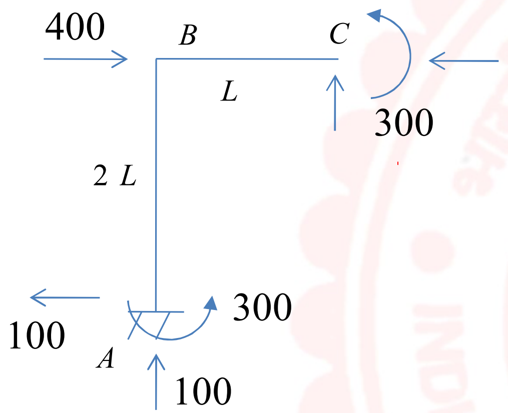


From horizontal /vertical equilibrium -

$$H_A = 100 \ kN$$

$$V_A = 100 \ kN$$

2 D FRAMES



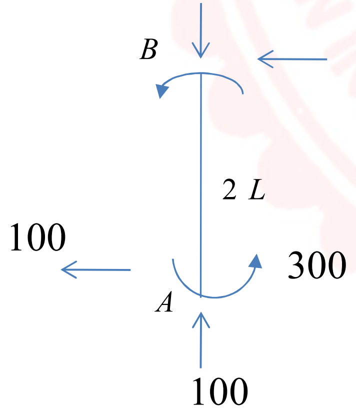
Check equilibrium of ABC

Check moment equilibrium about C

$$M_C = 100 \times L + 100 \times 2L - 300 - 300$$
$$= 0$$

2 m

Yield Check @ B -

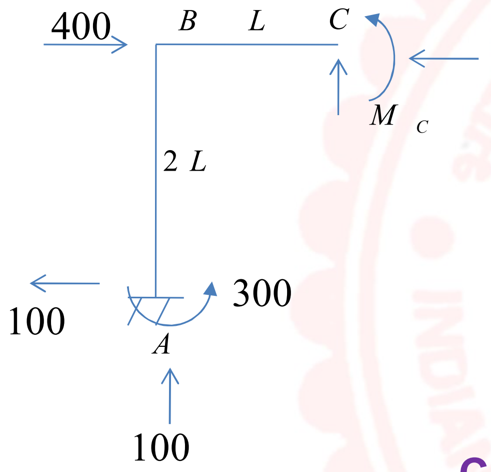


Moment equilibrium satsified.

2 m

$$M_B = 100 \times 2L - 300$$
$$= 100 \ kNm$$
$$< M_p \ ok$$

2 D FRAMES



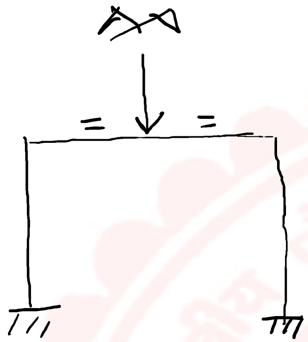
Yield Check @ C -

$$M_C = 100 \times 2L + 100 \times L - 300$$
$$= 400 + 200 - 300$$
$$= 300\ kNm\ (ok)$$

Check : Equilibrium & yield check for other mechanisms.

HOW DOES THE GOVERNING MECHANISM CHANGE WHEN THE STRUCTURE IS UNDER (A) ONLY VERTICAL LOADS (B) ONLY HORIZONTAL LOADS

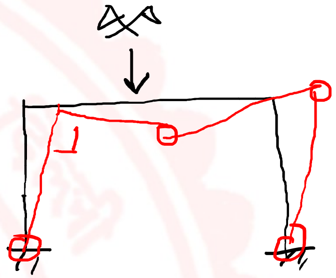
(A) Sway mechanism not possible (B) beam mechanism not possible

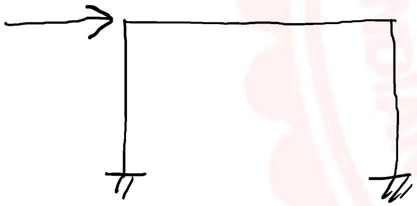


- Beam mech possible
- Sway mech. not possible

No sway frame

or Virtual work

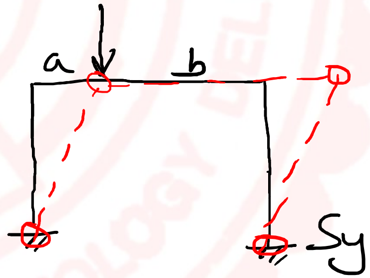




Pure sway mech. possible

Beam mech. not possible

$a \neq b$



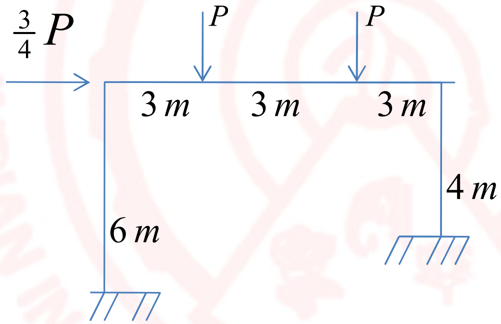
Symm. frame

Beam mech. possible

Sway mech → not possible (virtual work)

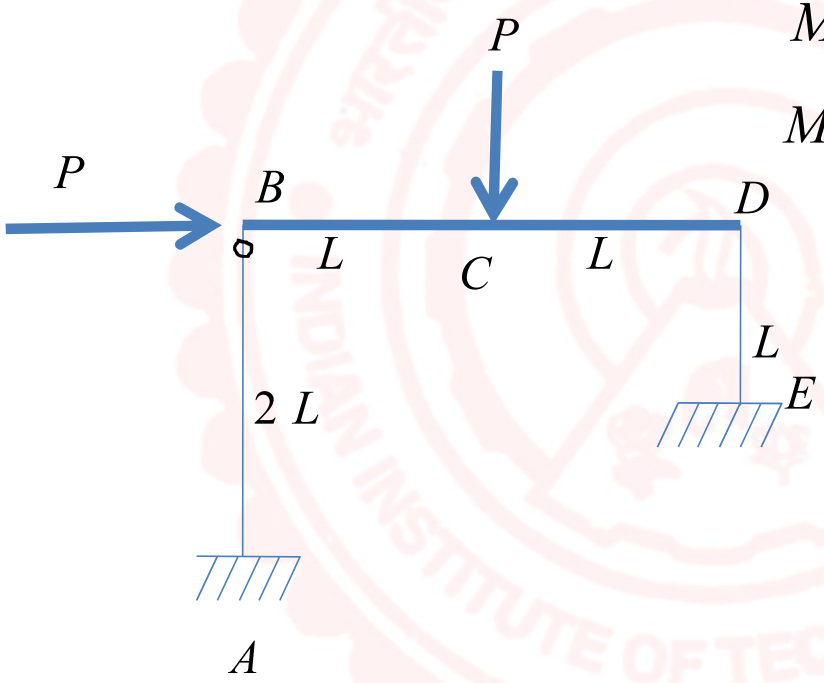
Combined mech. possible

PRACTICE PROBLEM



$$Mp = 300\ kN - m$$

EXAMPLE

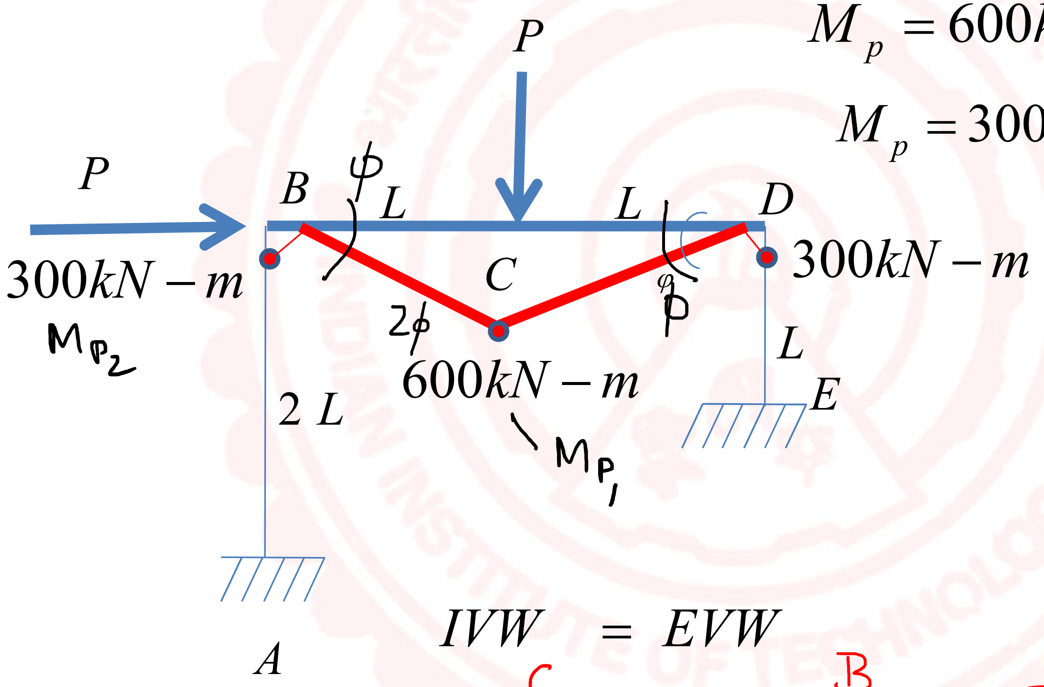


$$M_p = 600kNm(Beam) = M_B$$

$$M_p = 300kNm(Cols) = M_C$$

L = 2m

Beam Mechanism



$$M_p = 600kNm(Beam) = M_{P1}$$

$$M_p = 300kNm(Cols) = M_{P2}$$

L = 2m

$$IVW = EVW$$

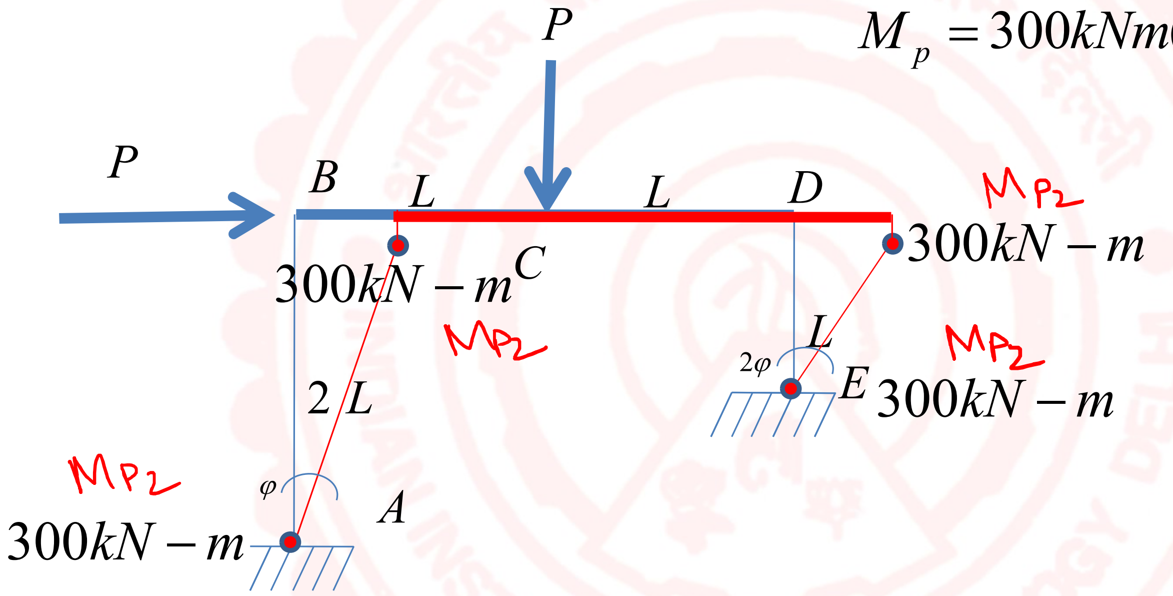
$$\overset{C}{M_{P1}}(2\varphi) + \overset{B}{M_{P2}}\varphi + \overset{D}{M_{P2}}\varphi = P_{U_1} L\varphi$$

$P_{U1} = ??$ 900kN

Sway Mechanism

$$M_p = 600kNm(Beam) = M_{P1}$$

$$M_p = 300kNm(Cols) = M_{P2}$$



$$IVW = EVW$$

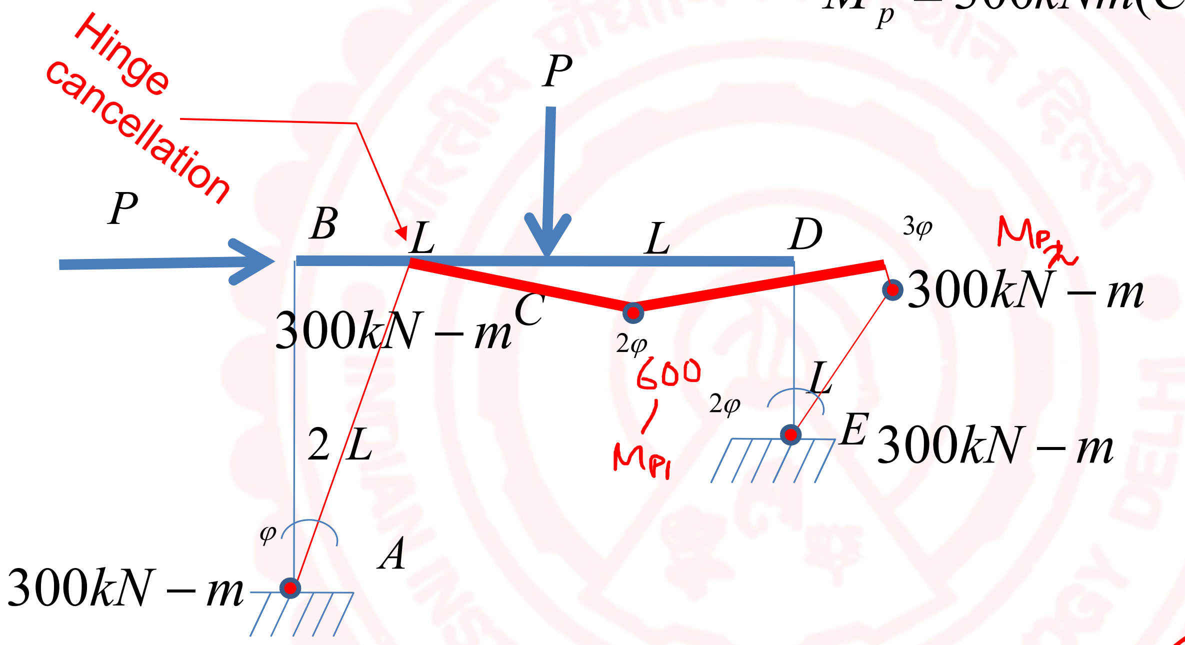
$$\overset{A}{M_{P2}}(\varphi) + \overset{B}{M_{P2}}(\varphi) + \overset{D}{M_{P2}}(2\varphi) + \overset{E}{M_{P2}}(2\varphi) = P_{U2}(2L\varphi)$$

P_{U1} = ?? 450 kN

Combined Mechanism

$$M_p = 600kNm(Beam) = M_{P1}$$

$$M_p = 300kNm(Cols) = M_{P2}$$



$IVW = EVW$

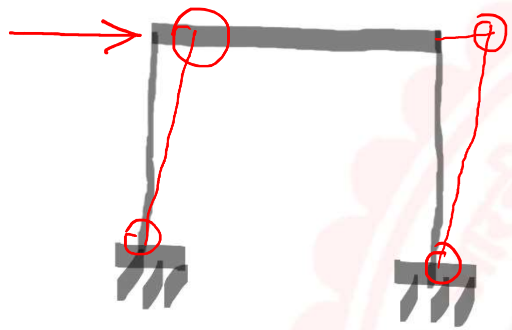
A E D C

$$M_{P2}(\varphi) + M_{P2}(2\varphi) + M_{P2}(3\varphi) + M_{P1}(2\varphi) = P_{U3}(2L\varphi) + P_{U3}(L\varphi)$$

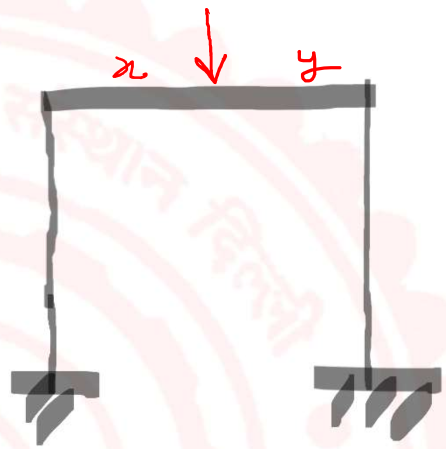
$P_{U3} = ??$ 500

CHECK FOR EQUILIBRIUM AND YIELD CRITERIA

Only horizontal loads



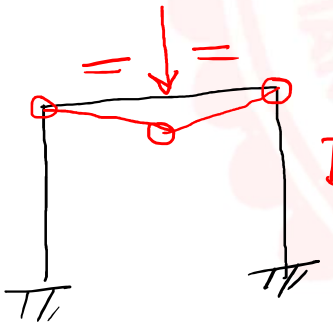
$m = 1$



$m = 2$

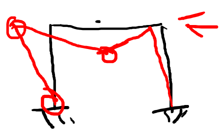
if $x = y$, structure symm.

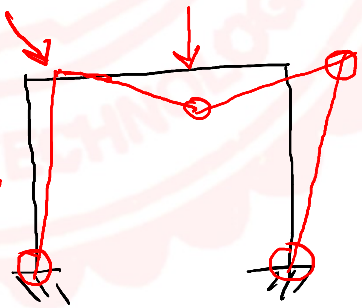
$m = 1$



Beam

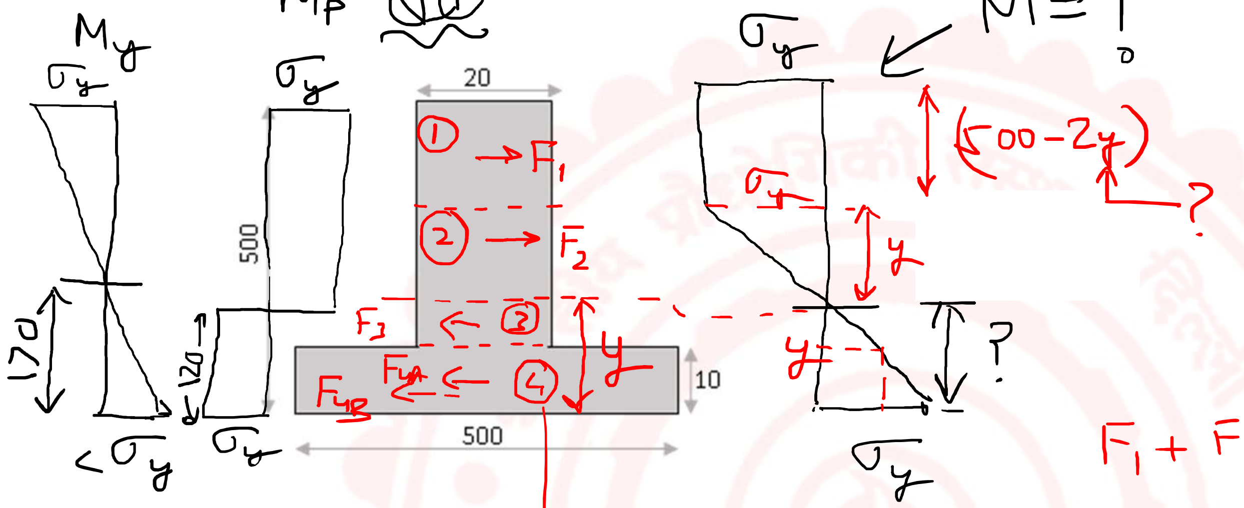
Hinge cancellation





Possibility of sway mech. under vertical loads is low.

M_p Q1



$M^* = ?$

2 mks

$F_1 + F_2 = F_3 + F_{4A} + F_{4B}$

Q2

4A uniform

4B Δ

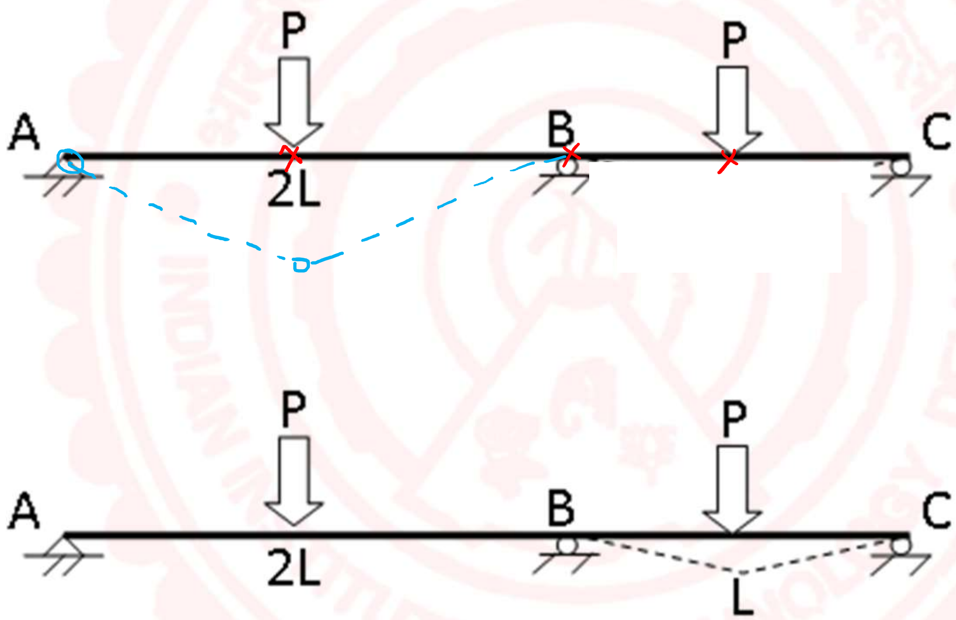
$y = (115, \cancel{\ldots})$?? Not valid

3 mks

$M^* = 502$ kNm

3 mks

EXAMPLE: To check for a mechanism to be critical



$n = ? = 3$

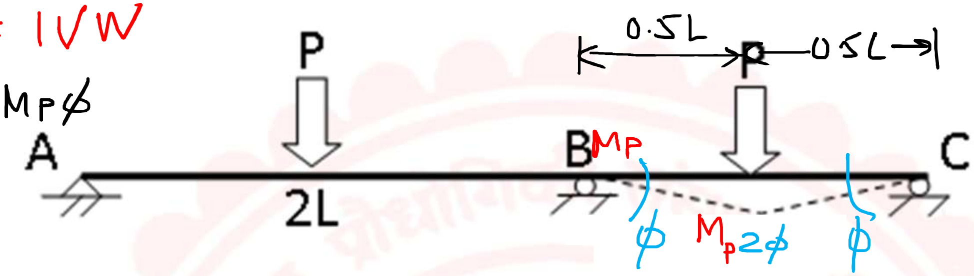
$r = ? = 1$

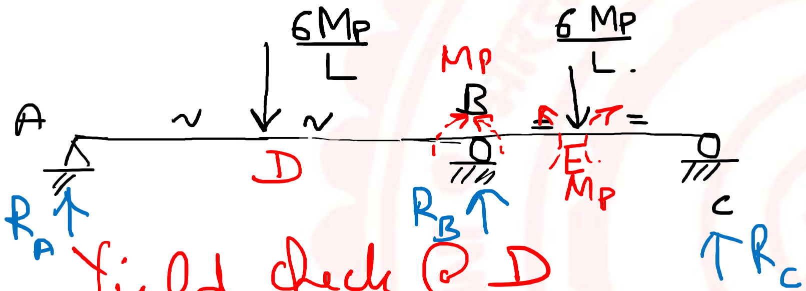
$m = 2$

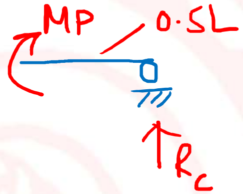
$EVW = IVW$

$$\frac{1}{2}P_u L\phi = 3M_p\phi$$

$$P_u = \frac{6M_p}{L}$$

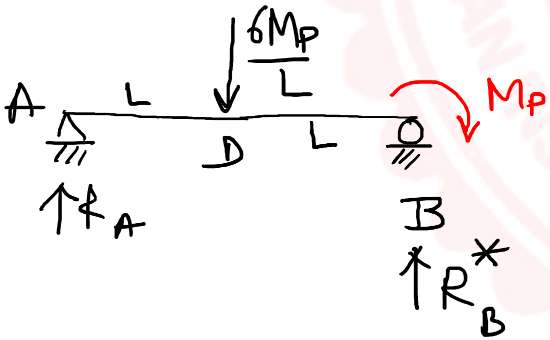






$$R_c = \frac{2M_p}{L}.$$

Yield check @ D



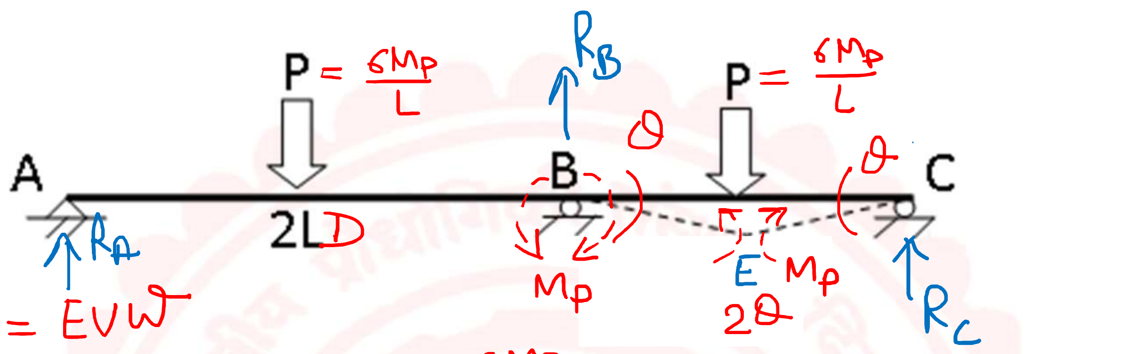
$$R_A \times 2L + M_p = \frac{6M_p}{L} \times L$$

$$R_A = \frac{2.5M_p}{L}$$

Vertical equilibrium of whole structure -

$$R_B = 7.5\left(\frac{M_p}{L}\right)$$

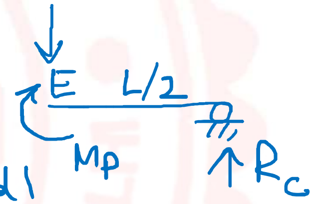
Internal BM@D = 2.5 Mp Yield criteria not satisfied.



IVW = EVW

$M_p\theta + M_p 2\theta = P_u \frac{L}{2}\theta \Rightarrow P_u = \frac{6M_p}{L}$

Lets do yield criteria check: (@ D)

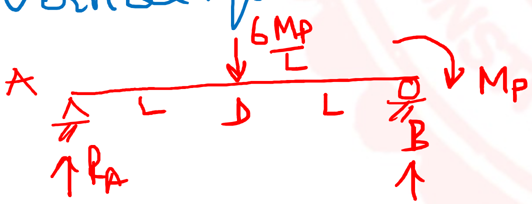


$R_C = \frac{2M_p}{L}$

R_A ? R_B ? Two equations needed!

Vertical equilibrium $\Rightarrow$ $R_A + R_B + R_C = 2P_u = \frac{12M_p}{L}$

$\Rightarrow R_A + R_B = 10\left(\frac{M_p}{L}\right)$



$R_A \times 2L + M_p = \frac{6M_p}{L} \times L \Rightarrow R_A = 2.5\left(\frac{M_p}{L}\right)$

$\Rightarrow R_B = 7.5\left(\frac{M_p}{L}\right)$

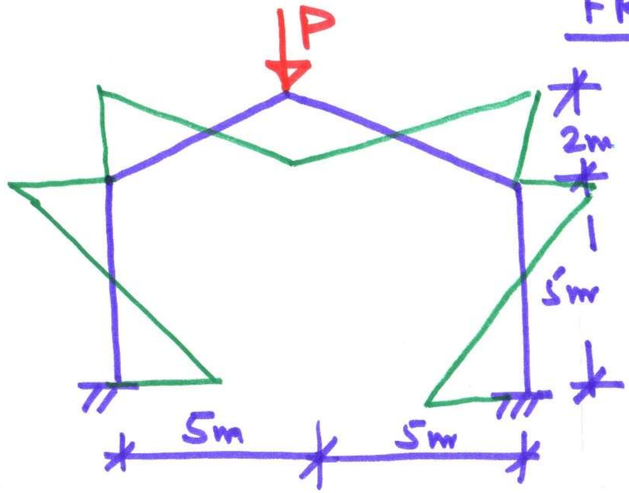
Check BM @ D — $2.5 M_p$ — Yield criteria not satisfied

$\Rightarrow$ look for other mech.

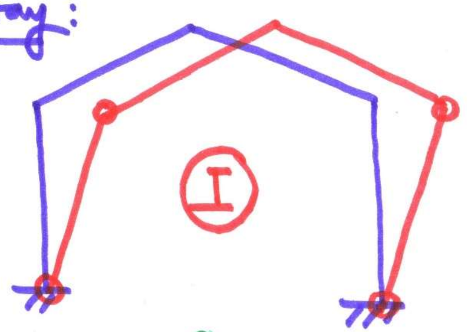
INCLINED PORTAL FRAMES

$$m = \underset{5}{n} - \underset{3}{r}$$

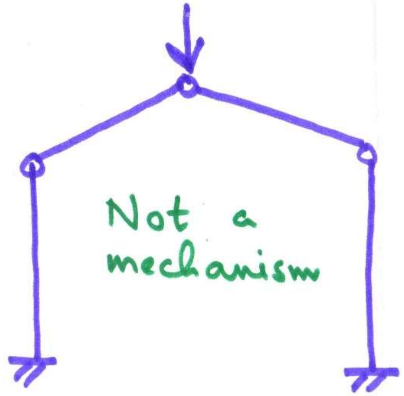
$$= 2$$

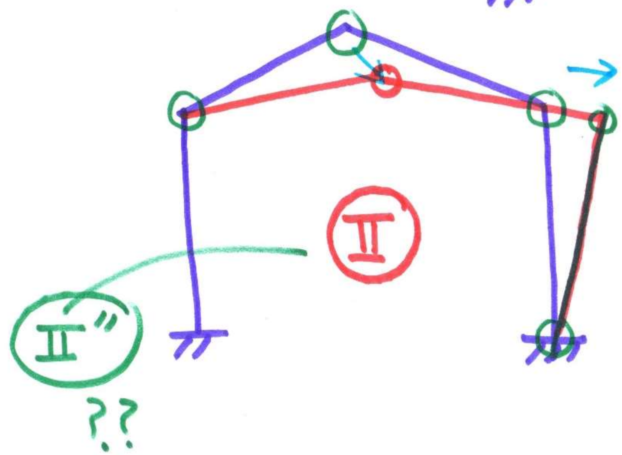


Sway:



Beam:



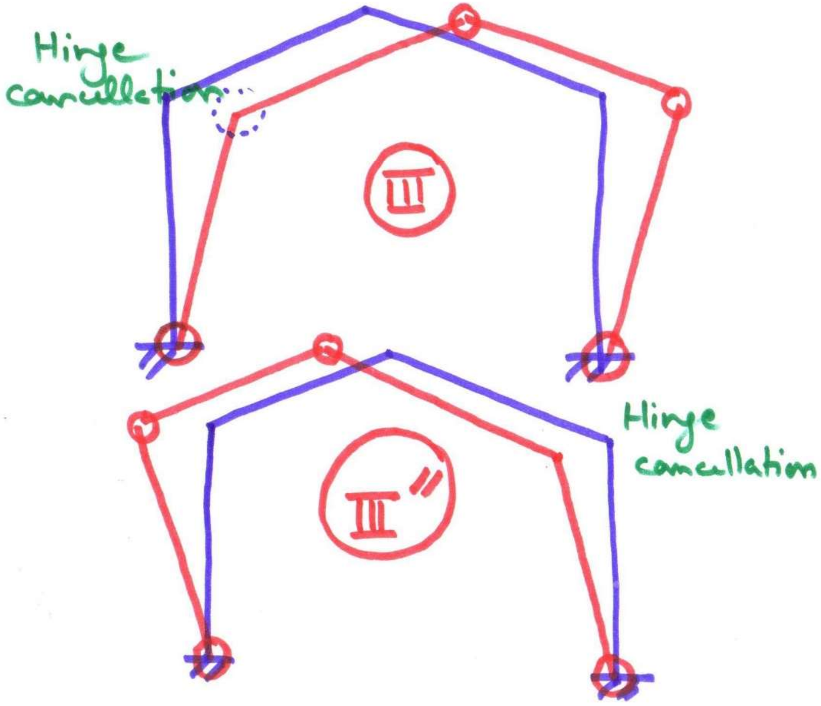


Superimpose

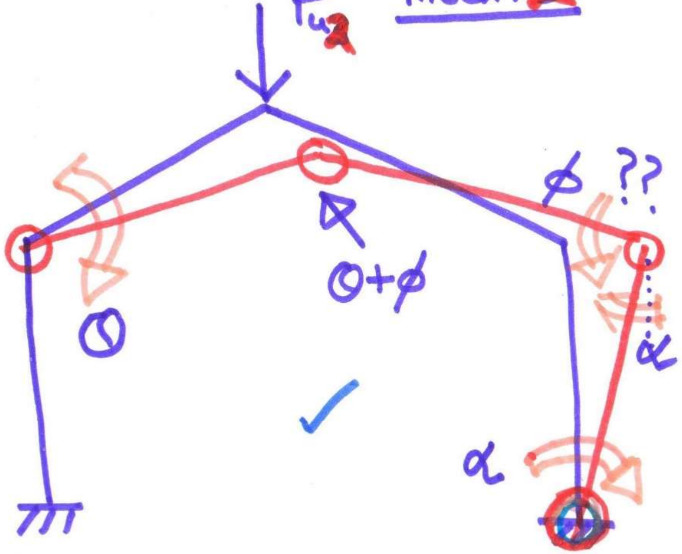
I & II

Combined mechanism

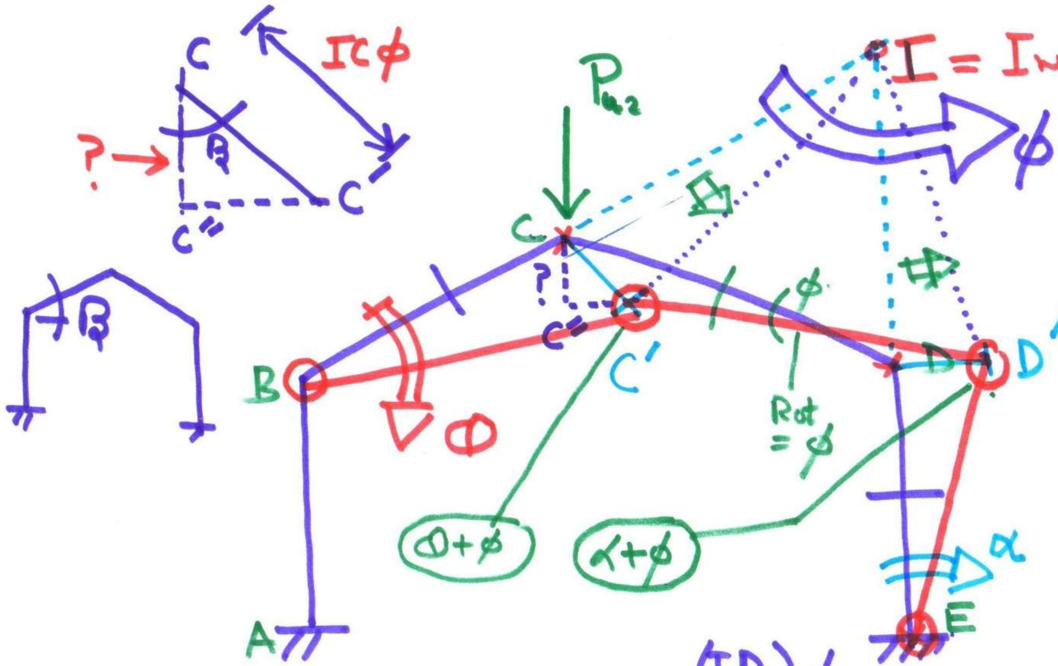
III



Mech. 2



1. All members are inextensible
2. Instantaneous centre of rotation.



I = Instantaneous centre of rotation

Apply rotation @ I such that Δ ICD undergoes rigid ~~by~~ body movement.

D — moves $\perp$ DE

C — moves $\perp$ BCI

$$DD' = ID\phi = ED\alpha \Rightarrow \alpha = \left(\frac{ID}{ED}\right)\phi$$

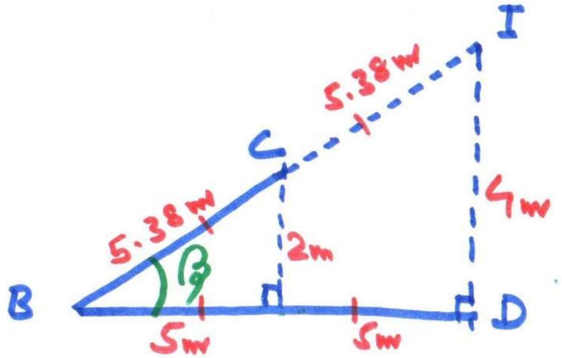
$$CC' = BC\Phi = IC\phi$$

$$\Phi = \left(\frac{IC}{BC}\right)\phi$$

CD undergoes rotation $= \phi$

ED — " " $= \alpha$

BC — " " $= \Phi$



$$\alpha = \left(\frac{ID}{ED}\right)\phi$$

$$\alpha = \frac{4}{5}\phi \quad \text{———} \quad \langle 1 \rangle$$

$$\theta = \left(\frac{IC}{BC}\right)\phi = \left(\frac{5.38}{5.38}\right)\phi$$

$$\theta = \phi \quad \text{———} \quad \langle 2 \rangle$$

Refer to $\Delta CC'C''$

BC

$$CC'' = IC\phi \cos\beta$$

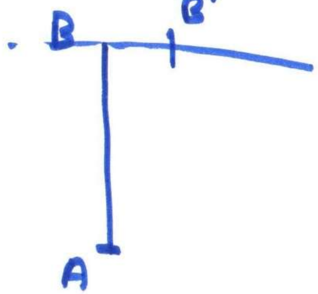
$$CC'' = 5\phi$$

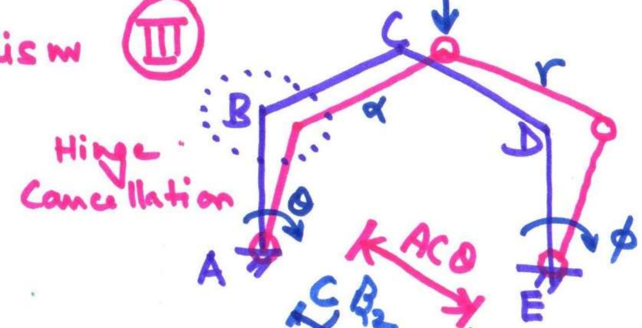
External V.W = Internal VW

$$P_{u2}\underbrace{CC''}_{5\phi} = \underbrace{M_p\theta}_{B} + \underbrace{M_p(\theta+\phi)}_{C} + \underbrace{M_p(\alpha+\phi)}_{D} + \underbrace{M_p\alpha}_{E}$$

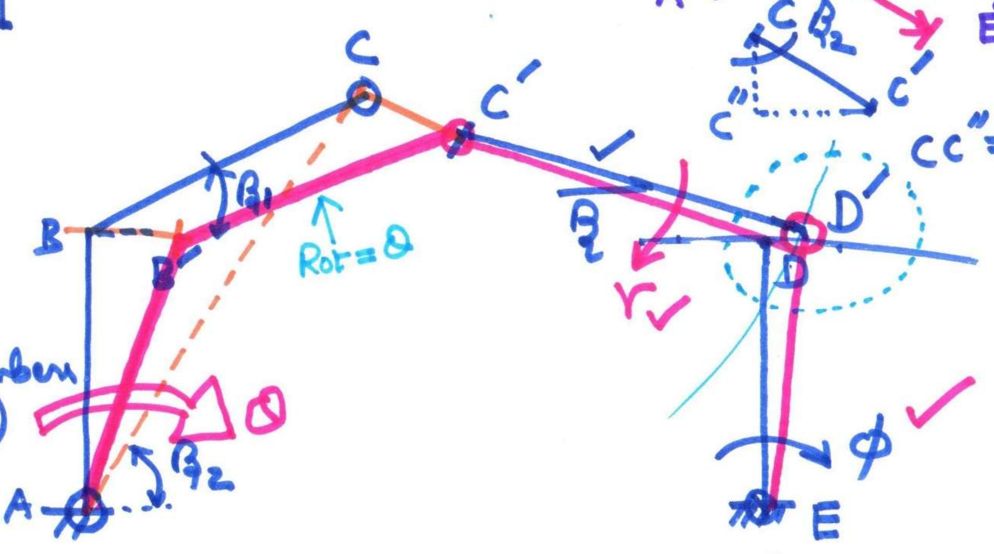
Solving $P_{u2} = 1.12\,M_p$

Mechanism III





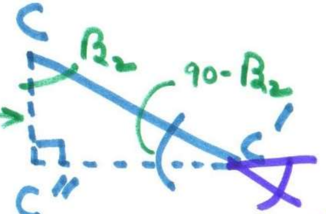
Consider ABC (disconnect other members temporarily)



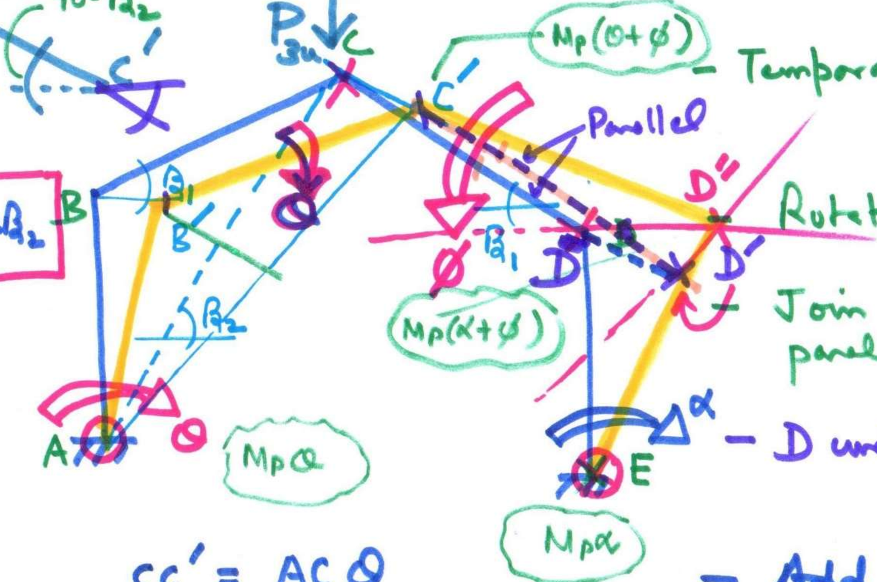
$CC'' = AC\theta \cos\beta_2$

Apply a small virtual rotation θ @ A

MECHANISM III



$CC'' = AC\theta \cos\beta_2$

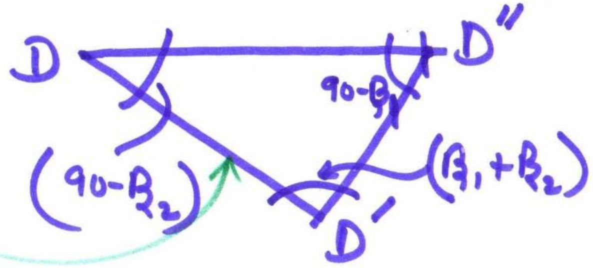


- Temporarily disconnect structure @ C, D
- Rotation θ about A
- Join CD such that it is parallel to earlier position
- D undergoes $DD' = CC'$ (both disp. & dir.)
- Add DE but need to find final location of D
- Located D'' final position of D
- Arc of ED with E = ⊙ & C'D' with C' = ⊙
- ΔABC is rigid undergoes rotation θ

$$CC' = AC\,\theta$$

$$\alpha = \frac{DD''}{ED} \qquad \phi = \frac{D'D''}{CD}$$

CC'' known



$$\frac{DD''}{\sin(\beta_1+\beta_2)} = \frac{D'D''}{\cos\beta_2} = \frac{DD'}{\cos\beta_1}$$

$$\frac{ED\alpha}{\sin(\beta_1+\beta_2)} = \frac{CD\phi}{\cos\beta_2} = \frac{AC''' \cancel{\cos\beta_2}\,\theta}{\cos\beta_1}$$

$$\alpha = \left[\frac{AC''' \sin(\beta_1+\beta_2)}{ED\cos\beta_1}\right]\theta$$

$$\phi = \left[\frac{AC\cos\beta_2}{CD\cos\beta_1}\right]\theta$$

Solve and obtain $P_{u3} = 1.5M_p$

Upper bound thm

$$P_u = \text{Smaller of } (P_{u2} \text{ \& } P_{u3})$$

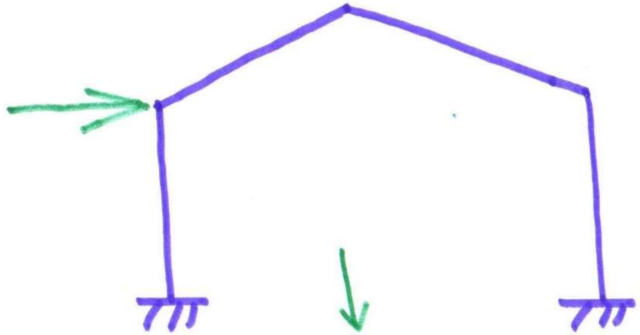
[eliminating sway mech. I]

Equilibrium & yield check to be done to rule out any mechanism having been missed out.

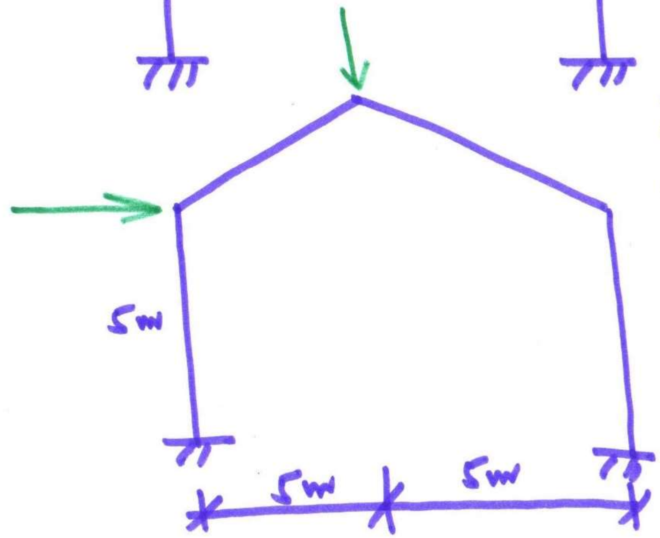
<H.W>

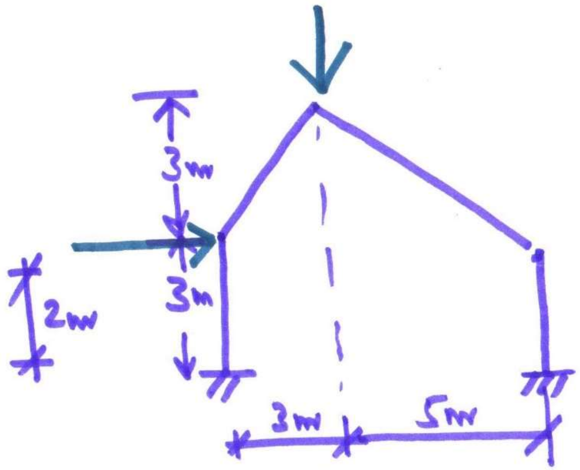
Practice Problems

I



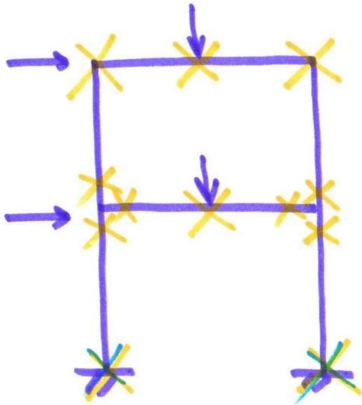
II



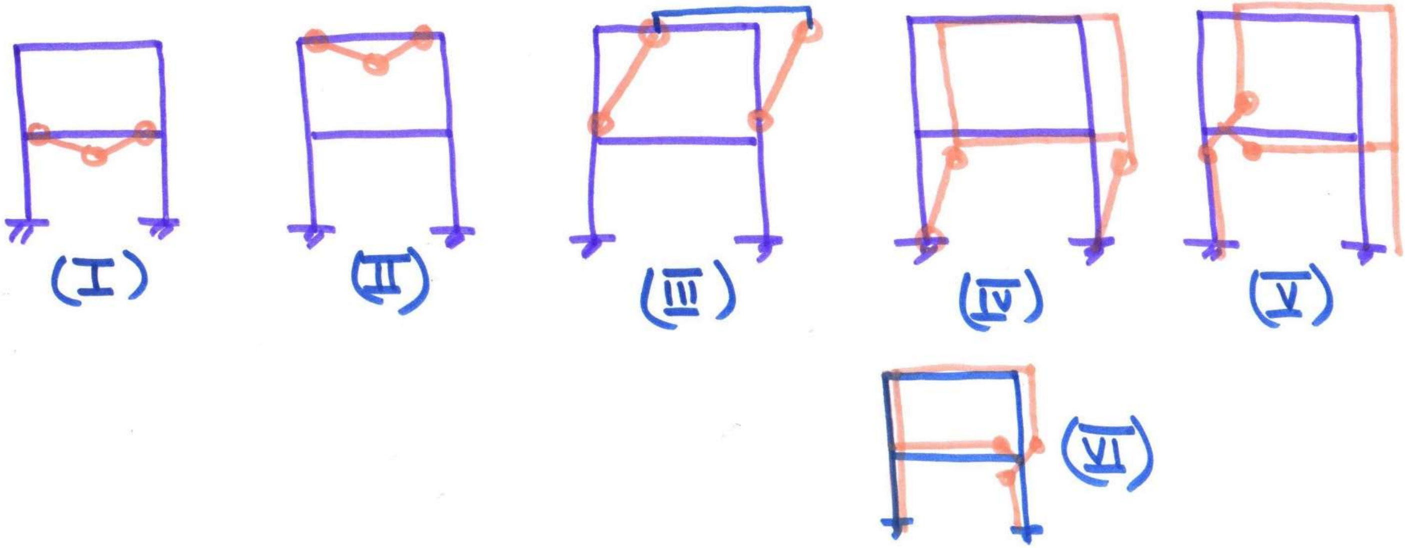


III

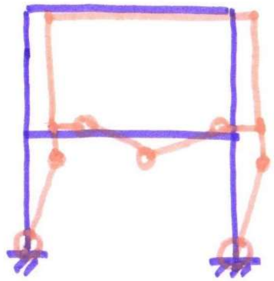
TWO STOREY FRAME



$$m = \underset{12}{n} - \underset{6}{r} = 6$$



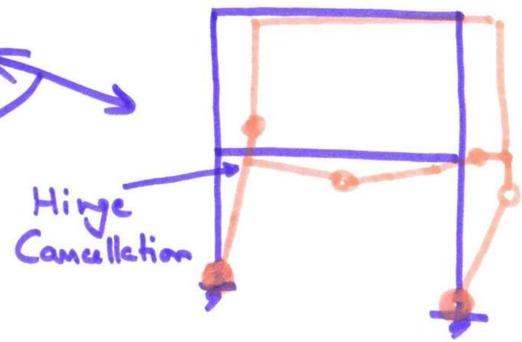
Combined mechanism $\overline{VII} = (I) + (\overline{IV})$



$\overline{VIII} = \overline{VII} + \overline{V}$

$\uparrow$ Joint mechanism

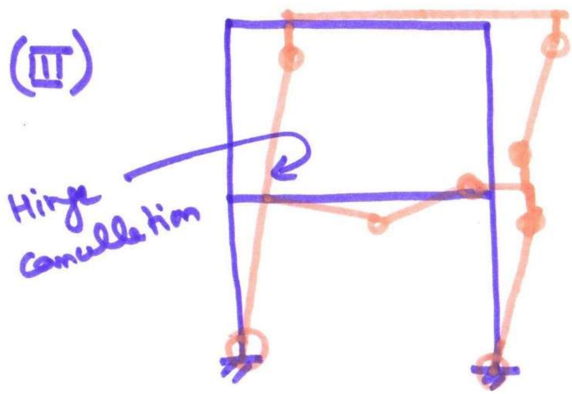
$\Downarrow$



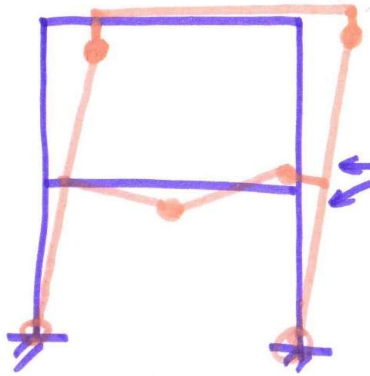
which to have lower P_u

Intuition tells one having lesser hinges

$(\overline{IX}) = (\overline{VIII}) + (\overline{III})$

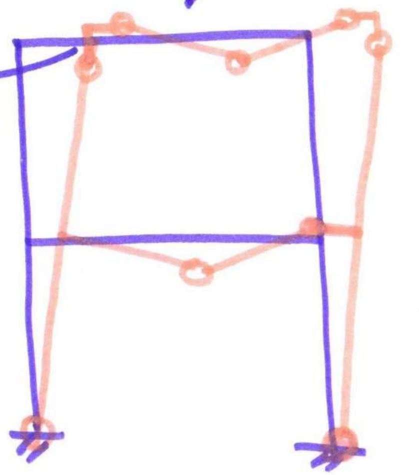


$(X) = (IX) + (VI)$

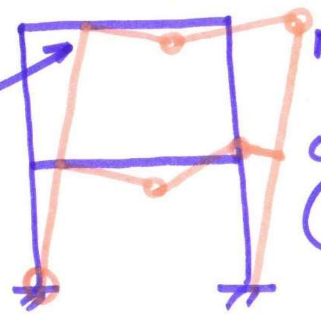


Hinge Cancellation

$(XI) = (X) + (II)$

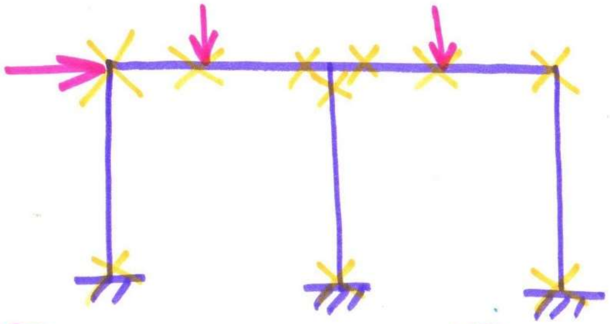


- Hinge will be either in beam / col.
- Try merging these segment length → 0



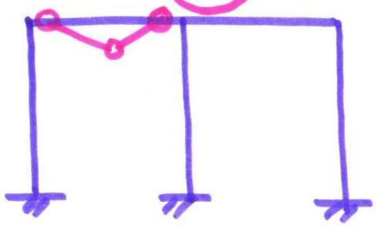
Hinge Cancellation

No cancellation (Irrespective of strength of beam or column)

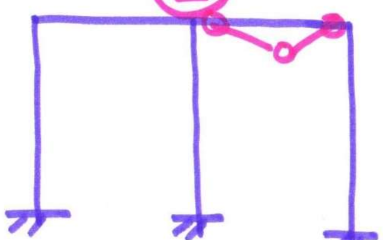


$$m = \underbrace{n}_{10} - \underbrace{r}_{6} = 4$$

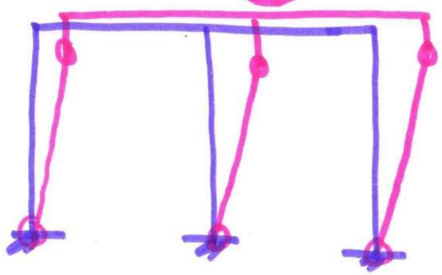
I

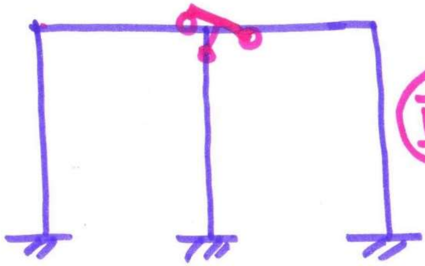


II



III

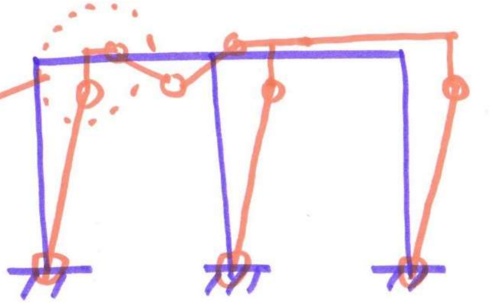




IV

(V) = (I) + (III)

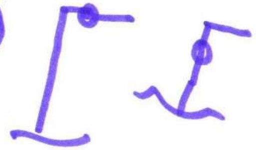
Hinge Cancellation.



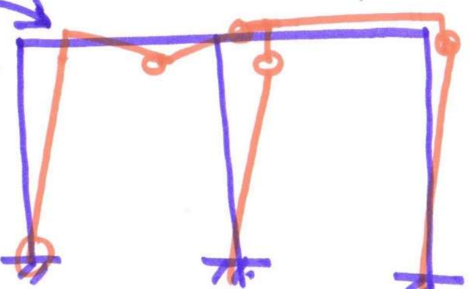
1 Merging

2 Rotation

3



Hinge Cancellation



VI = (V) + (II)