STRUCTURAL ANALYSIS:
REVIEW OF BASIC CONCEPTS

http://web.iitd.ac.in/~sbhalla/cvl756.html

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SB1

METHODS OF ANALYSIS

FORCE METHODS

Forces are unknowns

DISPLACEMENT METHODS

Displacements are unknowns

- Method of consistent deformations

Force method — Degree of Static Indeterminacy (DSI)

- Slope deflection method
- Matrix Stiffness method
- Direct stiffness method for computer applications
- Moment distribution method

Displacement methods — Degree of Kinematic Indeterminacy (DKI)

Slide 2

SB1 Suresh Bhalla, 26-09-2020

STABILITY AND DETERMINACY

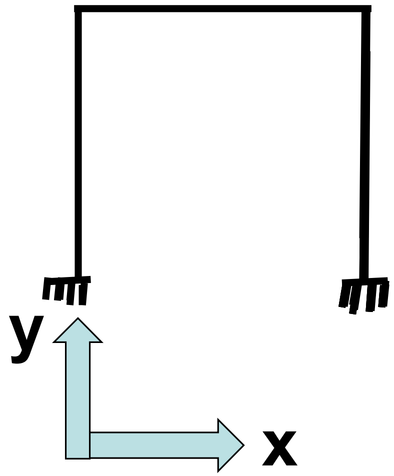
NECESSARY AND SUFFICIENT CONDITIONS

For overall stability (2D STRUCTURE):

(1) $r >= 3$ — r = No. of reactions

$$(\sum F_x = 0, \sum F_y = 0, \sum M_z = 0)$$

(2) No geometric instability



For 3D structure :

(1) $r >= 6$ $(\sum F_{x,y,z} = 0, \sum M_{x,y,z} = 0)$

(2) No geometric instability

FORCE METHOD

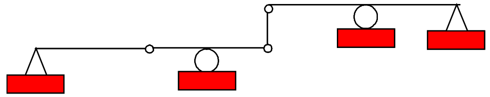
DEGREE OF STATIC INDETERMINACY (DSI)

It is the number of unknown forces, over and above the minimum required, to satisfy the conditions of equilibrium and stability for a structure.

DSI = 1

STABILITY AND DETERMINACY (BEAMS)

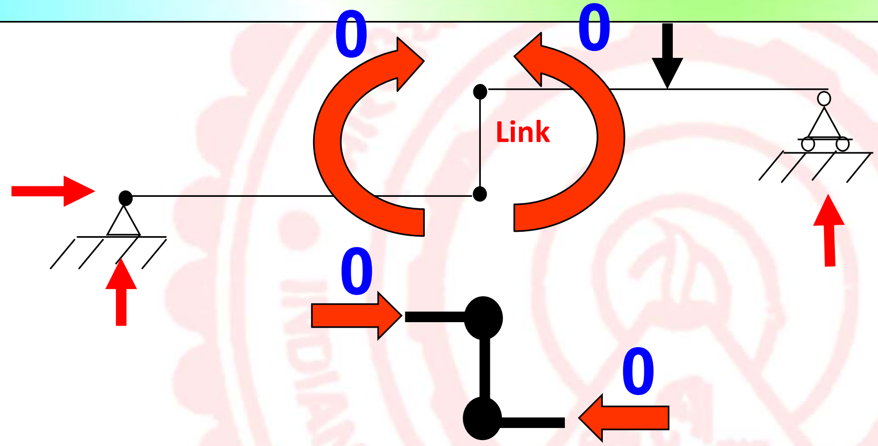
r = No. of reactions,
c = Conditions of construction



$r < c + 3$	Statically unstable
$r = c + 3$	Statically determinate provided no geometric instability .
$r > c + 3$	Statically indeterminate provided no geometric instability

UNKNOWNS VS NUMBER OF EQUATIONS

EXAMPLE 1



r Vs c + 3

3 2

c +3 = 5

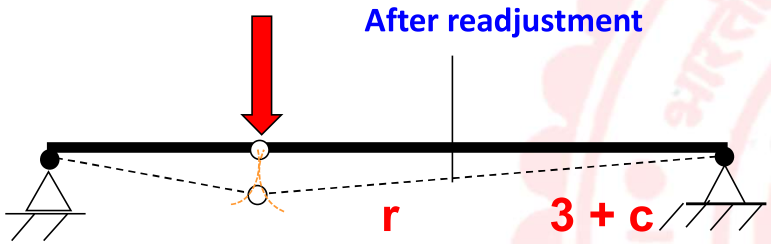
$r < 3 + c$

Hence unstable

Two equations of conditions :

1. Cannot resist moment at link.
2. Link is 2 force element, therefore cannot resist forces perpendicular to link.

EXAMPLE 2



r 3 + c

4 3 + 1 = 4

r = 3 + c

Each element behaves as link element. Therefore, cannot resist any force normal to itself……..Both members shall rotate about respective hinges….essentially large radius

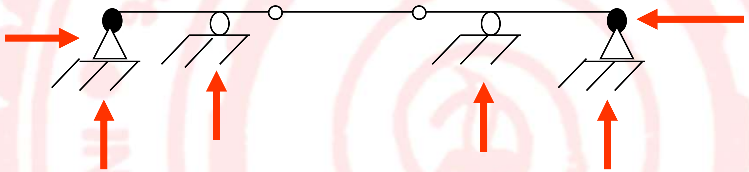
Stable and determinate ????

Number of reactions are adequate but the beam is still unstable <u>not due to inadequate arrangement of supports but an instability within the structure</u>

Hence, this is called as <u>internal geometrical instability</u>

Note: Structure will undergo large inelastic deformation but total collapse

EXAMPLE 3



r 3 + c

6 3 + 2 = 5

r > 3 + c

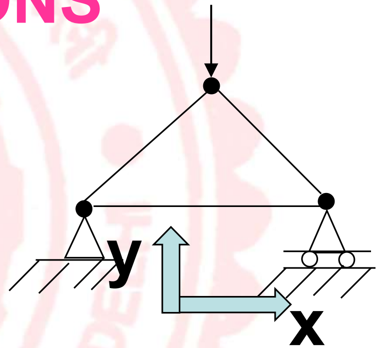
ANY GEOMETRIC INSTABILITY ??

Stable and indeterminate to first degree

STABILITY AND DETERMINACY (TRUSS)

UNKNOWNS VS NUMBER OF EQUATIONS

b = No of bars, r = no of reactions, j = no of joints



Unknowns = b+r

Equations = 2j

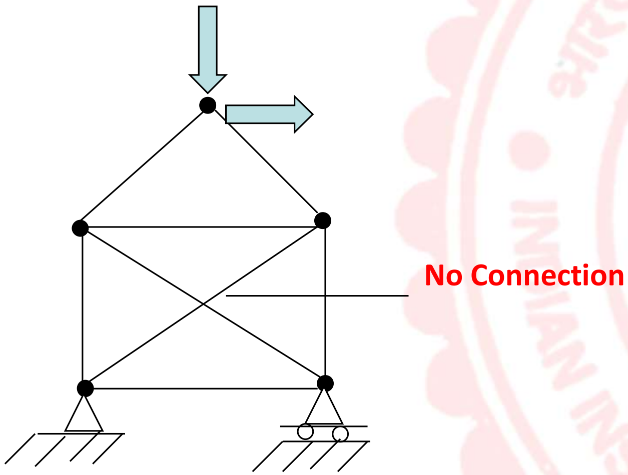
$b+r < 2j$ — Statically unstable

$b+r = 2j$ — Statically determinate provided no geometric instability

$b+r > 2j$ — Statically indeterminate provided no geometric instability

$\sum M_z$ not independent

EXAMPLE 4



b + r   2j

8 + 3   2x5

11   >   10

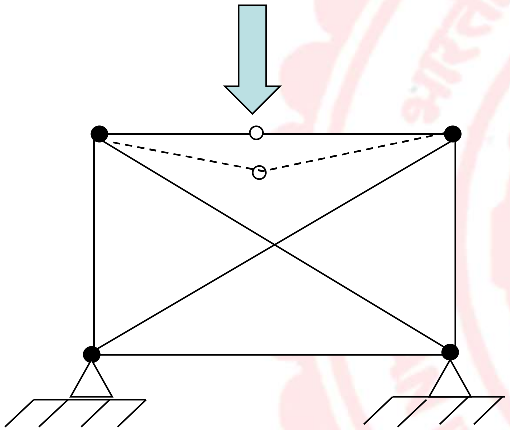
ANY GEOMETRIC INSTABILITY ??

Stable and indeterminate to first degree.

How to make stable and determinate????

Remove one bar only such that truss action not disturbed

EXAMPLE 5



b + r 2j

7 + 4 2x5

11 >10

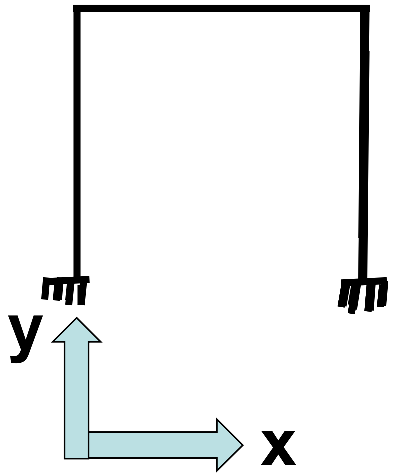
Stable and determinate ????

No…. Structure has
Internal Geometrical Instability...

Rework converting the right support into roller

STABILITY AND DETERMINACY (RIGID FRAMES)

b = No of elements, r = no of reactions, j = no of joints,
c = No of conditions of construction



UNKNOWNS VS NUMBER OF EQUATIONS

$3b+r < 3j+c$ Statically unstable

$3b+r = 3j+c$ Statically determinate provided no geometric instability

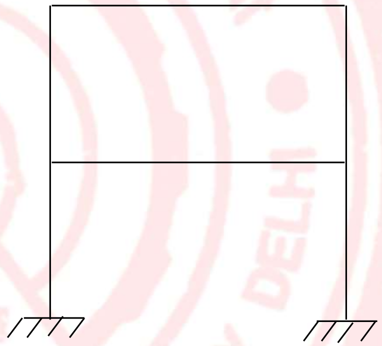
$3b+r > 3j+c$ Statically indeterminate provided no geometrical instability

Unknowns = $3b+r$

Equations = $3j+c$

EXAMPLE 6

3b+r		3j + c
3x6+6		3x6+0
24		18
3b+r	>	3j + c



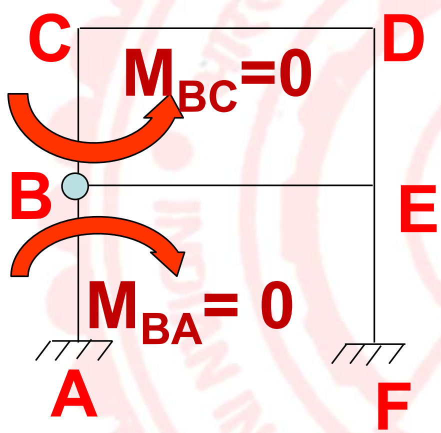
Stable and indeterminate to 6th degree

ANY GEOMETRIC INSTABILITY ??

EXAMPLE 7

3b+r VS 3j+c

Hinge



M_{BE} = 0 not required since $\sum M_B = 0$ (overall moment equilibrium of joint) is already considered and M_{BE} = 0 automatically results

Independent conditions of construction:

$M_{BC} = 0$, $M_{BA} = 0$. i.e **c = 2**

3b+r	3j + c
3x6+6	3x6+2
24	20
3b+r >	3j + c

ANY GEOMETRIC INSTABILITY ??

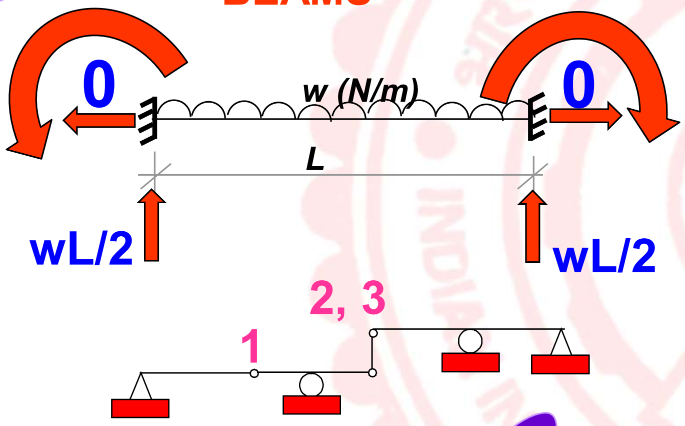
Stable and indeterminate to 4th degree

FORCE METHOD

3 (1)

First step: Degree of static indeterminacy (DSI)

BEAMS



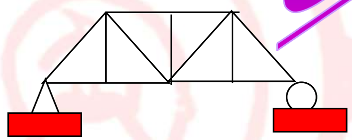
0

Unknowns = r

Equations = 3+c

TRUSSES

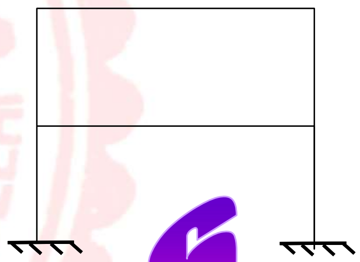
0



Unknowns = b+r

Equations = 2j

FRAMES



6

Unknowns = 3b+r

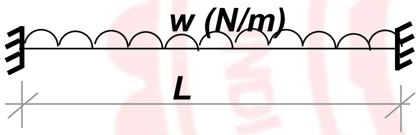
Equations = 3j+c

r = No. of reactions, c = Conditions of construction,

b = No of bars/ members, j = No. of joints

FORCE METHOD

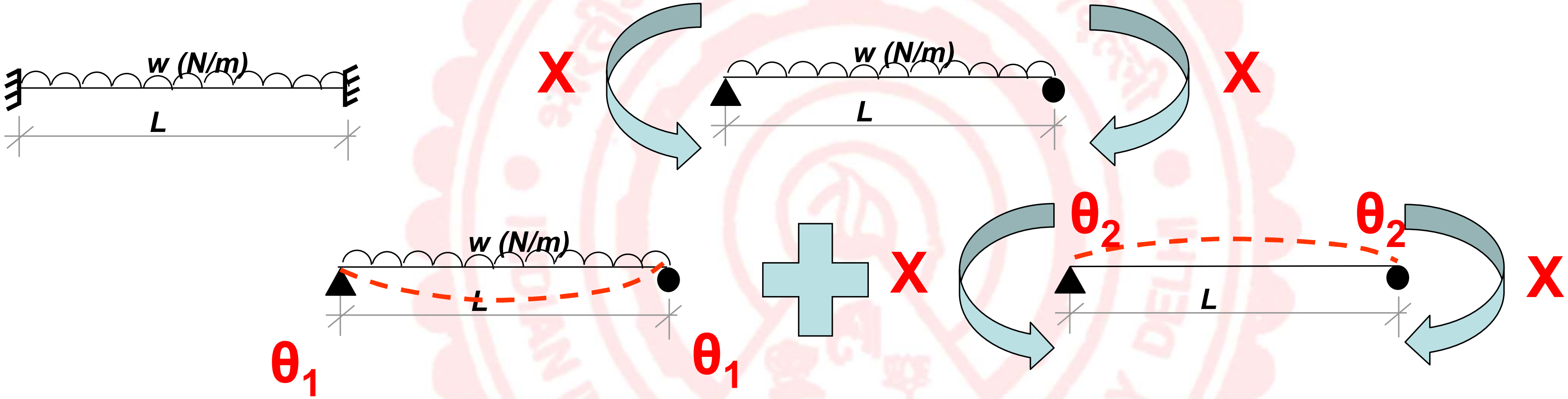
Redundant forces and reaction are unknowns



HOW TO ANALYSE???

- Choose redundants
- Form compatibility and equilibrium equations
- Solve

FORCE METHOD



Compatibility condition: $\theta_1 = \theta_2$

This enables determination of redundant "X"

Unknown reactions obtained by using Equilibrium condition.

DISPLACEMENT METHOD

Displacements are considered as unknowns

Degree of kinematic indeterminacy (DKI)

No. of independent displacements (degrees of freedom) possessed by the structure.

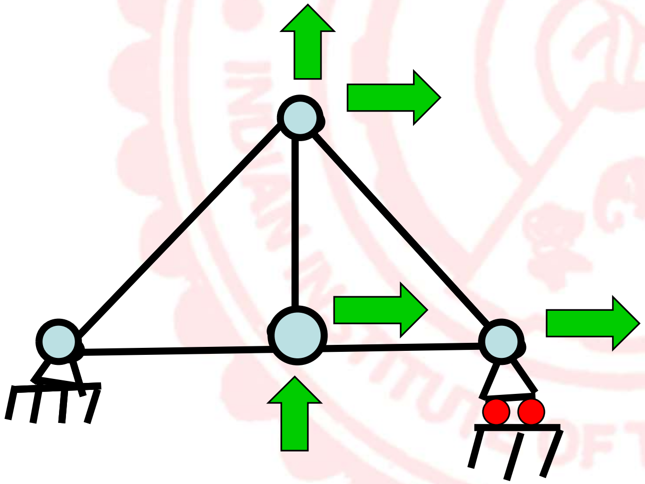
All other displacements can be expressed in terms of these key displacements.

DEGREE OF KINEMATIC INDETERMINACY (DKI): TRUSSES

PLANE TRUSS: DKI = 2j – r

j = No. of joints

r = No. of reactions

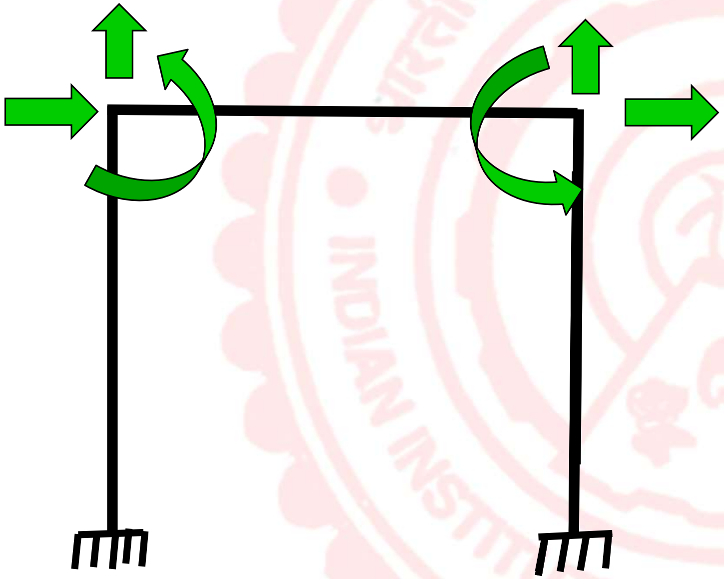


DKI = 2x4 - 3

= 5

SPACE TRUSS: DKI = 3j – r

DEGREE OF KINEMATIC INDETERMINACY (DKI): 2D FRAMES



PLANE FRAME: **DKI = 3j – r**

DKI = 3x4 - 6

= 6

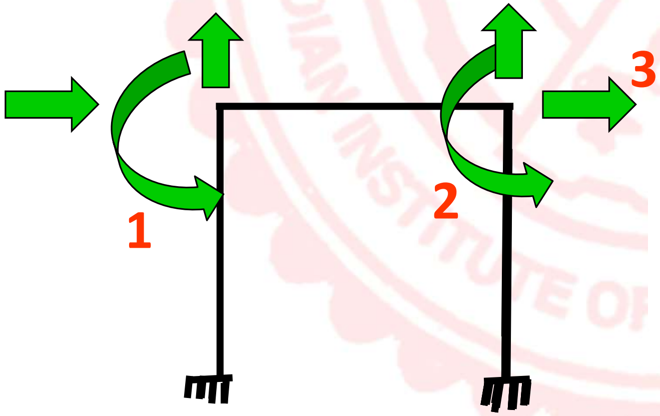
SPACE FRAME: **DKI = 6j – r**

DEGREE OF KINEMATIC INDETERMINACY (DKI):

2D FRAMES WITH INEXTENSIBLE MEMBERS

PLANE FRAME: $DKI = 3j - r - C_i$

C_i = No. of conditions of inextensibility, generally equal to the number of inextensible members



DKI = 3x4 - 6 - 3

= 3

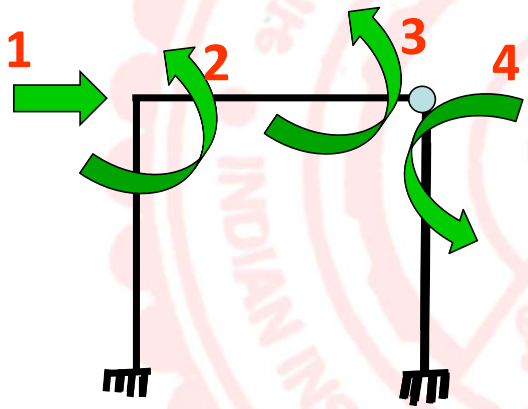
SPACE FRAME: $DKI = 6j - r - C_i$

DEGREE OF KINEMATIC INDETERMINACY (DKI):

2D FRAMES WITH INEXTENSIBLE MEMBERS AND RELEASES

PLANE FRAME: $DKI = 3j - r - C_i + f$

f = No. of releases



$DKI = 3 \times 4 - 6 - 3 + 1$

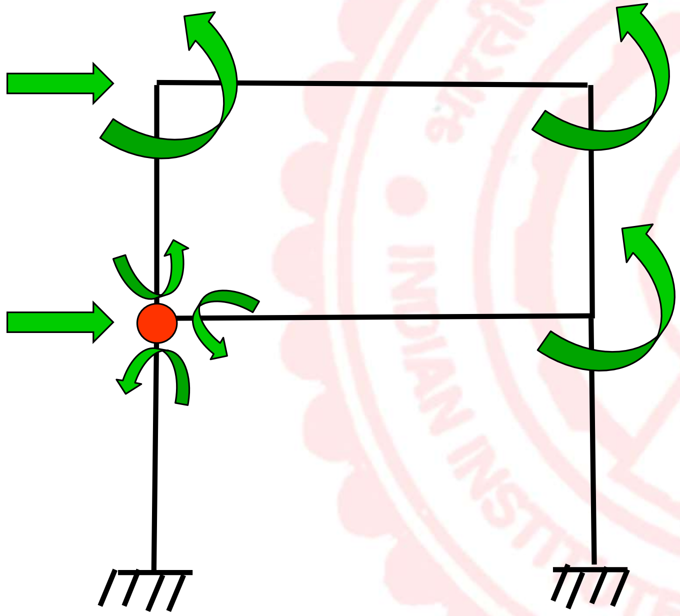
$= 4$

$f = N-1$,

N = No of members meeting at the joint

SPACE FRAME: $DKI = 6j - r - C_i + f$

EXAMPLE 8



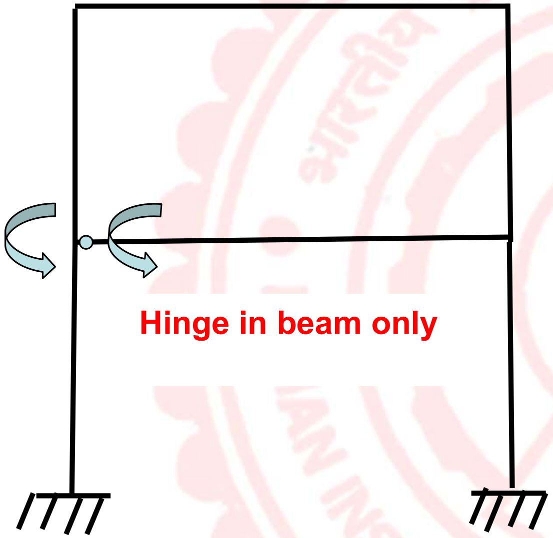
$DKI = 3j - r - C_i + f$

2

DKI = 8

(Members are inextensible)

EXAMPLE 9



(Members are inextensible)

$DKI = 3j - r - C_i + f$

$f = 1$

For space frames, $f = 3N-3$

THANK YOU

FOR DISCUSSION AND QUERIES:

PLEASE JOIN MS TEAM CHANNEL