DIRECT STIFFNESS METHOD FOR ANALYSIS OF SKELETAL STRUCTURES (PART 2)

http://web.iitd.ac.in/~sbhalla/cvl756.html

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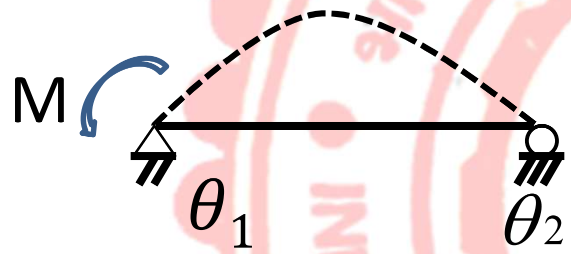
STATIC CONDENSATION: ANALOGY WITH MODIFIED SLOPE DEFLECTION EQUATIONS

In general, $$M_{AB} = (2EI/L)[2\theta_1 + \theta_2]$$



$$M = (3EI/L)\theta_1$$

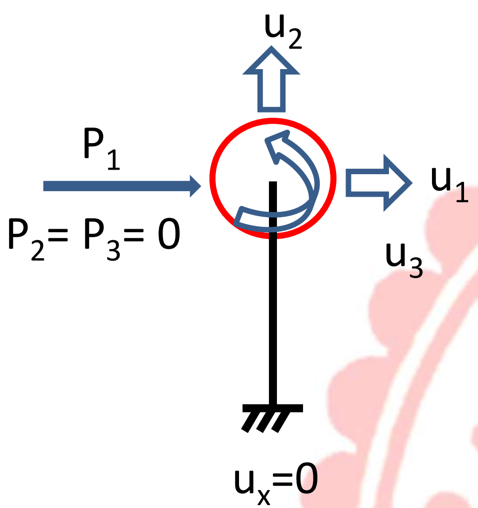
$\theta_2 \neq 0$ Its effect is automatically included

→ **The complete matrix may include d.o.f more than required**

→ **Stiffness matrix K_{pp} can be condensed by eliminating the unwanted d.o.f**

→ **Unwanted disps. are expressed in terms of prominent d.o.f**

STATIC CONDENSATION



$$\left[\begin{array}{c|cc} K_{11} & K_{12} & K_{13} \\ \hline K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{array}\right] \left[\begin{array}{c} u_1 \\ \hline u_2 \\ u_3 \end{array}\right] = \left[\begin{array}{c} P_1 \\ \hline P_2 \\ P_3 \end{array}\right]$$

Zero

CONDENSATION MEANS TO ACHIEVE

$$K^* \, u_1 = P_1{}^*$$

Such that effects of other stiffness terms and disps. are automatically taken care of

$$\left[\begin{array}{c|c} K_{11} & \overline{K}_{12} \\ \hline \overline{K}_{21} & \overline{K}_{22} \end{array}\right] = \left[\begin{array}{c} u_1 \\ \hline \overline{u}_2 \end{array}\right] \left[\begin{array}{c} P_1 \\ \hline \overline{P}_2 \end{array}\right]$$

Zero (usually)

$$K_{11}u_1 + \overline{K}_{12}\overline{u}_2 = P_1 \text{ — } (1)$$

$$K_{21}u_1 + K_{22}\overline{u}_2 = \overline{P}_2 \text{ — } (2)$$

Can we eliminate u_2 from above equations?

From (2) $\rightarrow$ $K_{22}\overline{u}_2 = (\overline{P}_2 - \overline{K}_{21}u_1)$ $\qquad$ $\overline{u}_2 = \overline{K}_{22}{}^{-1}(\overline{P}_2 - \overline{K}_{21}u_1) \longrightarrow (3)$

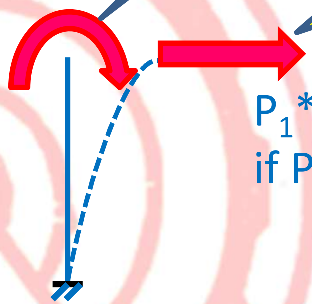
Substitute equation 3 into 1 ---

$$K_{11}u_1 + \bar{K}_{12}\bar{K}_{22}^{-1}(P_2 - K_{21}u_1) = P_1$$

$$\underbrace{(K_{11} - \bar{K}_{12}\bar{K}_{22}^{-1}\bar{K}_{21})}_{K_{11}^*}\, u_1 = \underbrace{P_1 - \bar{K}_{12}\bar{K}_{22}^{-1}\bar{P}_2}_{P_1^*}$$

$$K_{11}^*\, u_1 = P_1^*$$

Modified force vector



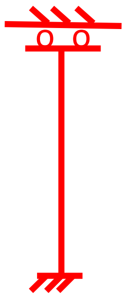
Dependent DOF

Preferred DOF

$P_1^* = P_1$

if $P_{2,3} = 0$

$$K_{11}^* = K_{11} - \begin{bmatrix} K_{12} & K_{13} \end{bmatrix} \begin{bmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{bmatrix}^{-1} \begin{bmatrix} K_{21} \\ K_{31} \end{bmatrix}$$



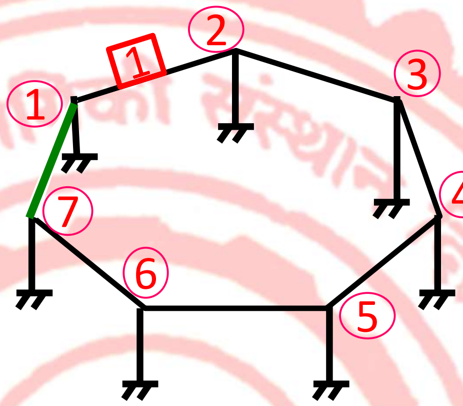
$u_2 = u_3 = 0$

Locking of dof is Not condensation

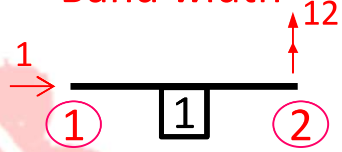
The resulting matrix is compact and suitable for dynamic analysis in this particular case

METHOD OF SUBSTRUCTURES

1) Ring like structures



Band width

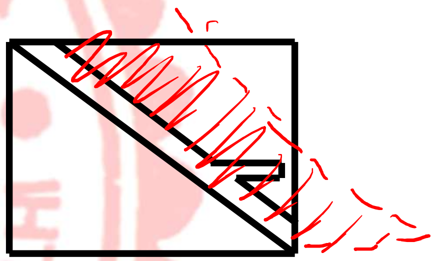


b = 12 – 1 + 1 = 12

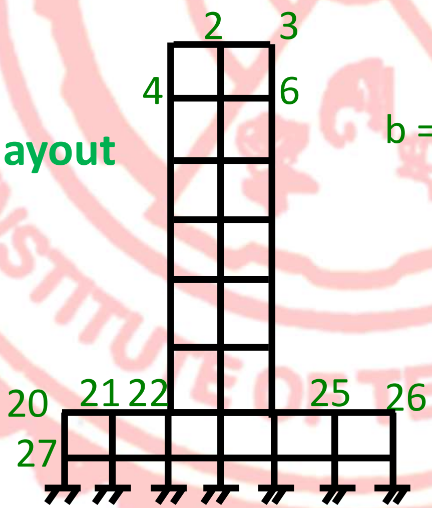
Join joints 1 and 7

New b = 42 - 1 + 1 = 42

K_{PP} →



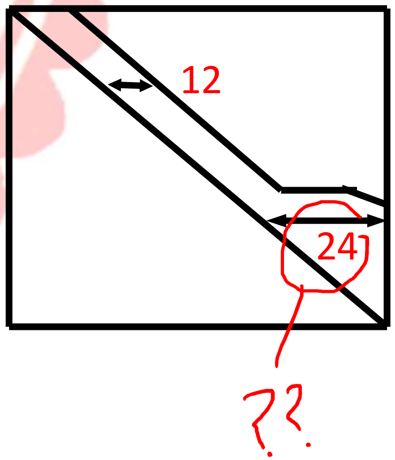
2) Sudden change of layout



b = 12 – 1 + 1 = 12

b = 12 – 1 + 1 = 24

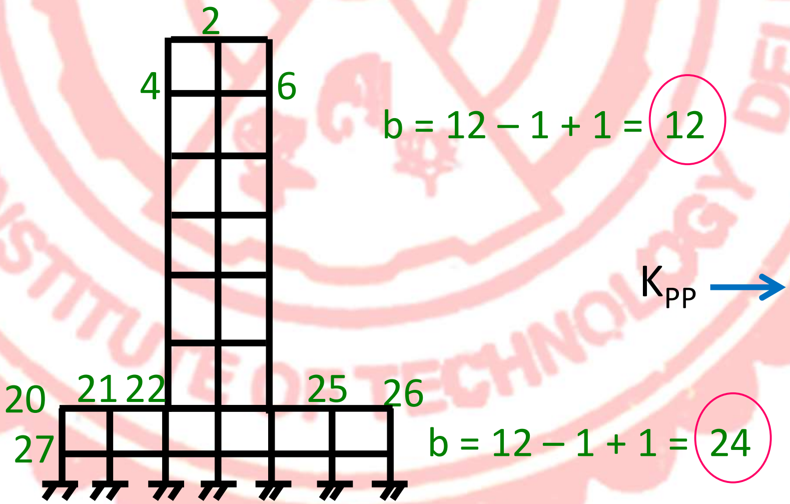
K_{PP} →

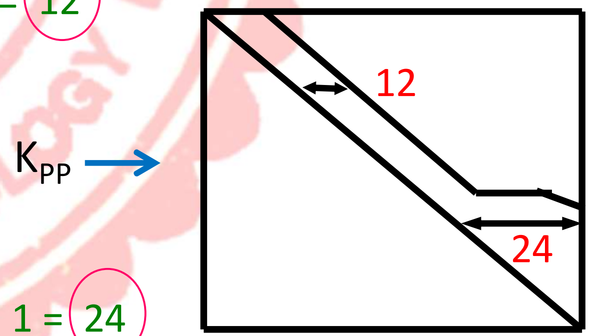


3) Very large structure to be analyzed on a small computer

METHOD OF SUBSTRUCTURES

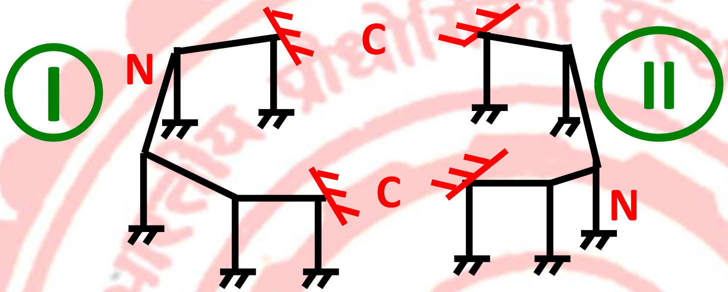
Method of substructures means that instead of solving the entire structure in one go, **we divide the structure into sub-parts**, **analyze individually**, **and then integrate**, **but without any approximation or loss of accuracy in final results**





COMPUTATIONAL APPROACH

Substr.



C : Common nodes

N : Nodes other than common

I

$$\left[\begin{array}{c:c} K_{NN}^{(1)} & K_{NC}^{(1)} \\ \hdashline K_{CN}^{(1)} & K_{CC}^{(1)} \end{array}\right] \left[\begin{array}{c} u_{N1} \\ \hdashline u_{C1} \end{array}\right] = \left[\begin{array}{c} P_{N1} \\ \hdashline P_{C1} \end{array}\right] \checkmark$$

K_{PP}

Internal forces @ common nodes

?? (Unknown)

II

$$\left[\begin{array}{c:c} K_{NN}^{(2)} & K_{NC}^{(2)} \\ \hdashline K_{CN}^{(2)} & K_{CC}^{(2)} \end{array}\right] \left[\begin{array}{c} u_{N2} \\ \hdashline u_{C2} \end{array}\right] = \left[\begin{array}{c} P_{N2} \\ \hdashline P_{C2} \end{array}\right] \checkmark$$

? ?

Unknown

By process of static condensation, we eliminate u_{N1} and u_{N2}

Compatibility $u_{c1} = u_{c2} = u_c$

Equilibrium $P_{c1} = -P_{c2}$

Condensed stiffness matrix I

Condensed force vector

common nodes

$$\overline{K}_{CC1}\, u_{C1} = \overline{P}_{C1} \rightarrow 1$$

u_C

Similarly, $\overline{K}_{CC2}\, u_{C2} = \overline{P}_{C2} \rightarrow 2$

u_C

Add 1 and 2 $\rightarrow (\overline{K}_{CC1} + \overline{K}_{CC2})\, u_C = \overline{P}_{C1} + \overline{P}_{C2}$

?

Solving u_C = ??

$$K^{(1)}_{NN} u_{N1} + K^{(1)}_{NC}\, \underset{\substack{\| \\ u_C}}{u_{C1}} = P_{N1}$$

Solve for u_{N1}

Similarly, we can compute u_{N2}

$$K^{(1)}_{NN} u_{N1} + K^{(1)}_{NC}\, u_{C1} = P_{N1}$$

$$u_{N1} = (K^{(1)}_{NN})^{-1} \left[P_{N1} - K^{(1)}_{NC}\, u_{C1}\right]$$

$$K^{(1)}_{CN}\, u_{N1} + K^{(1)}_{CC}\, u_{C1} = P_{C1}$$

$$\overline{K}_{cc1} = K^{(1)}_{cc} - K^{(1)}_{CN} (K_{NN})^{-1} K^{(1)}_{NC}$$

$$\overline{P}_{C1} = \overset{??}{P_{C1}} - K^{(1)}_{CN} (K^{(1)}_{NN})^{-1} P_{N1}$$

$$\overline{P}_{C2} = (\overset{??}{-P_{C1}}) - K^{(2)}_{CN} (K_{NN})^{-1} P_{N2}$$

$\overline{P}_{C1} + \overline{P}_{C2}$ = known

Since P_{C1} and P_{C2} get cancelled out

How to obtain Internal forces at common nodes??

$\bar{P}_{C1} = \bar{K}_{CC1}\, u_C$

Known

$\bar{P}_{C1} = P_{C1} - K_{CN}^{①} \left[K_{NN}^{①} \right] P_{N1}$

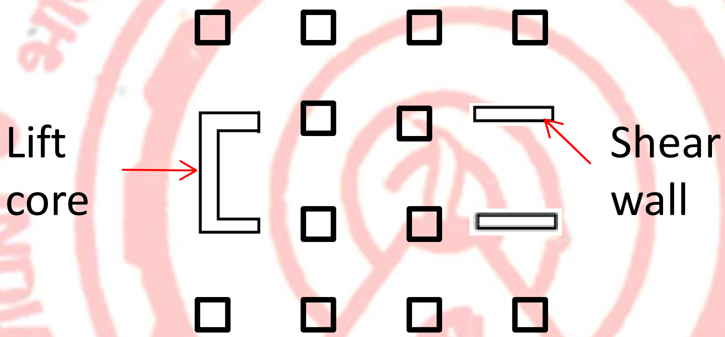
determine

Similarly P_{C2} can also be computed

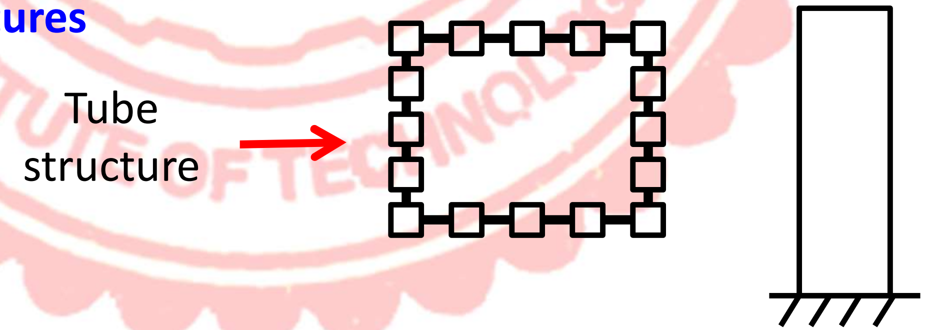
ANALYSIS OF BUILDING FRAMES

1) **8-10 storeys** : Simple frames (gravity + lateral loads)

2) **8-10 storeys onwards (upto 20 storeys)** : Frames + Shear walls (Lift core)



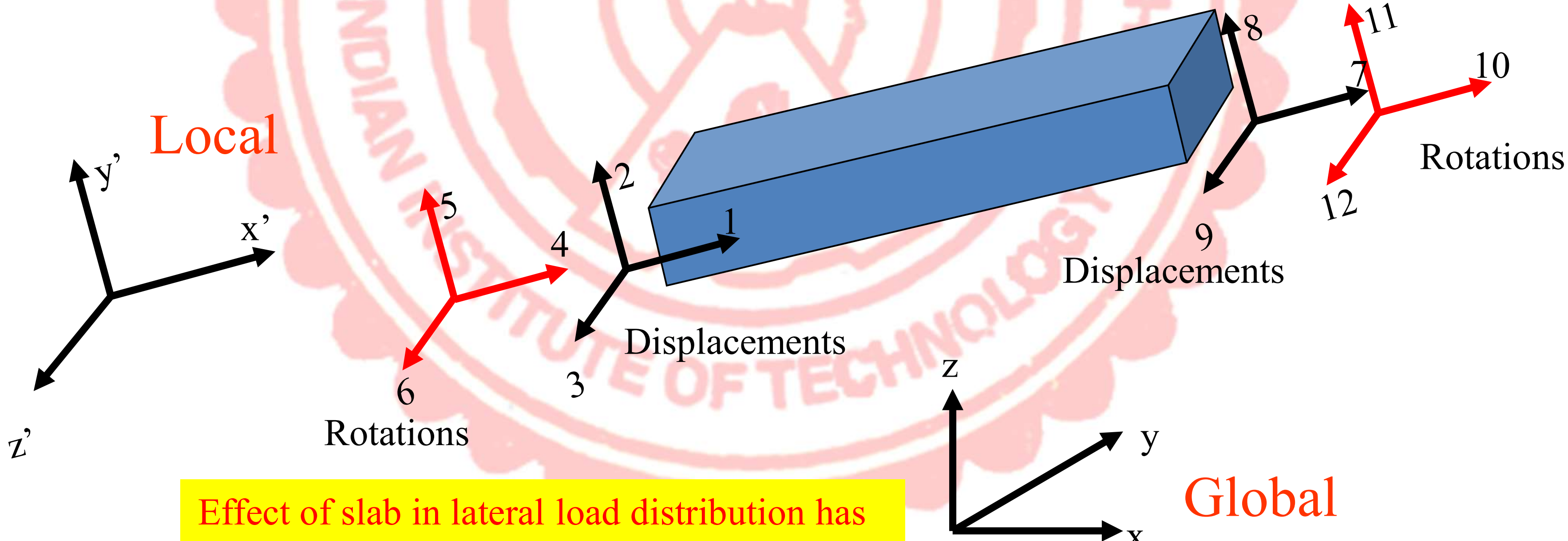
3) **Very tall structures**



MATRIX FORMULATIONS FOR 3D STRUCTURES (DONE TILL NOW)

The 2D formulations can be extended into 3D

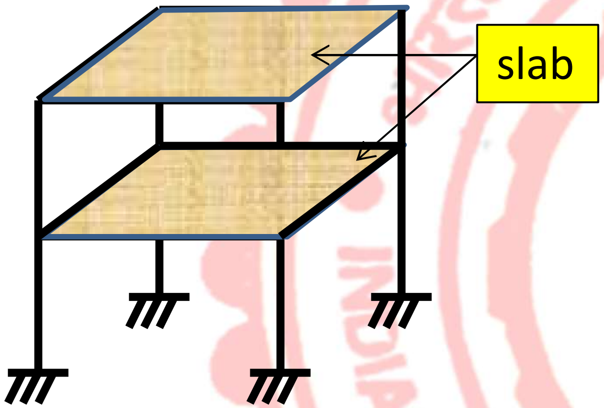
Additional term (dof 4, 10) $$T = \left[\frac{GJ}{L}\right]\theta$$



Effect of slab in lateral load distribution has been ignored so far…..

GENERAL 3D ANALYSIS

1) Distribution of vertical and lateral loads is automatically taken care of.
2) Rigorous
3) Output very compact



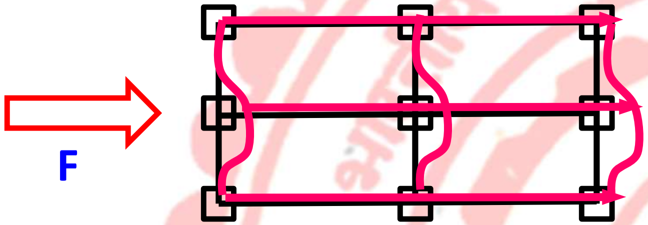
Usual 3D analysis

Ignores presence of slab

Structural functions of floor system

1) Distribution of vertical loads to beams through bending.
2) Distribution of lateral loads by in plane action.
3) Compatibility condition

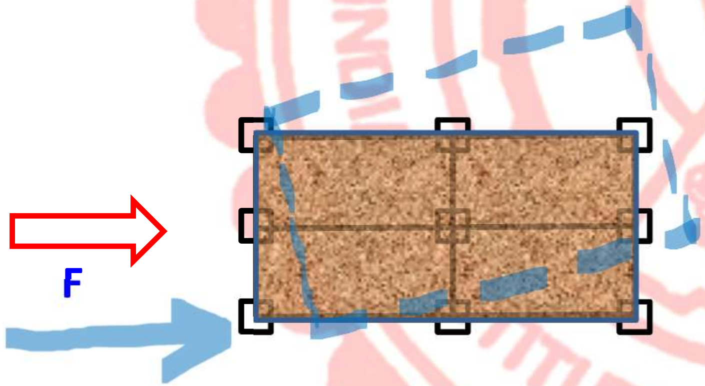
HOW DOES THE PRESENCE OR ABSENCE OF SLAB AFFECT THE VERTICAL AND LATERAL LOAD ANALYSIS??



Symmetrical Building without slab

- ✓Under vertical loads
- ✓Under horizontal loads

(Axial forces in beams, non-uniform distribution, special compatibility condition)

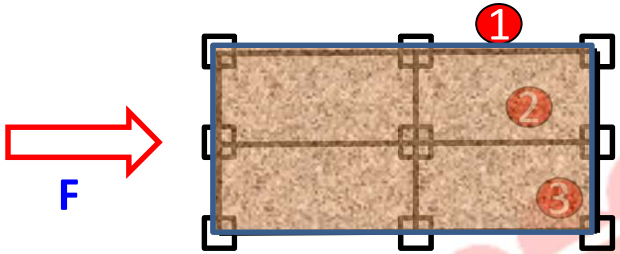


Symmetrical Building with slab

- ✓Uniform translation
- ✓No rotation of floor

✓Forces will be distributed in proportion to stiffness of frames

What will happen in above two cases if the force "F" were acting unsymmetrically



Symmetrical Building

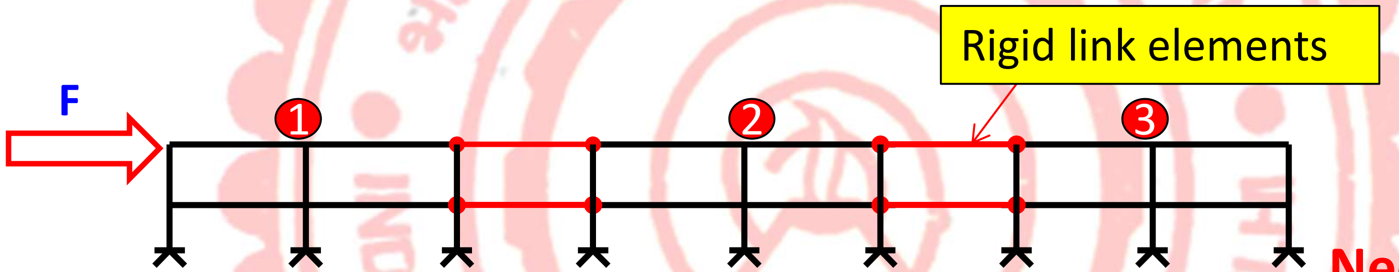
✓**Uniform translation**

✓**No rotation of floor**

✓Forces will be distributed in proportion to stiffness of frames

A SIMPLIFIED ANALYSIS APPROACH

(symmetrical building and symmetrical loading)

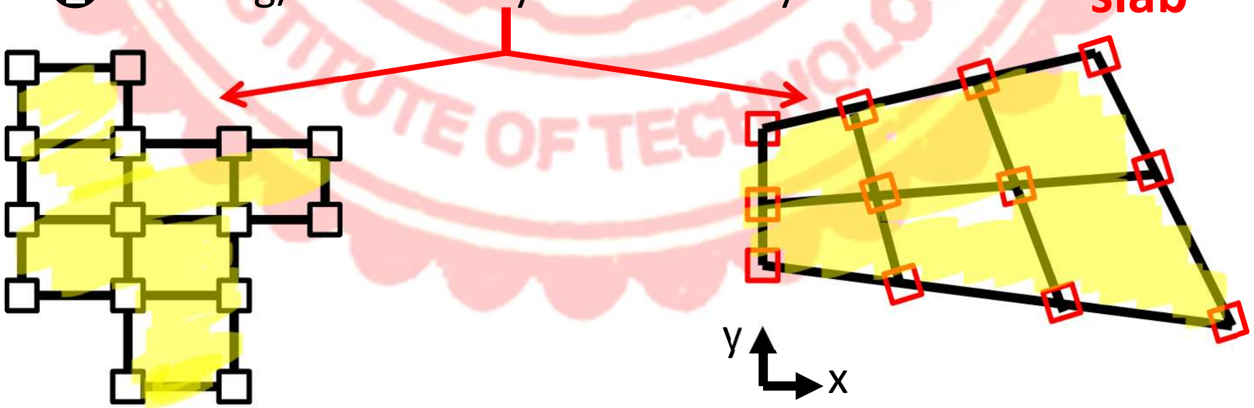


Need full 3D analysis rigorously taking the effect of slab

Not valid

❶ Unsymmetrical lateral loading (Torsion)

❷ Building/structural system is not symmetrical



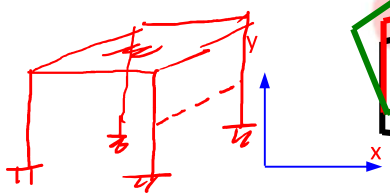
3D ANALYSIS TAKING RIGIDY OF FLOOR SLAB INTO ACCOUNT

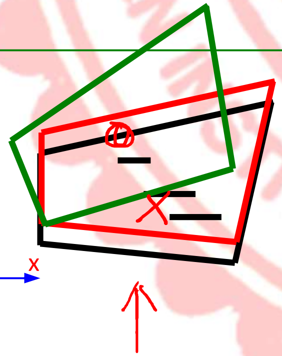
ASSUMPTIONS

1. Slab is monolithic with beam and cols.
2. Continuous slab ⟹ NO LARGE CUTOUTS
3. All joints (unrestrained joints) lie on floor slabs HOW??
4. Slab sufficiently thick so that rigid diaphragm action results
5. All other assumption of direct stiffness approach

Valid for RC structures with floor slab (Not valid for steel frames with sheet based roof)

Basically need the translation and rotation at a reference point





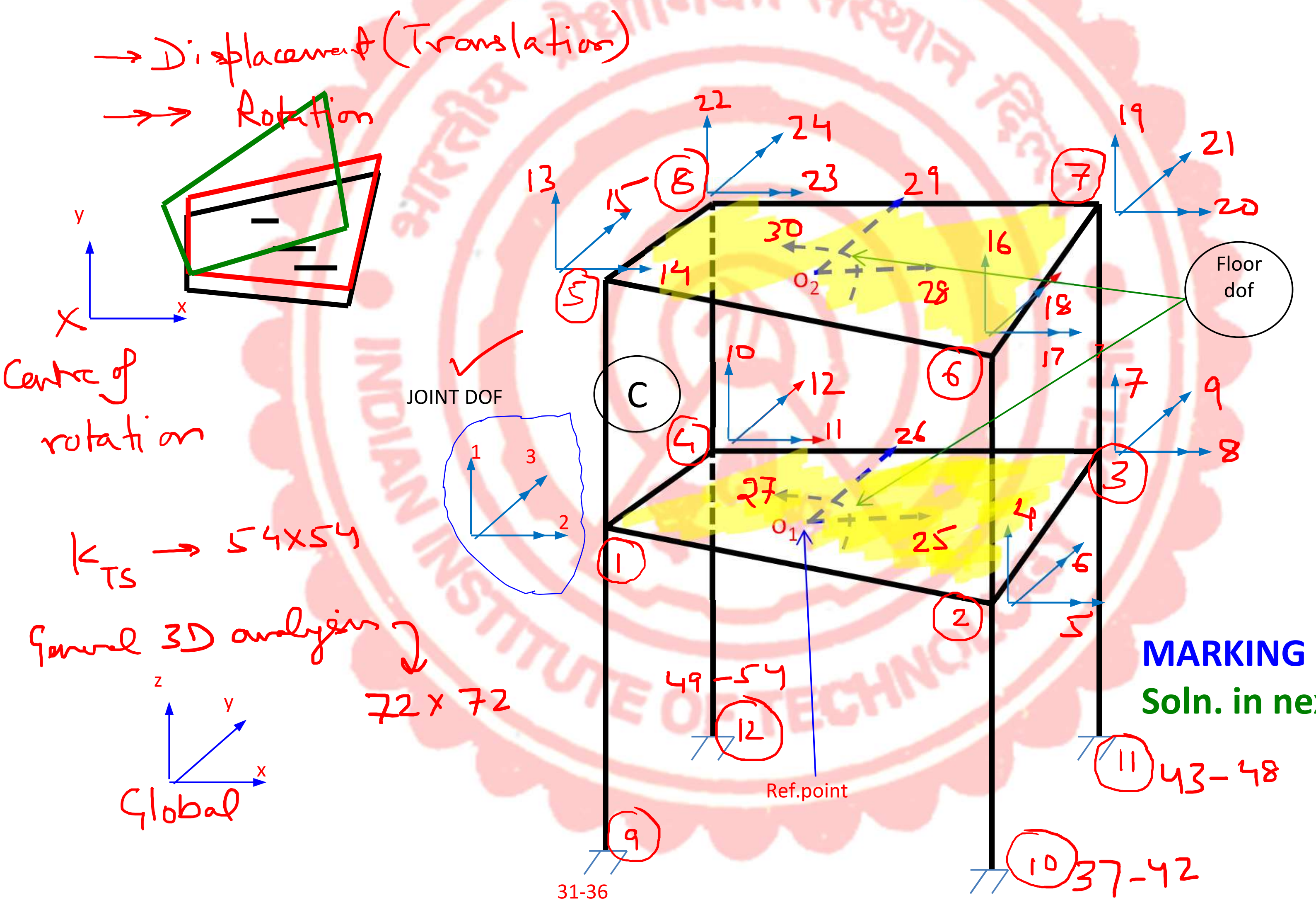
∴ we consider dof @ ref point

Implications:

1. Points lying on slab undergo rigid body translation and rotation.
2. Beams cannot have any bending in plane of slab.
3. No axial deformation in beams.

Formulations will implement (1) to (3)

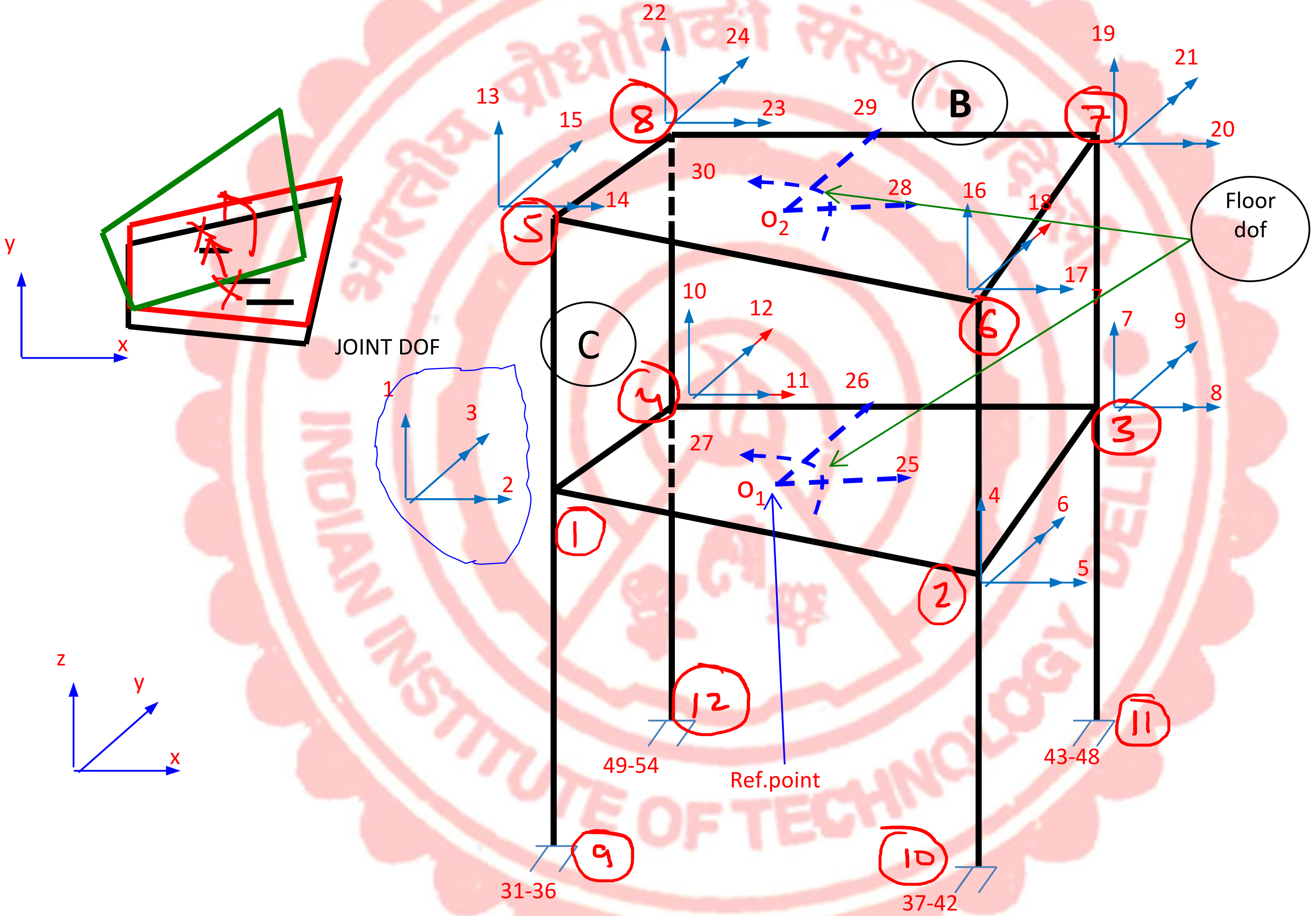
NUMBERING APPROACH TO TAKE CARE OF SLAB ACTION



MARKING EXERCISE…

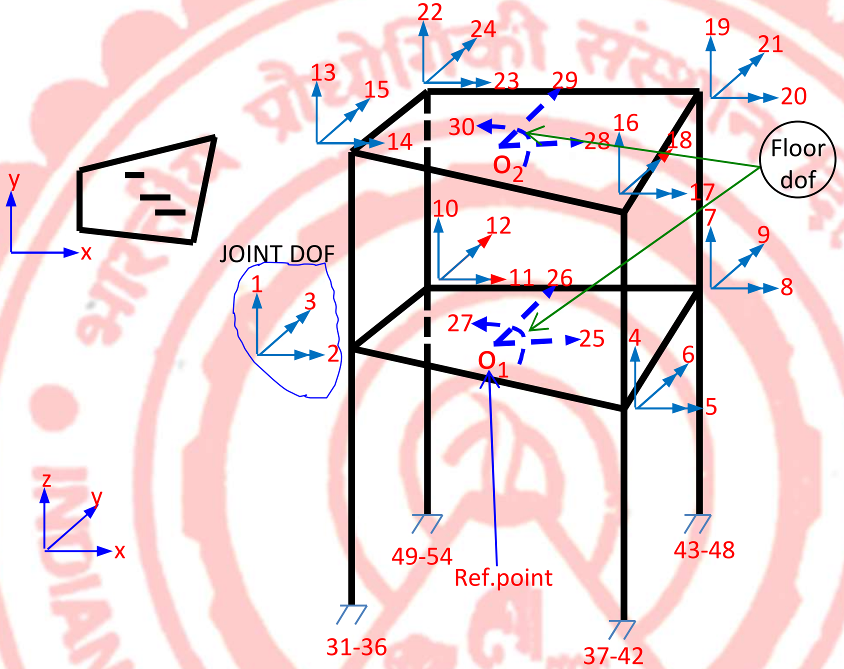
Soln. in next slide

NUMBERING APPROACH TO TAKE CARE OF SLAB ACTION



Choice of references O_1 and O_2 is arbitrary…All displacements and forces shall be transformed to this point…

TOTAL DEGREES OF FREEDOM OF STRUCTURE



No. of unrestrained joints

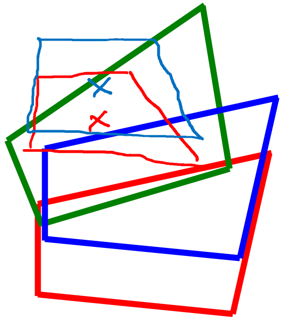
No. of floors

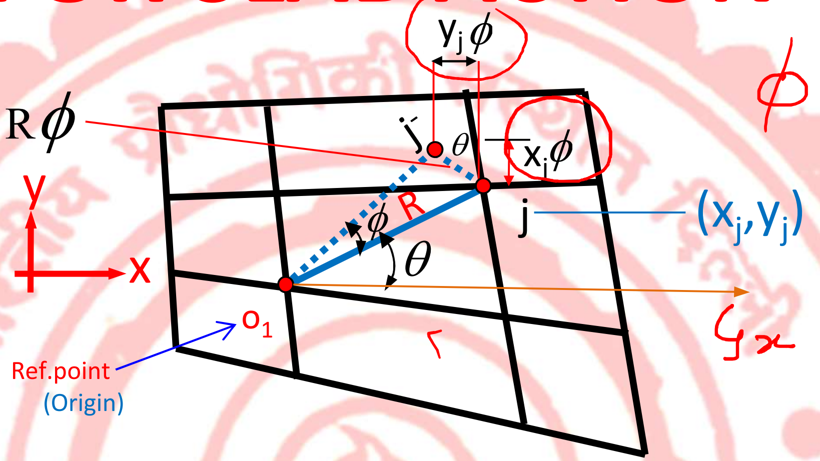
$$\underline{\text{DOF}} = 3N_u + 3F + 6N_r < 6N$$

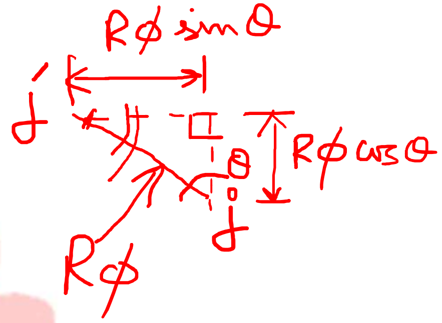
Regular 3D

No. of restrained joints

GEOMETRIC TRANSFORMATION FOR SLAB ACTION





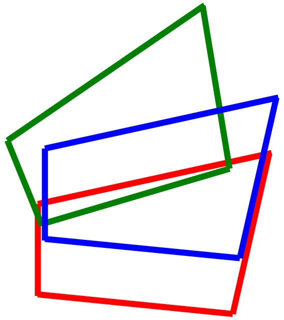


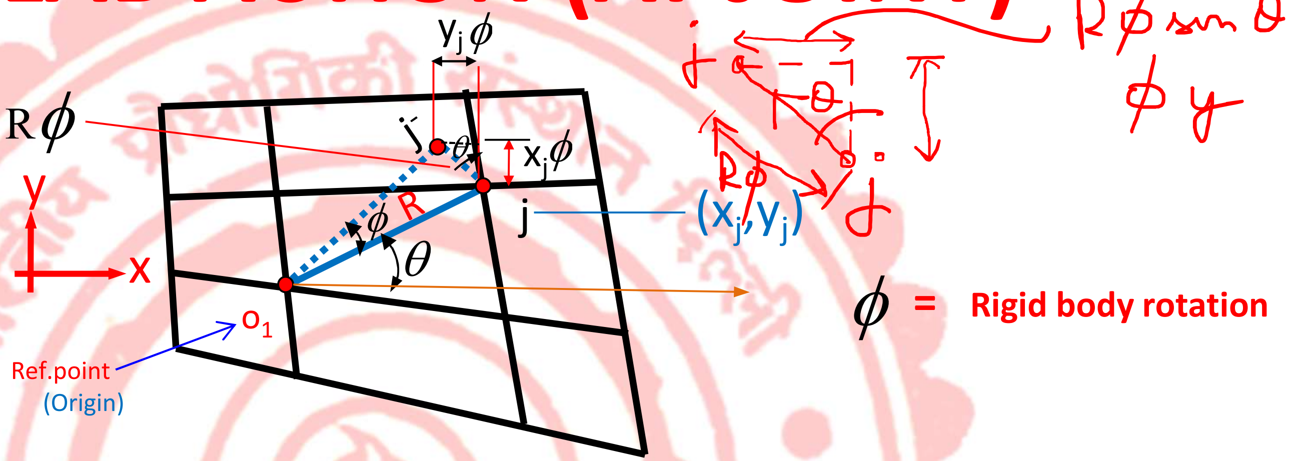
ϕ = **Rigid body rotation**

All displacements and forces corresponding to floor degrees of freedom need to be transformed to the floor reference point O_1

O_1 should not be confused as the centre of rotation of the floor

GEOMETRIC TRANSFORMATION FOR SLAB ACTION (AT JOINT)





EXPRESSING DISPLACEMENTS OF POINT "j" IN TERMS OF DISPLACEMENTS OF REFERENCE PONT ". O_1"

Joint j　　Ref. O_1

x dir　　$D_{1j} = D_1^{*} - y_j\,\phi$

y dir　　$D_{2j} = D_2^{*} + x_j\,\phi$

Rotation (Z)　　$D_{6j} = \phi = D_6^{*}$

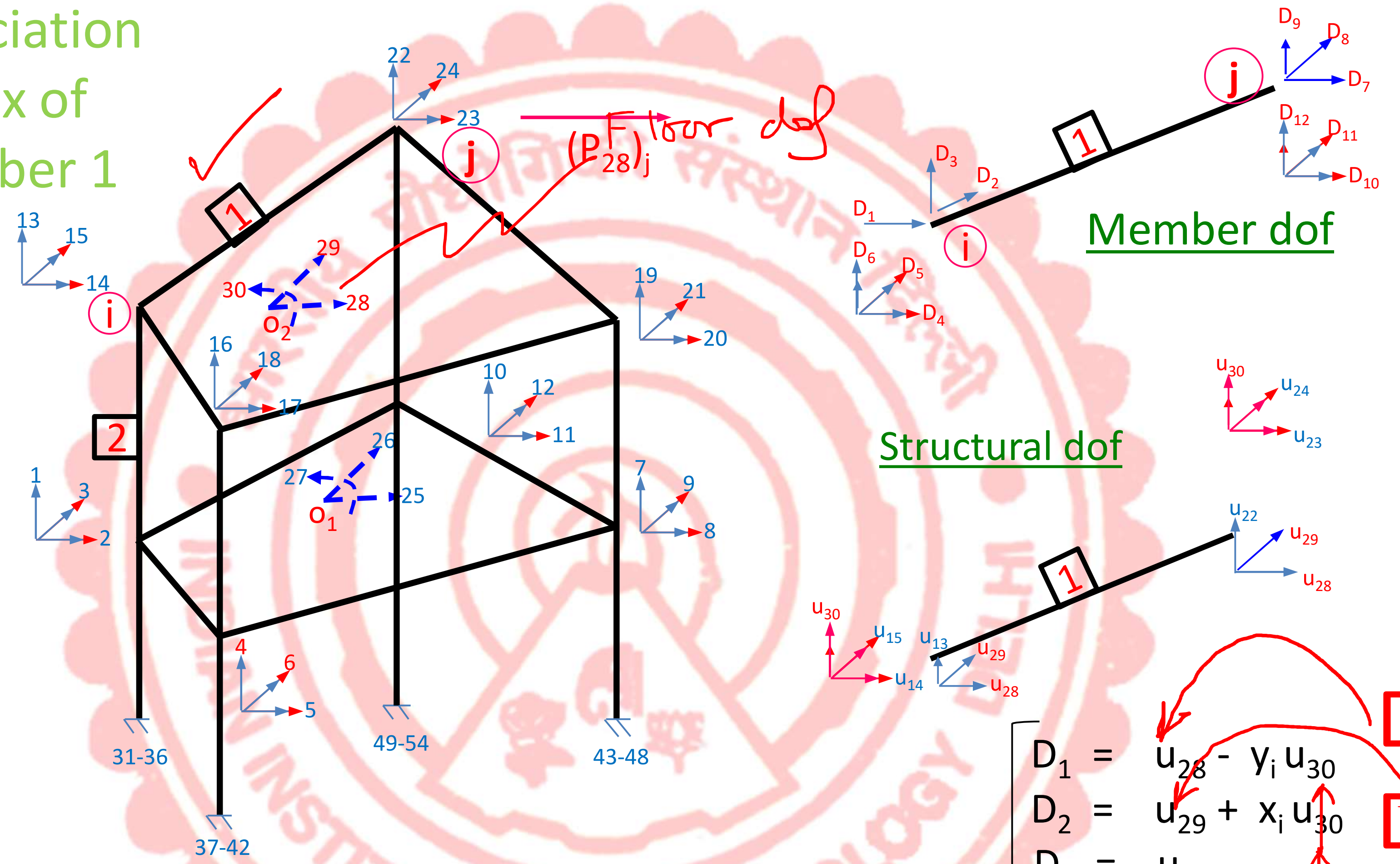
Similar transformations needed at other end of the member

TRANSFORMATION FOR A BEAM ELEMENT

Association matrix of member 1

$$\begin{bmatrix} 28 \\ 29 \\ 13 \\ 14 \\ 15 \\ 30 \\ 28 \\ 29 \\ 22 \\ 23 \\ 24 \\ 30 \end{bmatrix}$$

Joint dof



$$\begin{aligned} D_1 &= u_{28} - y_i\, u_{30} \\ D_2 &= u_{29} + x_i\, u_{30} \\ D_6 &= u_{30} \end{aligned}$$

D_1^* D_2^* D_6^*

$$\begin{aligned} D_7 &= u_{28} - y_j\, u_{30} \\ D_8 &= u_{29} + x_j\, u_{30} \\ D_{12} &= u_{30} \end{aligned}$$

D_7^* D_8^* D_{12}^*

TRANSFORMATION OF DISPLACEMENTS FOR BEAM

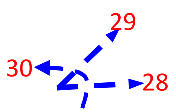
$$
\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \end{Bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -y_i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & x_i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -y_j \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & x_j \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix} D_1^* \\ D_2^* \\ D_3 \\ D_4 \\ D_5 \\ D_6^* \\ D_7^* \\ D_8^* \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12}^* \end{Bmatrix}
\begin{array}{l} \leftarrow u_{28} \\ \leftarrow u_{29} \\ \\ \\ \\ \leftarrow u_{30} \\ \leftarrow u_{28} \\ \leftarrow u_{29} \\ \\ \\ \\ \leftarrow u_{30} \end{array}
$$

*Indicates that the Displacement has been transformed to the reference point

$$\{D\} = [C]\{D^*\}$$

Disp. transformation matrix

This inplane rigidity is automatically considered now



Axial force=0

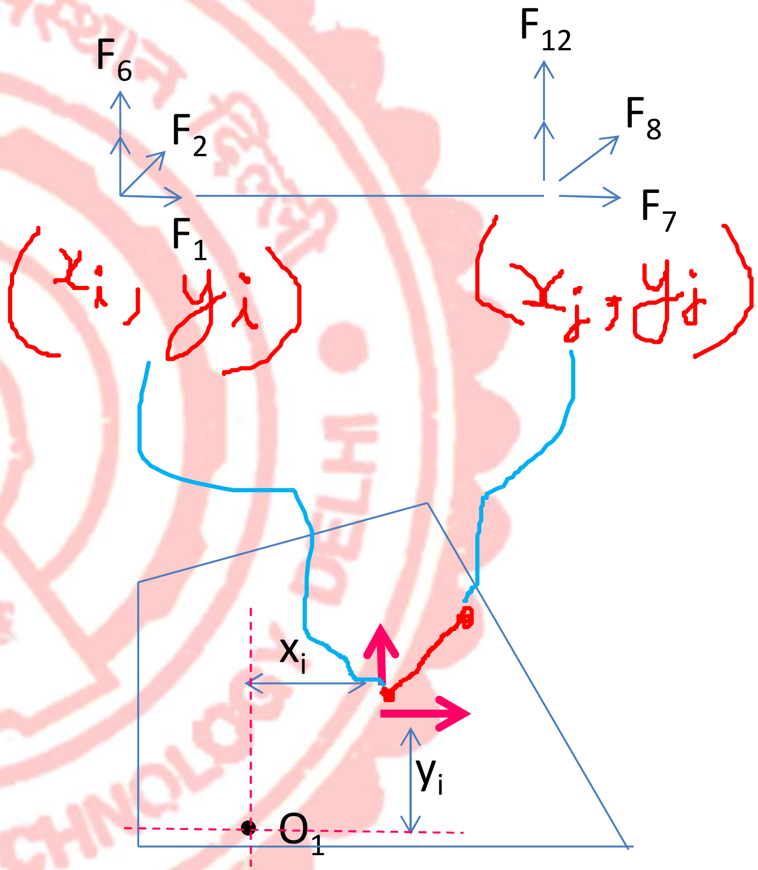
M_y(lateral BM)=0

BEAM MEMBER FORCE TRANSFORMATION

Member end forces corresponding too floor degrees of freedom should also be transformed to point ‘O’ (reference point)

$$F_1^* = F_1$$
$$F_2^* = F_2$$
$$F_6^* = F_6 - y_i F_1 + x_i F_2$$

$$F_7^* = F_7$$
$$F_8^* = F_8$$
$$F_{12}^* = F_{12} - y_j F_7 + x_j F_8$$



TRANSFORMATION OF FORCES FOR BEAM

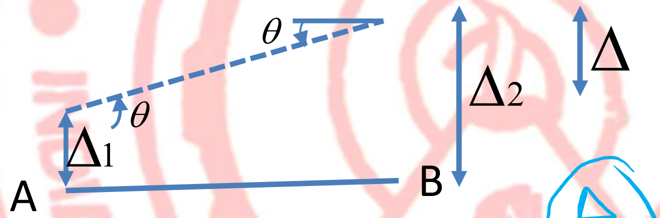
$$
\begin{bmatrix} F_1^* \\ F_2^* \\ F_3 \\ F_4 \\ F_5 \\ F_6^* \\ F_7^* \\ F_8^* \\ F_9 \\ F_{10} \\ F_{11} \\ F_{12}^* \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-y_i & x_i & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -y_j & x_j & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_{10} \\ F_{11} \\ F_{12} \end{bmatrix}
$$

$$\{F^*\} = [C]^T \{F\}$$

Force transformation matrix

RIGID BODY MOVEMENT FOR BEAMS

1) No axial force
2) No lateral bending moment



$\frac{\Delta}{L}$

$$M_{AB} = 2EI/L(2\theta_A + \theta_B - 3\Delta/L) \;;\Delta = \theta\, L$$

$$= 0$$

θ

$D = CD^*$

12x12 Transformation matrix

For beam $\Longrightarrow$

❶ $[K]_L$ 12x12

❷ $[K]_G = [T]^T [K]_L [T]$

❸ $\{D\} = [C] \{D^*\}$

$\therefore \{F\} = [K]_g \{D\}$

$= [K]_g [C] \{D^*\}$

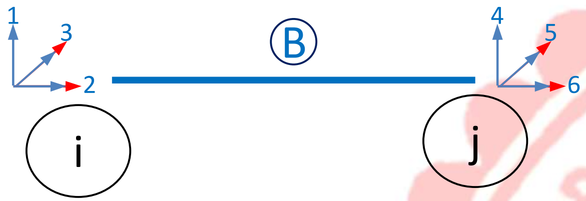
$\{F^*\} = [C]^T \{F\}$

$\{F^*\} = \underbrace{[C]^T [K_g] [C]}_{\Downarrow \; [K_g]^*} \{D^*\}$

Important

$[K_g]^*$ Transform $[K_g]$ to take presence of slab into account

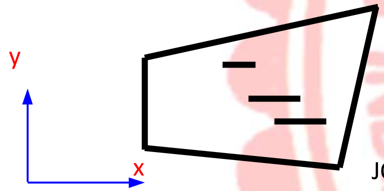
IMPLICATIONS ON BEAM

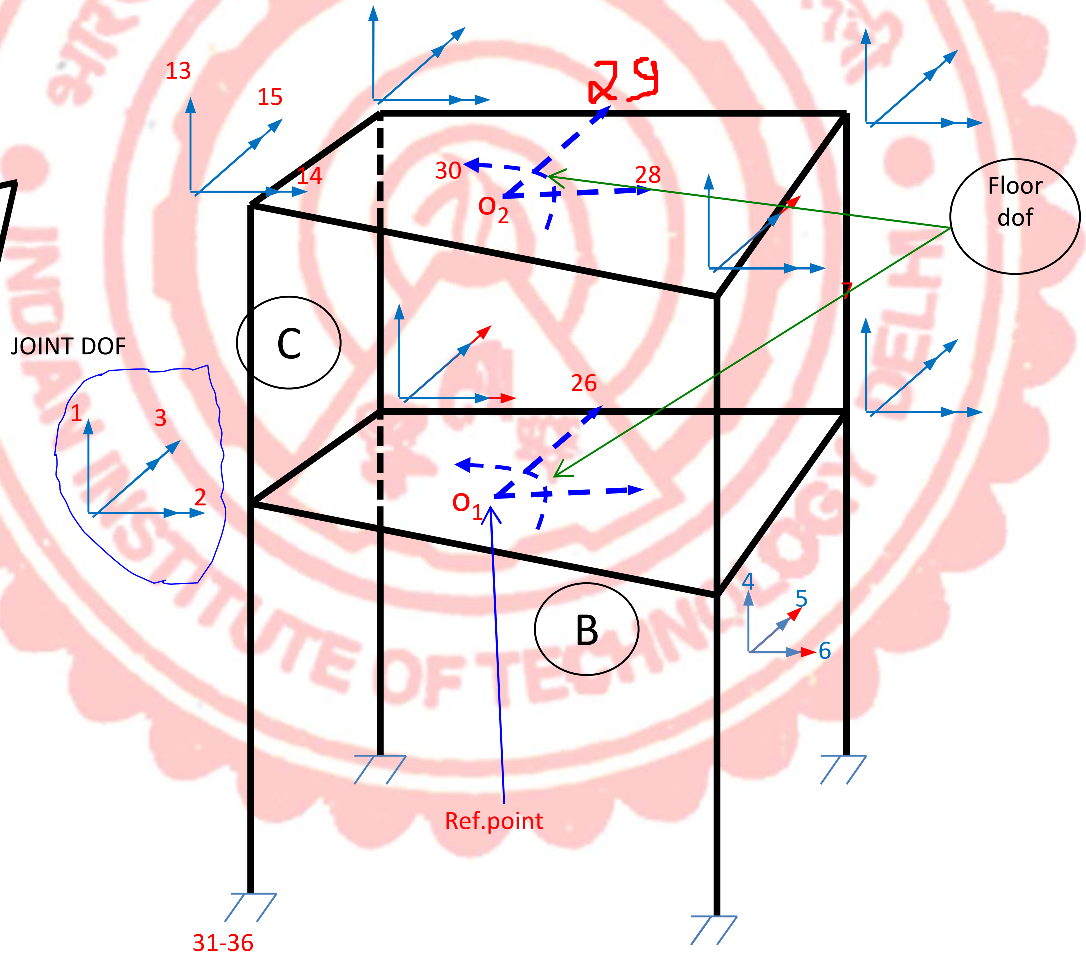


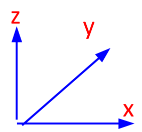
Beam Member effectively has 6 d.o.f

However, we are achieving this effect on 12 x12 matrix through geometric transformation

TORSION EXISTS….







TRANSFORMATION FOR COLUMNS

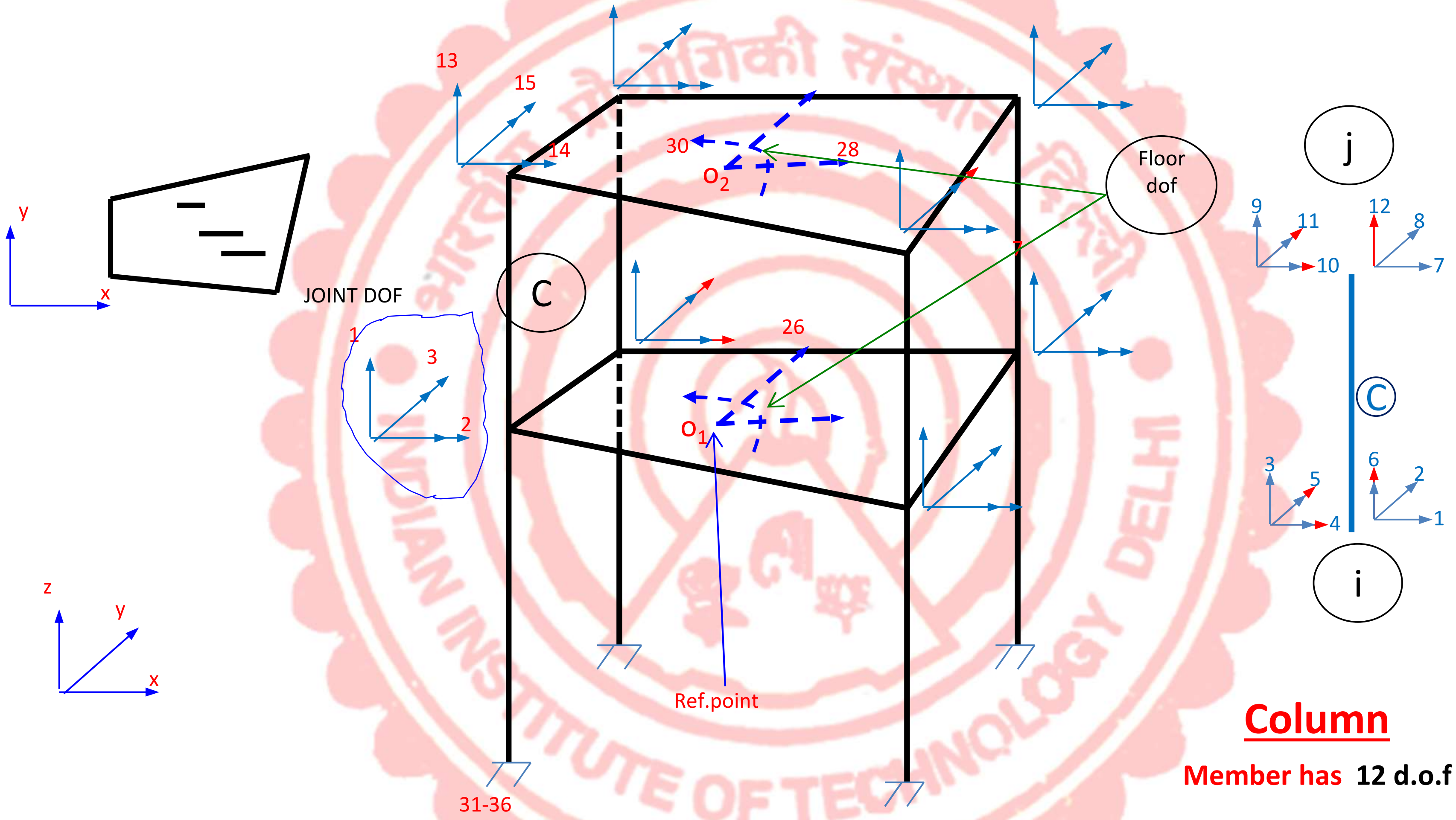
End points lie on different floors…..hence, unlike beams, they shall be subjected to biaxial moments in addition to axial forces

Member end forces and displacements need to be transformed to the reference points.

$$\left\{ \begin{bmatrix} C \end{bmatrix}^{T} \begin{bmatrix} K_g \end{bmatrix} \begin{bmatrix} C \end{bmatrix} \right\} \Downarrow \begin{bmatrix} K_g \end{bmatrix}^{*}$$

COLUMN IS A VERTICAL MEMBER….WHAT IS UNIQUE IN TRANFORMATION????

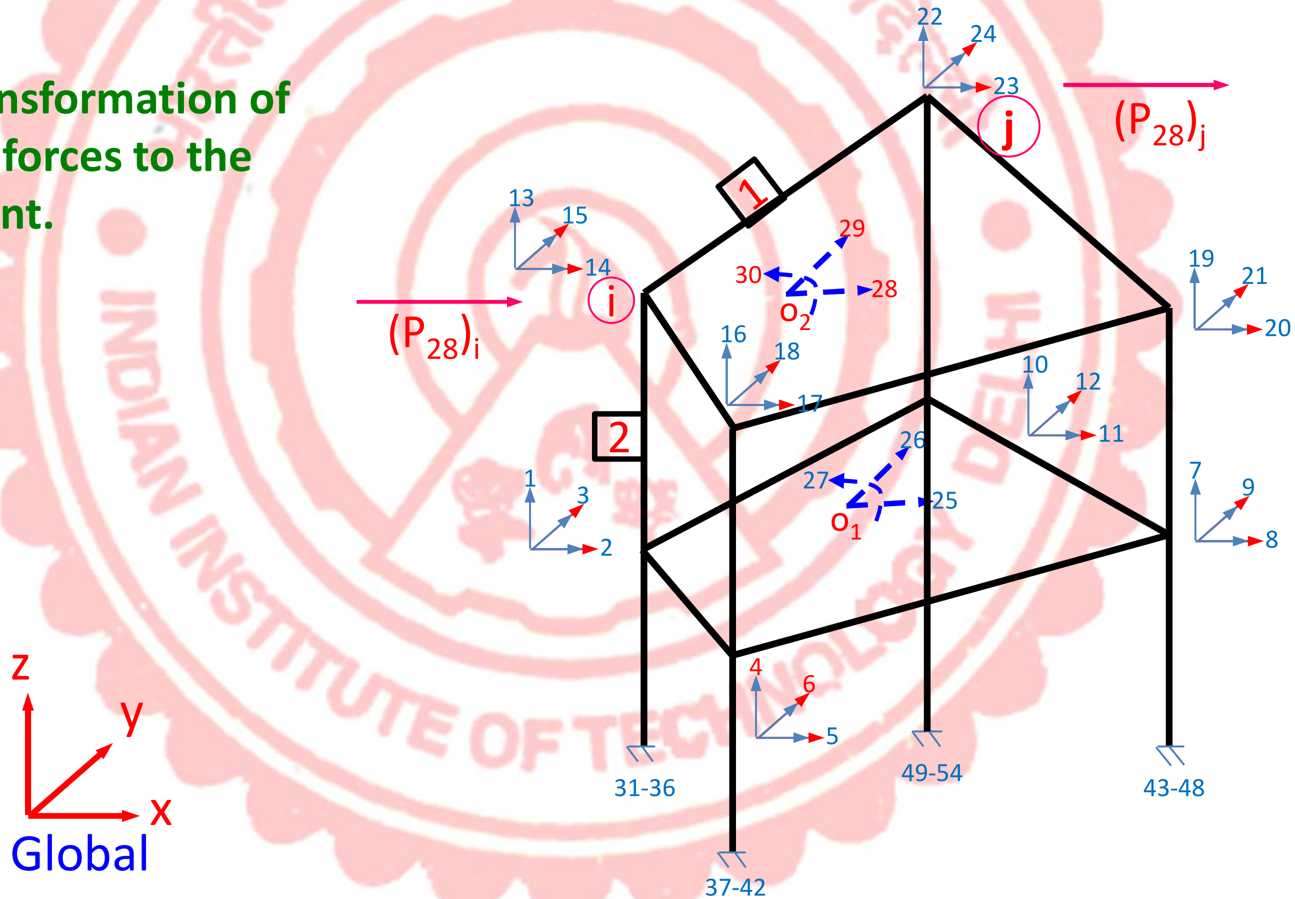
IMPLICATIONS ON COLUMS



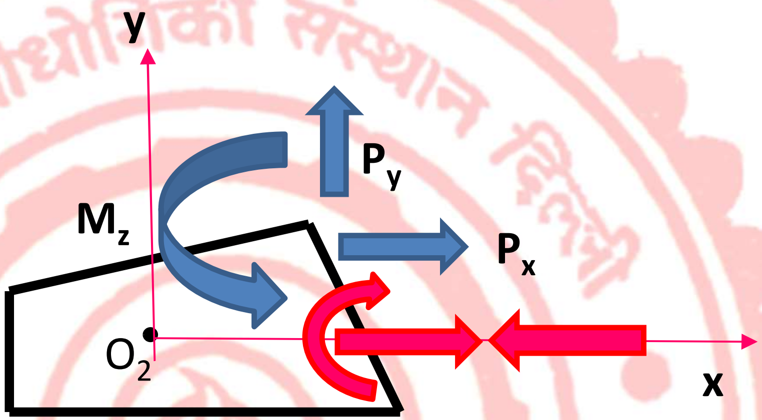
x_i and x_j same……. y_i and y_j same…….

TRANSFORMATION FOR DIRECT JOINT LOADS

Similar to transformation of member end forces to the reference point.



TRANSFORMATION OF DIRECT JOINT LOADS RELATED TO FLOOR D.O.F



$$P_x^* = P_x$$

$$P_y^* = P_y$$

$$M_z^* = M_Z - Y_i P_x + X_i P_y$$

BANDWIDTH

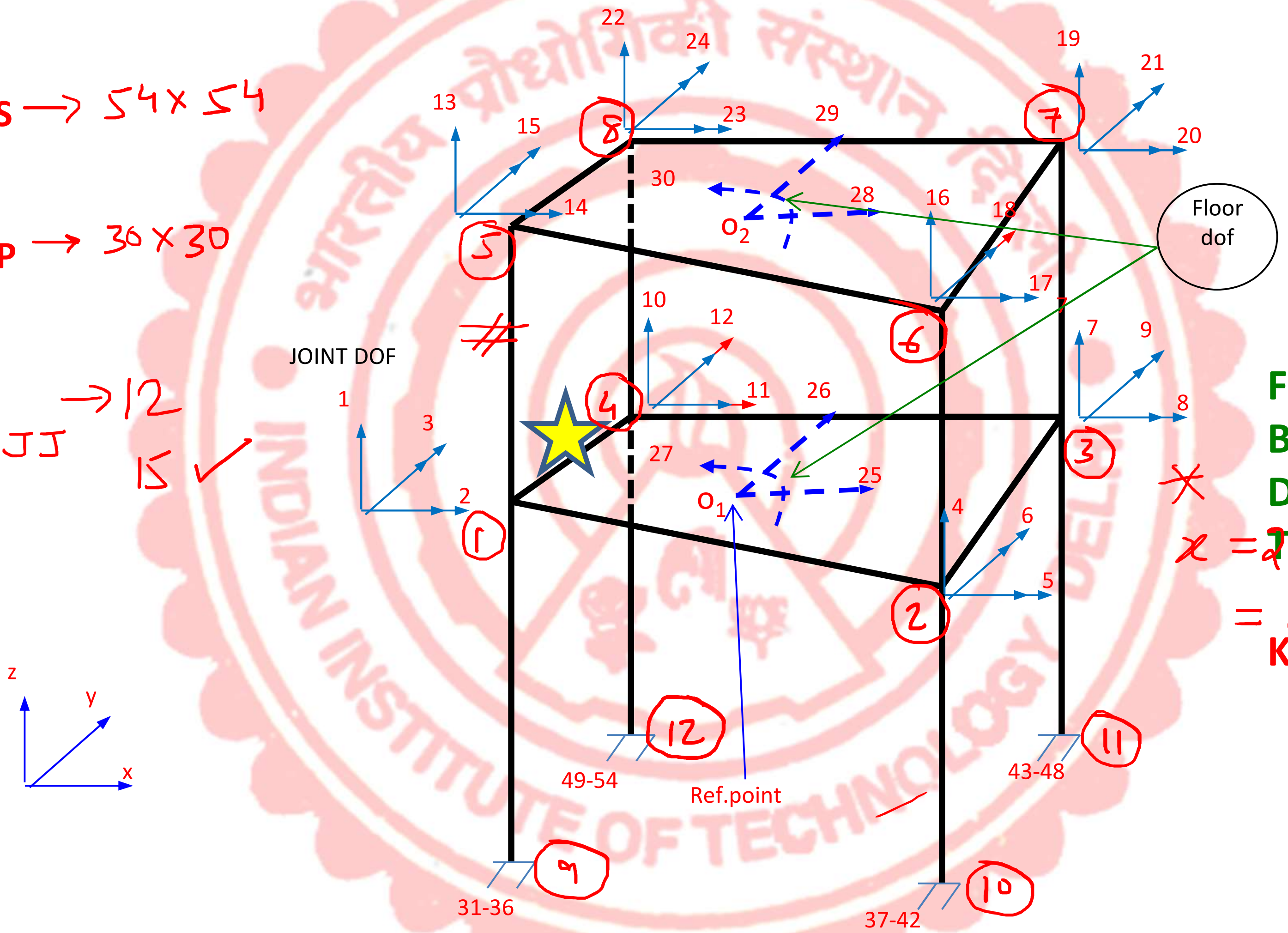
$\# \; x = 30 - 1 + 1$
$= 30$

$K_{TS} \rightarrow 54 \times 54$

$K_{PP} \rightarrow 30 \times 30$

$K_{JJ} \rightarrow 12$

15 ✓



FIND THE BANDWIDTH DICTATED BY THIS MEMBER

$x = 27 - 1 + 1$
$= 27$

K_{PP}

Band width is very large

We must follow alternate approach............

IMPLICATIONS ON SOLUTION PROCEDURE

30x30 K_{TS}

JDOF = 24

FDOF = 6

$$\begin{bmatrix} P \\ \cdots\cdots \\ X \end{bmatrix} = \left[\begin{array}{c:c} K_{PP} & K_{PX} \\ \hdashline K_{XP} & K_{XX} \end{array}\right] \begin{bmatrix} u_P \\ \cdots\cdots \\ u_X \end{bmatrix}$$

24x24

$$P = \begin{bmatrix} P_J \\ \cdots\cdots \\ P_F \end{bmatrix} \qquad K_{PP} = \left[\begin{array}{c:c} K_{JJ} & K_{JF} \\ \hdashline K_{FJ} & K_{FF} \end{array}\right]$$

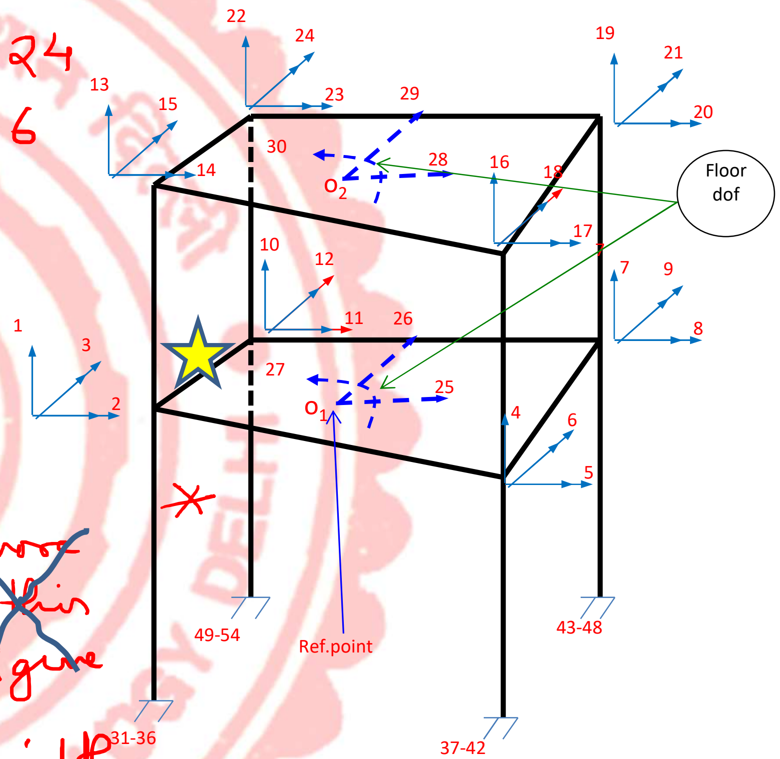
24x24

6x6

b = Band width of K_{JJ}

Ignore this figure

$$u_P = \begin{bmatrix} u_J \\ \cdots\cdots \\ u_F \end{bmatrix}$$



J: Joint degrees of freedom

F: Floor degrees of freedom

SOLUTION :

$$\left[\begin{array}{c} P \\ \hline X \end{array}\right] = \left[\begin{array}{c|c} K_{PP} & K_{PX} \\ \hline K_{XP} & K_{XX} \end{array}\right] \left[\begin{array}{c} u_P \\ \hline u_x \end{array}\right]$$

Banded

$$K_{PP} = \left[\begin{array}{c|c} K_{JJ} & K_{JF} \\ \hline K_{FJ} & K_{FF} \end{array}\right]$$

Prescribed disps.

$$P = K_{PP}u_P + K_{Px}u_x$$

$$\underbrace{\left(P - K_{Px}\, u_x\right)} = K_{PP}u_P$$

??

$$\left\{ P = K_{PP}u_P \text{ if } u_x = 0 \right\}$$

$$P^* = K_{PP}u_P$$

$$\underline{u_P = K_{PP}^{-1}\, P}$$ Displacements

This approach SHALL NOT BE effective/efficient

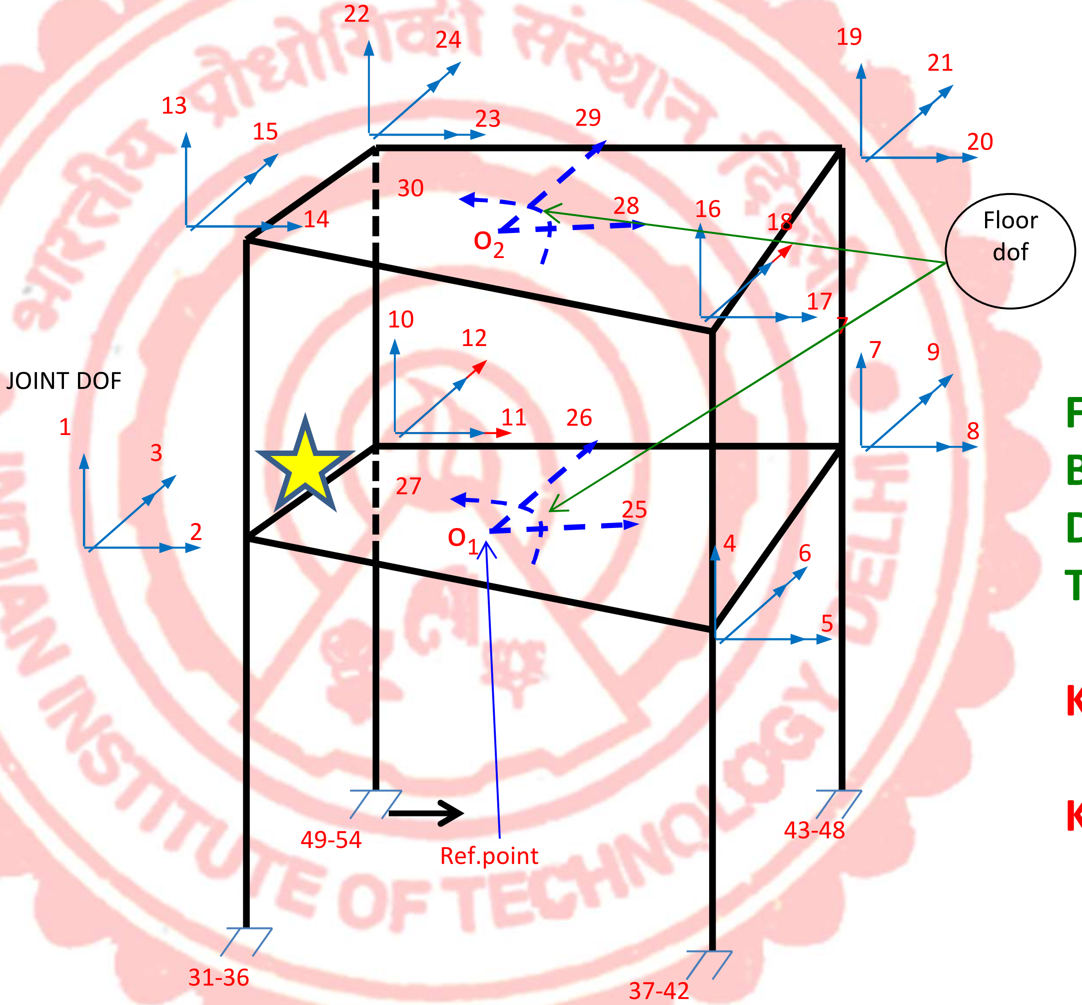
WHY???

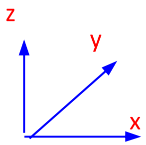
Very large bandwidth…….hence computation not effective

NUMBERING APPROACH TO TAKE CARE OF SLAB ACTION

K_{TS} 54x 54

K_{PP} 30 x 30





FIND THE BANDWIDTH DICTATED BY THIS MEMBER

K_{PP} (30)

K_{JJ} (15)

To take advantage of banded nature of K_{JJ} K(ad not K_{PP}) , we must use <u>CONDENSATION</u>, i.e. condense the joint degrees of freedom into floor degrees of freedom

APPLICATION OF CONDENSATION FOR 3D BUILDING WITH RIGID SLAB

$$\begin{bmatrix} P \\ \cdots \\ X \end{bmatrix} = \begin{bmatrix} K_{PP} & \vdots & K_{PX} \\ \cdots & & \cdots \\ K_{XP} & \vdots & K_{XX} \end{bmatrix} \begin{bmatrix} u_P \\ \cdots \\ u_X \end{bmatrix}$$

Known

$$(P - K_{Px}u_x) = K_{PP}u_P$$

$$P^* = K_{PP}u_P$$

To be eliminated

$$P^* = \begin{bmatrix} P^*_J \\ \cdots \\ P^*_F \end{bmatrix} = \begin{bmatrix} K_{JJ} & \vdots & K_{JF} \\ \cdots & & \cdots \\ K_{FJ} & \vdots & K_{FF} \end{bmatrix} \begin{bmatrix} u_J \\ \cdots \\ u_F \end{bmatrix}$$

J: Joint degrees of freedom

F: Floor degrees of freedom

Can we eliminate u_J from above equations?

$$u_J = K_{JJ}^{-1} (P^*_J - K_{JF}u_F)$$

$$P^*_F = K_{FJ}K_{JJ}^{-1} (P^*_J - K_{JF}u_F) + K_{FF}\, u_{FF}$$

$$(P^*_F - K_{FJ}K_{JJ}^{-1} P^*_J) = (K_{FF} - K_{FJ}K_{JJ}^{-1} K_{JF})\; u_F$$

Condens force vector

$$P_F^{**} = K^*_{FF}\, u_F$$

The whole matrix is condensed in terms of the horizontal degrees of freedom of structure. Suitable for dynamic analysis of structure.

C. Stiffness matrix

INVERSE OF K_{JJ}

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A B I

A ? B[Col 1] ? $\begin{bmatrix} 1 \\ \cdots \\ 0 \\ 0 \end{bmatrix}$

A X B[Col i] ? $\begin{bmatrix} 0 \\ \cdots \\ 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A \quad A^{-1}$

Solve using Chosleski's algorithm

$$A \; X = \begin{array}{c|c|l} & 0 & \\ & \cdots & \\ & 1 & i^{th} \text{ row} \\ & 0 & \end{array}$$

X = i^th col of A^-1

$X = i^{th}$ col of A^{-1}

Repeat $i = 1$ to N

SUMMARY: SOLUTION APPROACH

can take advantage and make banded matrix

1. K_{JJ}^{-1} $\quad \left\{ K_{JJ} \longrightarrow \text{Banded} \right\}$

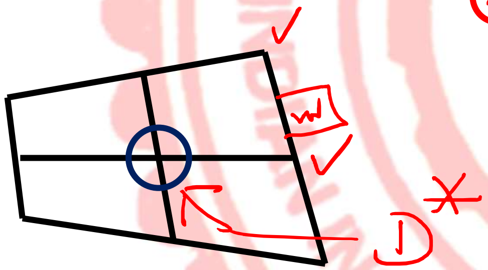
use Cholesky's algorithm, How? Already explained

2. $P_F^{**} = P^*_F - K_{FJ}K_{JJ}^{-1} P^*_J$

3. $K^*_{FF} = (K_{FF} - K_{FJ}K_{JJ}^{-1} K_{JF})$

4. ✓ $P_F^{**} = K_{FF}^{*} u_F$

Condensed matrix (corresponding to floor dof)



✓ $u_F = K_{FF}^{*-1} P_f^{**}$

5. ✓ $u_J = K_{JJ}^{-1}(P^*_J - K_{JF} u_F)$

6. Use code number approach $\quad D = CD^*$

7. $d = TD$ $\quad$ 8. $\boxed{f = K_L d + f^F}$

HOW TO OBTAIN INVERSE OF K_{JJ}

Multiply a matrix with its inverse…..as an example

In order to get the i^{th} column of the inverse , solve the following equation using Cholesky's approach for unknowns X

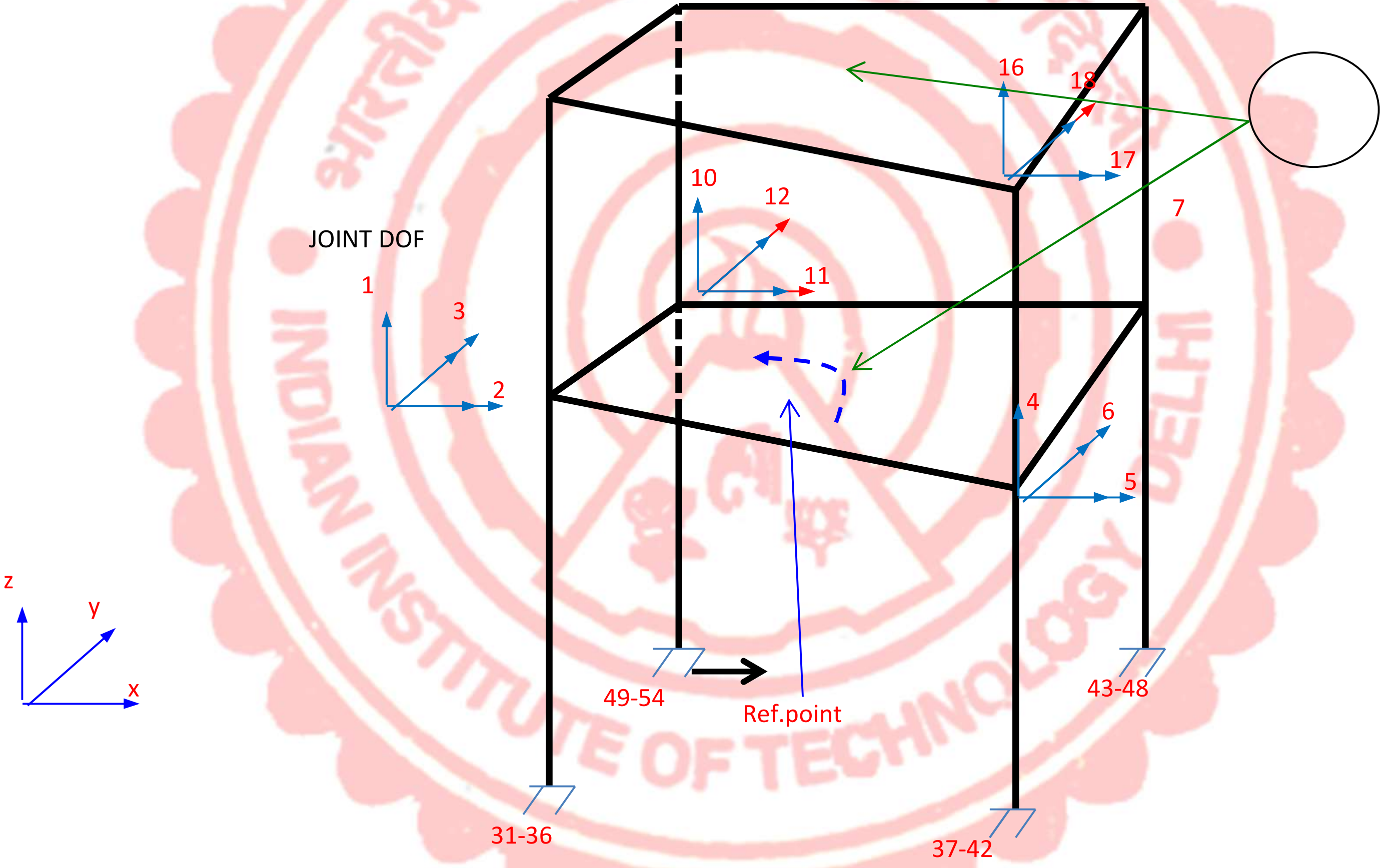
$$P = K_{JJ} \; X$$

X will be the j^{th} column of K_{JJ}^{-1}

where $P = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ 1 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$ At i^{th} row

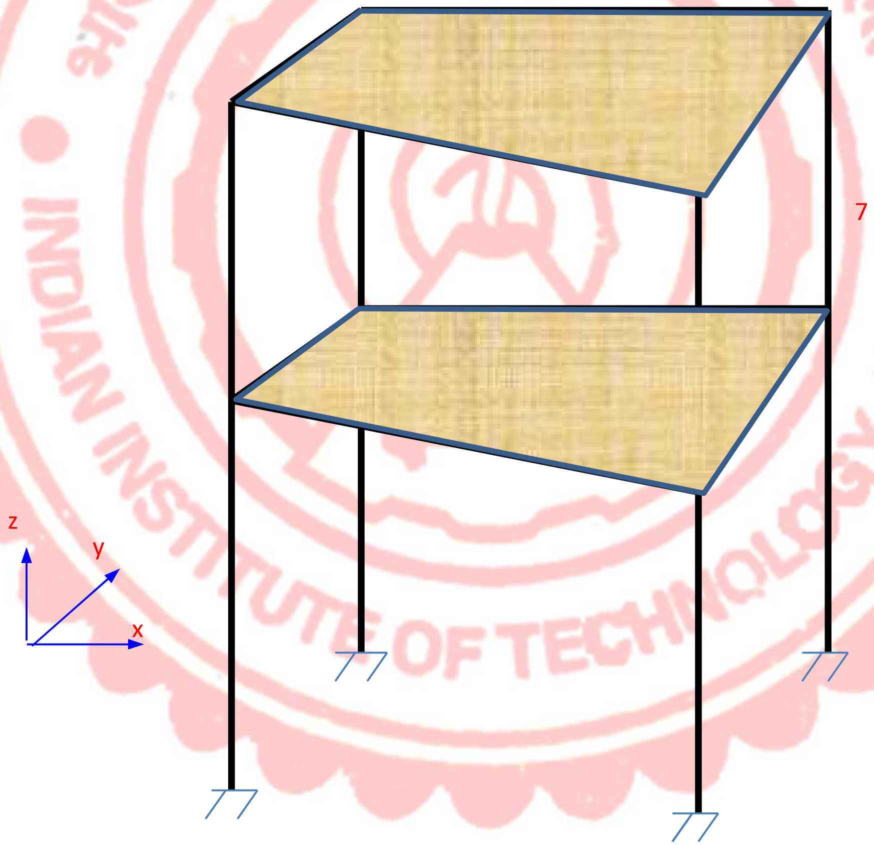
Convince yourself mathematically! Multiply a matrix with i^{th} col its inverse (begin with i = 1)

USE OF STANDARD SOFTWARE TO RIGOROUSLY CONSIDER SLAB ACTION



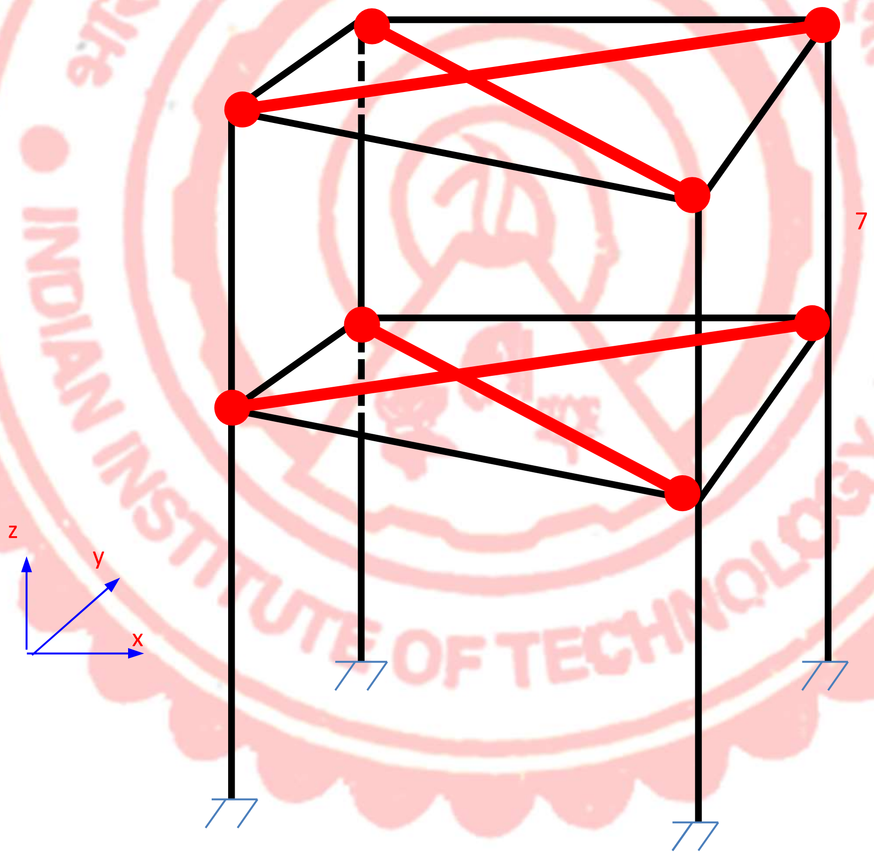
USE OF STANDARD SOFTWARE TO RIGOROUSLY CONSIDER SLAB ACTION

OPTION 1: INCLUDE PLATE ELEMENT IN ANALYSIS



USE OF STANDARD SOFTWARE TO RIGOROUSLY CONSIDER SLAB ACTION

OPTION 2: ADD LINK ELEMENTS AS HORIZONTAL BRACES IN ALL BAYS (ASSIGN HIGH STIFFNESS)



Link elements to have very high stiffness as compared to beams