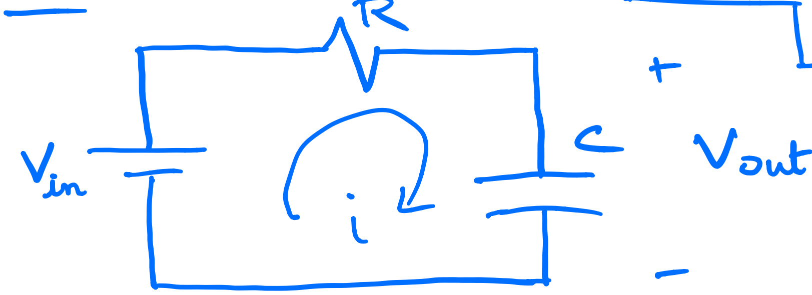
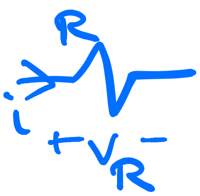
ELL333

Multivariable Control

31.07.2018

MATHEMATICAL MODELS



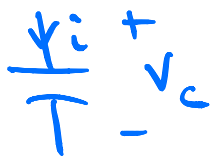


$V_R = iR$

Kirchoff's Voltage Law

$-V_{in} + V_R + V_c = 0$

$-V_{in} + iR + V_c = 0$



$i = c\dfrac{dV_c}{dt}$

$$\Rightarrow \quad -V_{in} + RC\frac{dV_c}{dt} + V_c = 0$$

$$\Rightarrow \quad RC\frac{dV_c}{dt} + V_c = V_{in} \quad , \quad V_{out} = V_c \quad (*)$$

Knowing $V_c(0)$ and $V_{in}(t)$, we can obtain how V_c changes as a function of time.

Obtaining state-space model...

define state $x \triangleq V_c$

input $u \triangleq V_{in}$

output $y \triangleq V_{out}$

$$\Rightarrow RC\frac{dx}{dt} + x = u,$$
$$y = x$$

?

$$\Rightarrow RC\frac{dx}{dt} = -x + u, \quad y = x$$

$$\Rightarrow \dot{x} \equiv \frac{dx}{dt} = \overbrace{\left(\frac{-1}{RC}\right)}^{A} x + \overbrace{\left(\frac{1}{RC}\right)}^{B} u,$$
$$y = \underbrace{(1)}_{C} x + D u$$

in this case '0'

Usual state space representation

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$x \in \mathbb{R}^{n\times 1}\ ([\ ]_{n\times 1})$, $u \in \mathbb{R}^{m\times 1}\ ([\ ]_{m\times 1})$, $y \in \mathbb{R}^{p\times 1}\ ([\ ]_{p\times 1})$

$\Rightarrow$ dimensions of $A \rightarrow n\times n$

$B \rightarrow n\times m$

$C \rightarrow p\times n$

$D \rightarrow p\times m$

$$\dot{x} = -\frac{1}{RC}x + \frac{1}{RC}u$$
$$y = x$$

Is this a linear system?

a) If $u_1 \to y_1$, $u_2 \to y_2 \Rightarrow u_1 + u_2 \to y_1 + y_2$

b) If $u \to y \Rightarrow \alpha u \to \alpha y$

$y = mu + c \to$ eqn. of line, not linear

Is it a time-invariant system?

If $u(t) \to y(t)$, then $u(t-\tau) \to y(t-\tau)$

Related word is 'autonomous'

loosely translates to parameters not being functions of time.

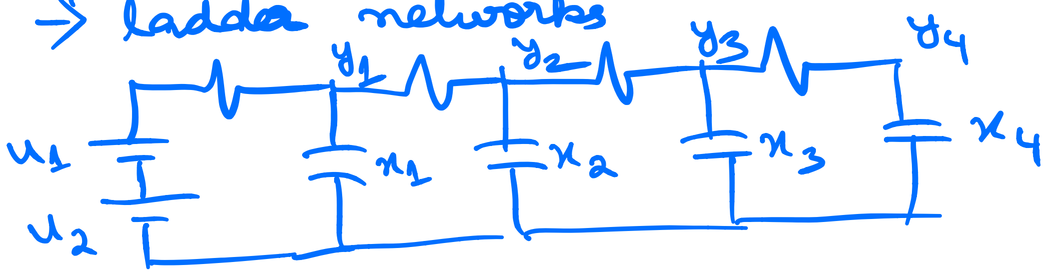
If $R = R(t)$, then non-autonomous

L T I $\to$ time-invariant.

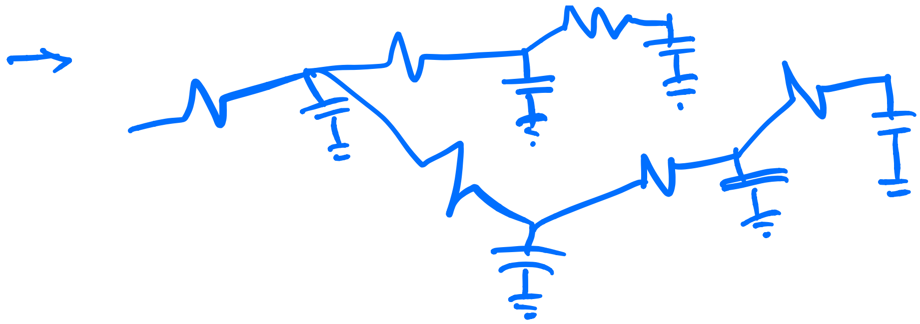
$\hookrightarrow$ linear

'multivariable' generalizations

$\to$ ladder networks



Often used as models of transmission lines.



'branched ladder'

also used as models of axons, where action potentials propagate.

↑ Example

Another example.