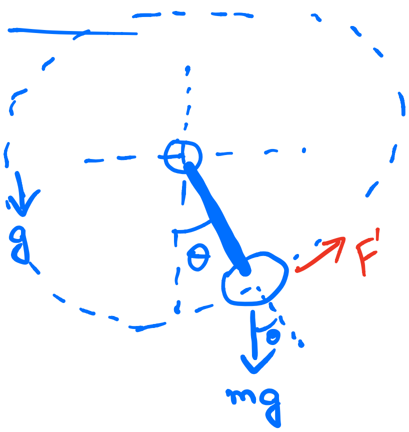
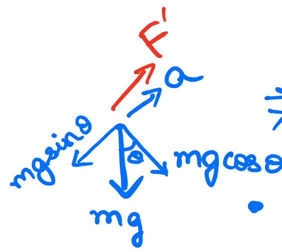
ELL 333

MULTIVARIABLE CONTROL

01.08.2018





- Tangential component gives rise to force for the motion

"F = ma"

$$F' \Rightarrow -mg\sin\theta = ma$$

$$a \rightarrow \frac{d^2x}{dt^2} \text{ or } \ddot{x}$$

- $l\theta = x$

$$\Rightarrow l\ddot{\theta} = \ddot{x}$$

$$\Rightarrow F' - mg\sin\theta = ml\ddot{\theta}$$

$$\Rightarrow l\ddot{\theta} = -g\sin\theta + F'/ml$$

$$\Rightarrow \ddot{\theta} = -\frac{g}{l}\sin\theta + F'/ml$$

$$\Rightarrow \ddot{\theta} + \frac{g}{l}\sin\theta = 0 + F'/ml$$

external force

Compare with $RC\frac{dV_c}{dt} + V_c = V_{in}$

What is the state-space representation of this?

Output $y \triangleq \theta$

Input $u \triangleq \frac{F'}{ml}$

States $x_1 = \theta$

$$x_2 = \dot{\theta}$$

$$\Rightarrow \quad \dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{g}{l}\sin x_1 + u$$

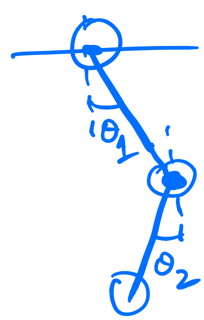
$$y = x_1$$

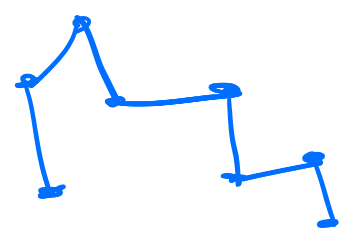
not as $A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ because of '$\sin x_1$' term

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{l}\sin x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Possible generalization of this





protein structures?

states: $\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2$

This system is nonlinear.

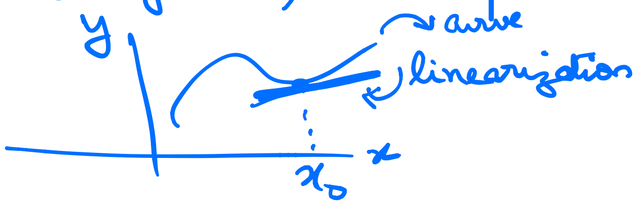
Check using $u_1 \to y_1$ Does $u_1 + u_2 \to y_1 + y_2$?

$u_2 \to y_2$

Such nonlinearities can lead to interesting behaviour, for example, chaos in double pendulum.

Linearize the model.

just approximating a curve with its tangent at a point.



Linearization is always around an operating point.

linearization around $x_1 = 0$?

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{l}\sin x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = x_1$$

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} x_2 \\ -\frac{g}{l}x_1 \end{bmatrix}}_{\begin{bmatrix} 0 & 1 \\ \cdots \end{bmatrix}\begin{bmatrix} x_1 \\ \cdots \end{bmatrix}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix} -g/l & 0 \end{bmatrix} \begin{bmatrix} x_2 \end{bmatrix}$$

$$y = x_1$$

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linearization around $x_1 = \pi$?

$$f(x) = f(x_0) + \frac{\partial f}{\partial x}\Big|_{x=x_0}(x - x_0) + \cdots$$

$$\sin x_1 = \underbrace{\cancel{\sin \pi}}_{=0} + \underbrace{\cos x_1}_{-1}\Big|_{x_1=\pi}(x_1 - \pi) + \cdots$$

$$\approx -x_1 + \pi$$