ELL333

MULTIVARIABLE CONTROL

LECTURE 08.08.2018

General Form of Linearization of $\dot{x} = f(x)$?

$$\Delta \dot{x} = \overbrace{A \cdot \Delta x}^{\text{linearizing around a point/trajectory where } f(x_0)=0} + f(x_0) \text{ if } x_0 \text{ is not an equilibrium point}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \Delta x = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

$$\dot{x}_1 = f_1(x_1, x_2 \ldots x_n)$$

$$\dot{x}_2 = f_2(x_1, x_2, \ldots x_n)$$

$$\vdots$$

$$\dot{x}_n = f_n(x_1, x_2, \ldots x_n)$$

$n=2$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$n=3$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}$$

general 'n' case

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{n \times n}$$

From 3-14 of Astrom & Murray

$$\frac{d}{dt}\begin{bmatrix} p \\ \theta \\ \dot{p} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{p} \\ \dot{\theta} \\ \dfrac{-ml s_\theta \dot{\theta}^2 + mg(ml^2/J_t) s_\theta c_\theta - c\dot{p} - \left(\frac{\gamma}{J_t}\right) ml c_\theta \dot{\theta} + u}{(\cdot)} \\ \dfrac{-ml^2 s_\theta c_\theta \dot{\theta}^2 + M_t g l s_\theta - c l c_\theta \dot{p} - \gamma(M_t/m)\dot{\theta} + l c_\theta u}{(\cdot)} \end{bmatrix}$$

eq. point. $= 0$?

$$\dot{p} = 0 = \dot{\theta}$$

$$-ml s_\theta \underbrace{\cancel{\dot{\theta}^2}}_{=0} + mg\left(\frac{ml^2}{J_t}\right) s_\theta c_\theta - c\underbrace{\cancel{\dot{p}}}_{=0} - \underbrace{\cancel{\left(\frac{\gamma}{J_t}\right) ml c_\theta \dot{\theta}}}_{=0} + u = 0$$

$$\Rightarrow \quad mg\left(\frac{ml^2}{J_t}\right) s_\theta c_\theta + u = 0 \quad -(1)$$

$$-ml^2 s_\theta \underbrace{\cancel{c_\theta \dot{\theta}^2}}_{=0} + M_t g l s_\theta - \underbrace{\cancel{c l c_\theta \dot{p}}}_{=0} - \underbrace{\cancel{\gamma\left(\frac{M_t}{m}\right)\dot{\theta}}}_{=0} + l c_\theta u = 0$$

$$\Rightarrow \quad M_t g l s_\theta + l c_\theta u = 0 \quad -(2)$$

Can try eliminating u from (1) & (2)

either $\cos\theta = \sqrt{\dfrac{M_t J_t}{m^2 l^2}}$ or $\sin\theta = 0$ $\rightarrow$ check

If 'u' (the input) is taken into account in calculating the 'equilibrium point', then a better term is 'operating point'.

In Astrom & Murray, where is the role of input 'u' in linearization discussed?