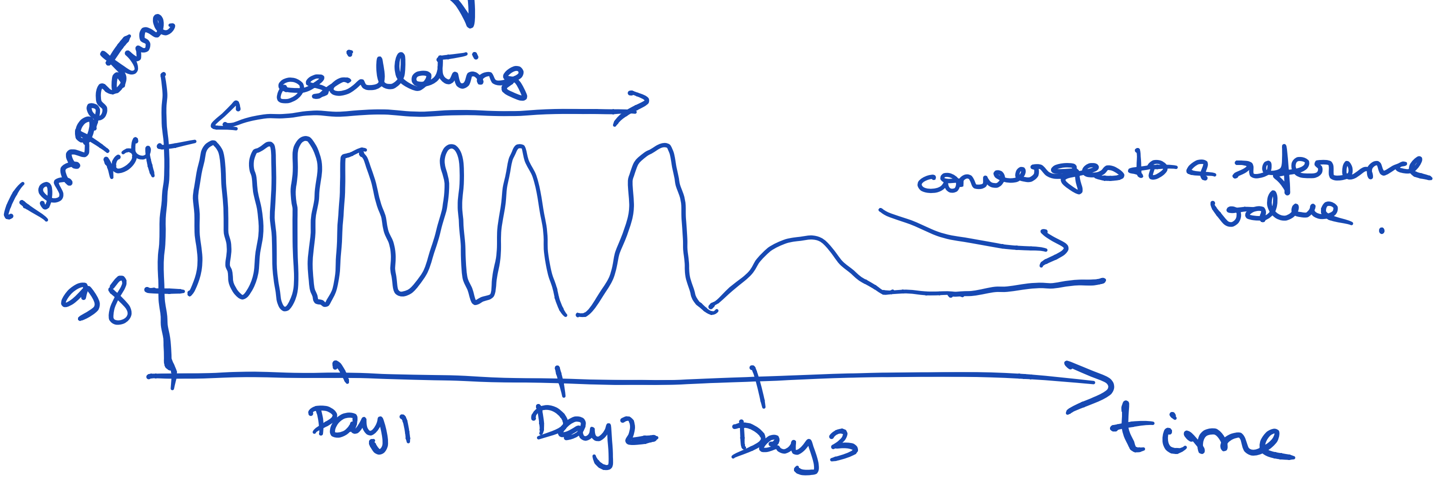
ELL333 04.09.2018

Multivariable Control

(restart ...)

We are looking at topic of MODELS, in particular subtopics of Dynamics and Stability



Measurable quantity (of interest) changes with time.

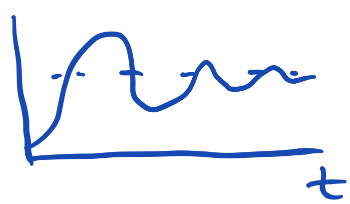
↳ What are different variations possible?

↳ Which ones can be denoted as 'stable'?

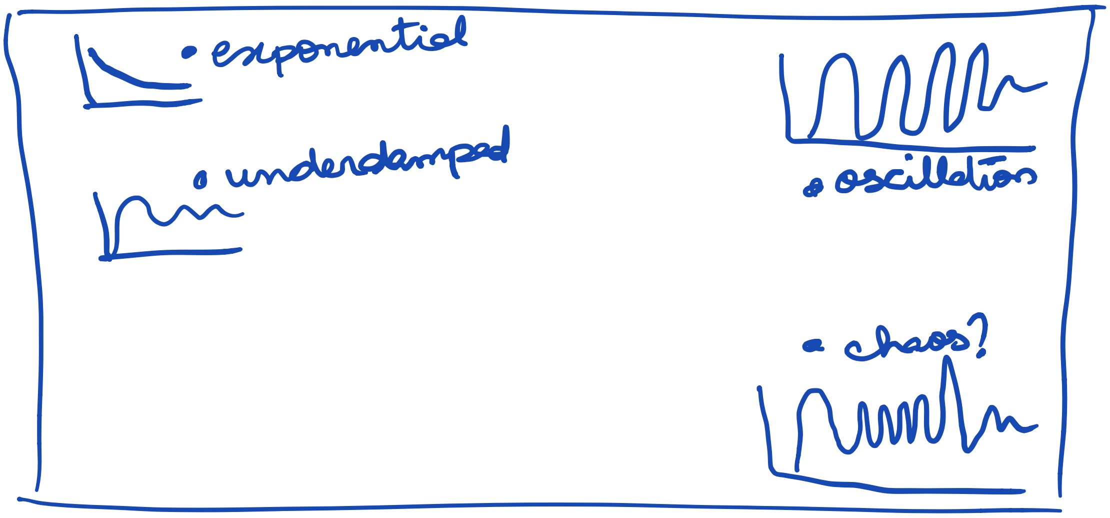
Other examples of (familiar) dynamic trajectories

- exponentials

- underdamped or overdamped



SET OF ALL POSSIBLE DYNAMIC RESPONSES



General System Model: $\dot{x} = f(x, u)$
$y = h(x, u)$

Linearization

General System Model: $\dot{x} = Ax + Bu$
$y = Cx + Du$

Goals:

We have introduced above models.

→ We want to study the kind of dynamics possible

→ We want to study 'stability' in these dynamics

Example 1. How to solve

this means studying about 'existence' and 'uniqueness' of solutions

$\dot{x} = -ax$? , $x \in \mathbb{R}$

$\dfrac{dx}{dt}$ → time

solution : $x(t) = c\, e^{-at}$

initial condition
$c = x(0)$

Here, solution exists (by inspection), and is unique if initial condition is specified.

Example 2: $\dfrac{dx}{dt} = x^2$, initial condition $x(0)$

$$\Rightarrow \int_{x(0)}^{x(t)} \frac{dx'}{x'^2} = \int_0^t dt'$$

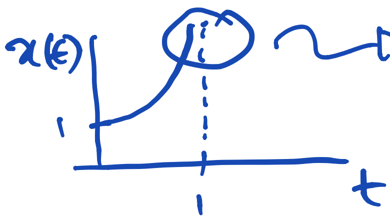
$$\Rightarrow -\frac{1}{x'}\Bigg|_{x(0)}^{x(t)} = t'\Big|_0^t$$

$$\Rightarrow -\frac{1}{x(t)} + \frac{1}{x(0)} = t$$

$$\Rightarrow \frac{1}{x(t)} = \frac{1}{x(0)} - t$$

or $\quad x(t) = \dfrac{1}{\frac{1}{x(0)} - t}$

if $x(0) = 1$,
$\Rightarrow x(t) = \dfrac{1}{1-t}$



for this time, the solution does not exist

Example 3: $\dfrac{dx}{dt} = \sqrt{x}$, initial condition $x(0)$

$$\Rightarrow \int_{x(0)}^{x(t)} x'^{-1/2}\, dx' = \int_0^t dt'$$

$$\Rightarrow \frac{x'^{1/2}}{1/2}\Bigg|_{x(0)}^{x(t)} = t$$

$$\Rightarrow 2\sqrt{x(t)} - 2\sqrt{x(0)} = t$$

$$\Rightarrow \sqrt{x(t)} = \frac{t}{2} + \sqrt{x(0)}$$

$$\Rightarrow x(t) = \left(\frac{t}{2} + \sqrt{x(0)}\right)^2$$

issues

- for $x(0) < 0$, solution is hard to define
- Uniqueness?