ELL333

MULTIVARIABLE CONTROL

26.09.2018

Stability in Linear Systems $\dot{x} = Ax$? ↳ $n \times n$

of equilibrium point $x = 0$.

↳ Solution is $x(t) = e^{At}\, x(0)$ ← initial condition

find T

$$AT = TJ \Rightarrow e^{At} = T e^{Jt} T^{-1}$$

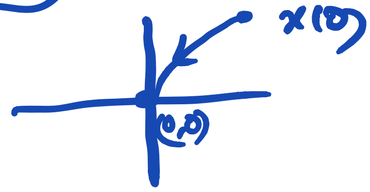
general form of e^{Jt}

has a block diagonal structure

$$\begin{bmatrix} e^{\lambda_1 t} & & & \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ & & & e^{\lambda_n t} \end{bmatrix} \qquad \begin{bmatrix} e^{\lambda_1 t} & & \\ & \begin{matrix} e^{\lambda_2 t} & t e^{\lambda_2 t} \\ 0 & e^{\lambda_2 t} \end{matrix} & \\ & & e^{\lambda_3 t} \end{bmatrix}$$

if $J = \mathrm{diag}(\lambda_1, \lambda_2, \ldots \lambda_n)$

if $J = \begin{bmatrix} \lambda_1 & & \\ & \begin{matrix} \lambda_2 & 1 \\ 0 & \lambda_2 \end{matrix} & \\ & & \lambda_3 \end{bmatrix}$ for $(n=3)$



does this follow?

If real part of all eigenvalues is negative, then system is asymptotically stable.

What if

- one of the eigenvalues is zero?

because $e^{\lambda t} = 1$ if $\lambda = 0$ so it adds constant to the solution.

☑ stable ☐ asymptotically stable

☐ something else

- two or more of the eigenvalues is zero?

→ unstable because of terms such as t, t^2 that may arise. These diverge as $t \to \infty$ away from $x = 0$.

- any one of the eigenvalues has positive real part?

→ unstable because $e^{\lambda t}$ blows up if real part of λ is positive