ELL333

MULTIVARIABLE CONTROL

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Stability in nonlinear systems $\dot{x} = f(x)$.
of equilibrium point $x = x^*$, $f(x^*) = 0$

Example.


$\theta = 0$

Standing
Bicycle
upright position

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"FACT" : If the linearization of the nonlinear system around the equilibrium point has **NO** eigenvalues with zero real part, then

whatever conclusion of stability is there for the linear system also holds for the full nonlinear system.

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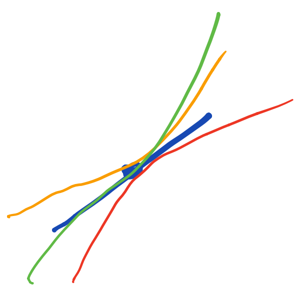
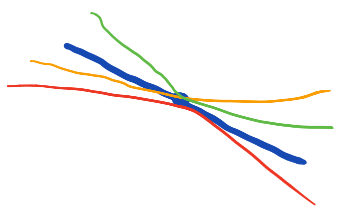
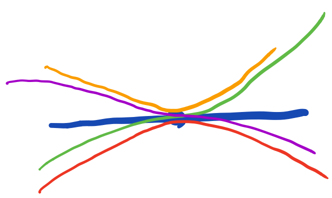






# Examples

(a) $\dot{x} = -x$

(b) $\dot{x} = -x^2$

(c) $\dot{x} = -x^3$

(d) $\dot{x} = -x - x^3$ (HW)

Determine the stability of equilibrium point $x=0$ in these systems. Can this be done by looking at their linearizations?

(a) $\dot{x} = -x$

eigenvalues = '$-1$'

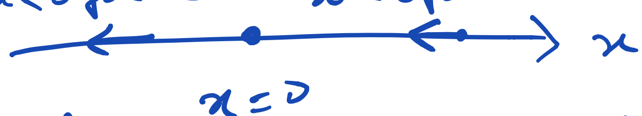
$\Rightarrow$ stable

(b) $\dot{x} = -x^2$

linearized system has Jacobian $= -2x\big|_{x=0} = 0$

$\Rightarrow$ linearly stable (Lyapunov sense / neutrally)

$\dot{x} < 0$ for $x < 0$ $\quad$ $\dot{x} < 0$ for $x > 0$




$\Rightarrow$ nonlinear system unstable at $x=0$

(c) $\dot{x} = -x^3$

Jacobian of linearized system $= -3x^2\big|_{x=0} = 0$

$\Rightarrow$ linearly stable

$\dot{x} > 0$ for $x < 0$ $\quad$ $\dot{x} < 0$ for $x > 0$


$\Rightarrow$ nonlinear system is stable at $x=0$ (asymptotically)

"Lyapunov Functions" $\longrightarrow V$

$\hookrightarrow$ generalized energy

For above examples, compute

(a) $\dot{x} = -x$, $V(x) = \frac{1}{2}x^2 > 0$ if $x \neq 0$, $V(0) = 0$

RC circuit equation,
RC=1

$$\frac{\partial V}{\partial t} = \dot{V} = \frac{\partial V}{\partial x} \cdot \frac{dx}{dt}$$

$$= \frac{\partial V}{\partial x} \cdot \dot{x}$$

$$= x \quad (-x)$$

$$= -x^2 < 0 \text{ if } x \neq 0$$

$x = 0$ is equilibrium point

$V$ is positive and $\dot{V}$ is negative

$\Rightarrow V \longrightarrow 0$ as $t \to \infty$

$\Rightarrow x \to 0$ as $t \to \infty$

(c) $\dot{x} = -x^3$, $V(x) = \frac{1}{4}x^4 > 0$ if $x \neq 0$, $V(0) = 0$

$\dot{V} = -x^6 < 0$