ELL333

MULTIVARIABLE CONTROL

01.10.2018

Lyapunov's Stability Theorem.

Consider nonlinear system $\dot{x} = f(x)$ with $x = x^*$ as an equilibrium point.

- Suppose a function $V(x)$ can be found such that $V(x^*) = 0$ and $V(x) > 0$ in a neighbourhood of $x = x^*$.
- And suppose that $\dot{V} = \frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt}$ $\to \dot{x}$ is such that $\dot{V} \leq 0$ in a neighbourhood of $x = x^*$

proof

→ Then $x = x^*$ is stable.

→ Further if $\dot{V} < 0$, then $x = x^*$ is asymptotically stable.

* Function 'V' is called the Lyapunov Function

↳ hard to find

↳ there can be many

Example: $\dot{x} = -x^3$,

$x = 0$ is equilibrium point

Lyapunov Function candidate

$V(x) = \frac{1}{4}x^4$, $V(0) = 0$, $V(x) > 0, x \neq 0$

$$\dot{V} = \frac{\partial V}{\partial x} \cdot \dot{x}$$

$$= x^3(-x^3)$$

$$= -x^6 < 0, \quad x \neq 0$$

$x = 0$ is asymptotically stable [From Lyapunov's Stability theorem

Example: $\dot{x} = -\sin x$, $x=0$ is eq. pt.

your point : $V(x) = x^2/2$

$$\dot{V} = x . \dot{x} = -x \sin x$$

$\dot{V} < 0$ if $x \in (-\epsilon, +\epsilon)$

$\Rightarrow x=0$ is asymptotically stable.

$\dot{x} = f(x)$, x^* is eq. pt.

$$V(x) = \frac{1}{2}(x - x^*)^2$$

$$\dot{V} = (x - x^*) . \dot{x}$$

$$= (x - x^*) f(x)$$

Stability as a performance specification

↳ define stability for $\dot{x} = f(x)$

↳ Tests for stability

- linear systems – calculate eigenvalues, as they determine behaviour of the solution
- nonlinear systems
  - → linearize (only works if real part of eigenvalues are non-zero)
  - → Lyapunov Function based

Minor Test-2, Saturday 06.10.2018
9:30–10:30, LH 318